NNLO Contributions for ϵ_K and rare Kaon decays

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based on Collaborations with
A. Buras, J. Brod, U. Haisch, M. Stamou, U. Nierste

Motivation & Content

- Rare Kaon decays and ϵ_K can provide for a precision test of the CKM picture of ~~CP~~ in $s \to d$ transition

$K^+ \to \pi^+ \bar{\nu} \nu$

$K_L \to \pi^0 \bar{\nu} \nu$

ρ

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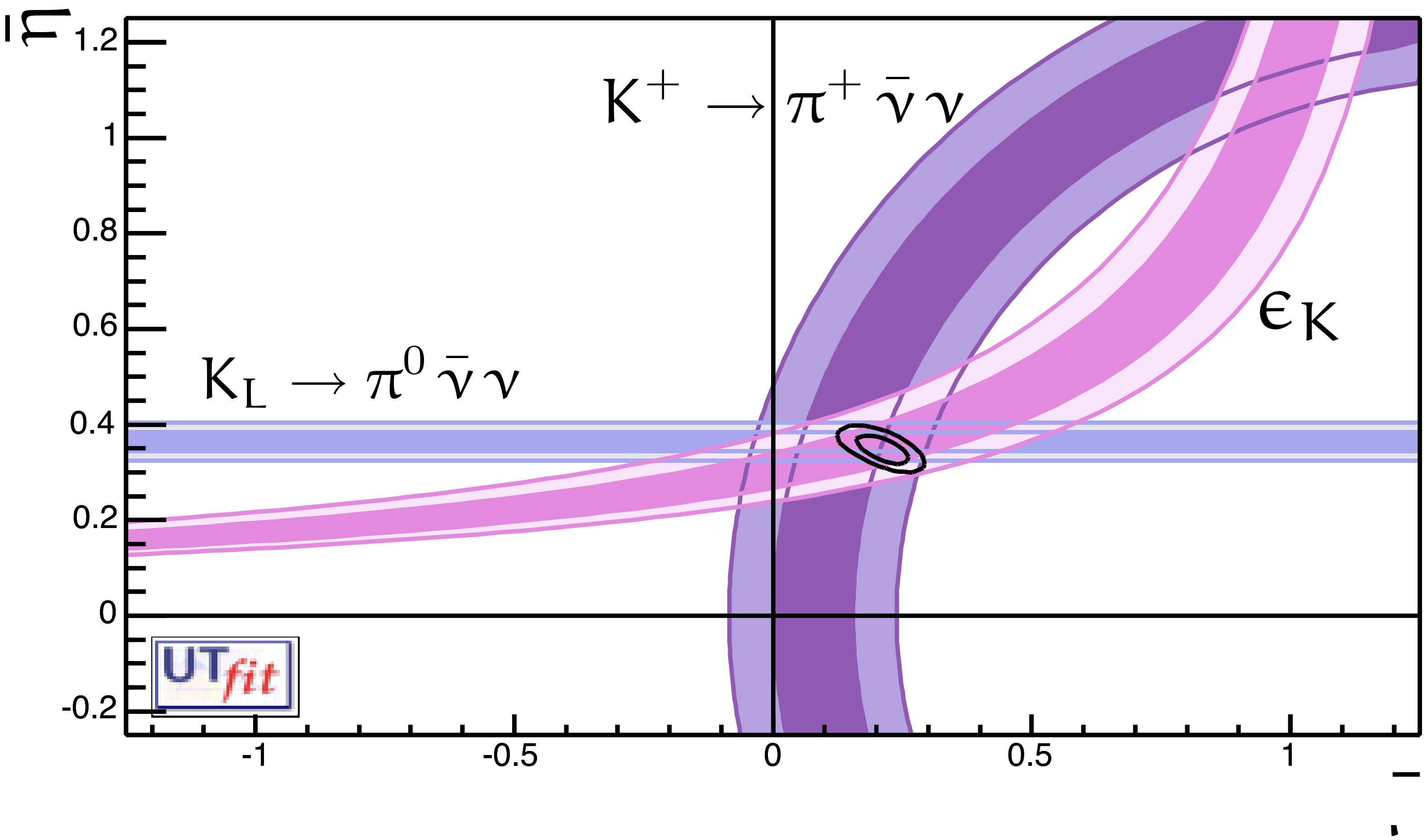
Future scenario [Buras, MG, Haisch, Nierste'06]



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- Rare Kaon decays and ϵ_K can provide for a precision test of the CKM picture of ~~CP~~ in $s \to d$ transition
- in $s \to d$: $|V_{ts}^* V_{td}| \propto \lambda^5$ Cabibbo suppression enhances sensitivity to generic new physics

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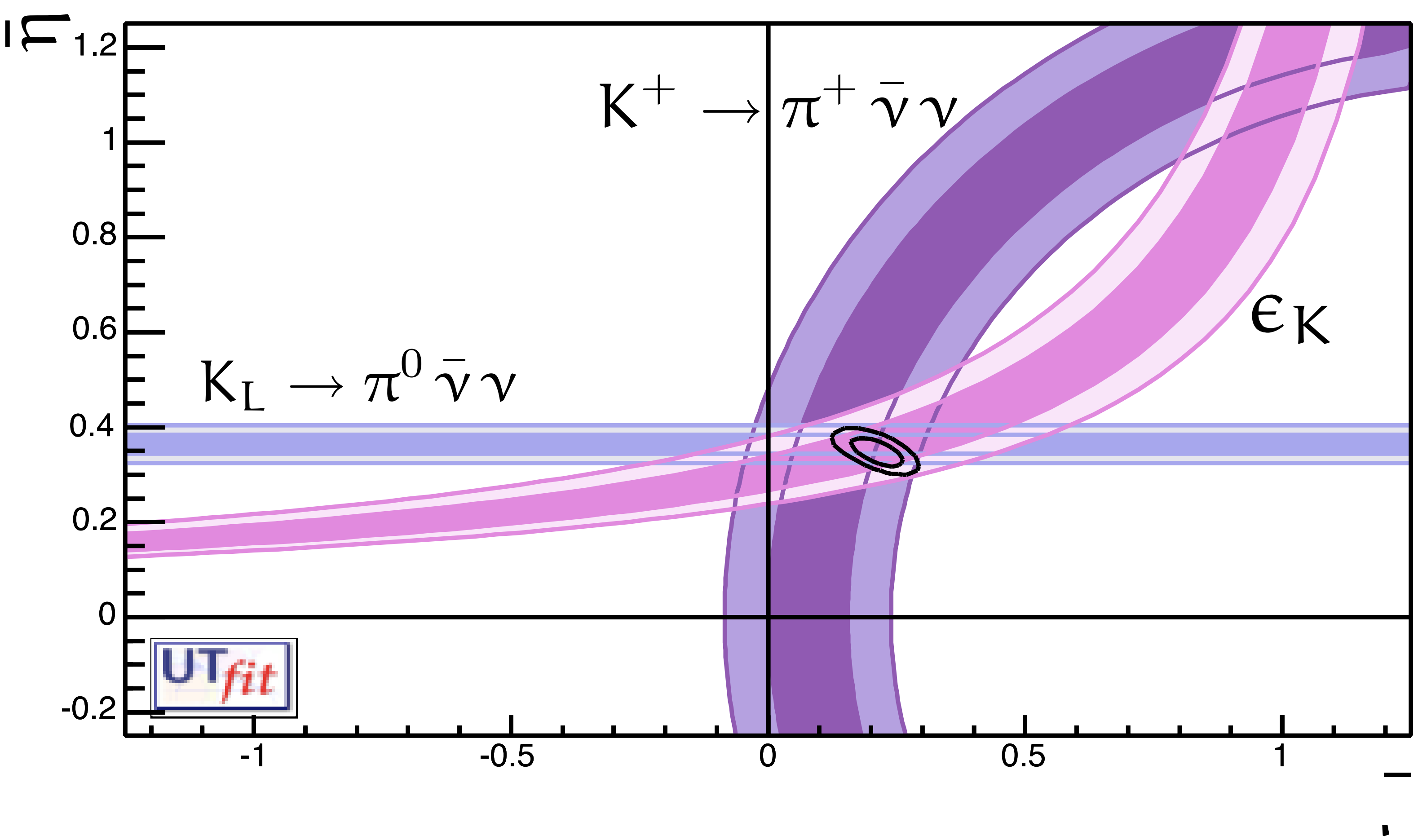

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- Rare Kaon decays and ϵ_K can provide for a precision test of the CKM picture of ~~CP~~ in $s \to d$ transition
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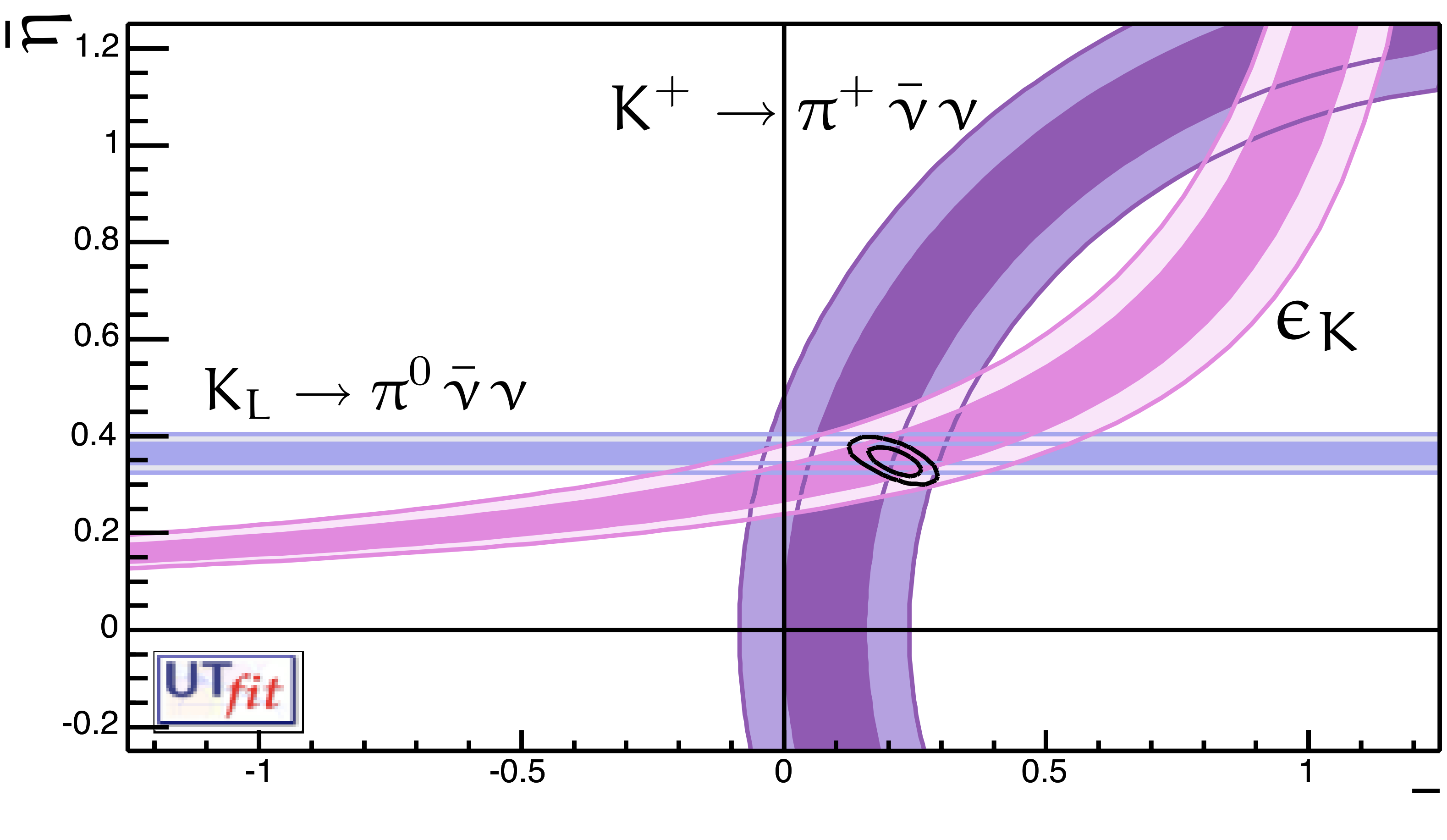
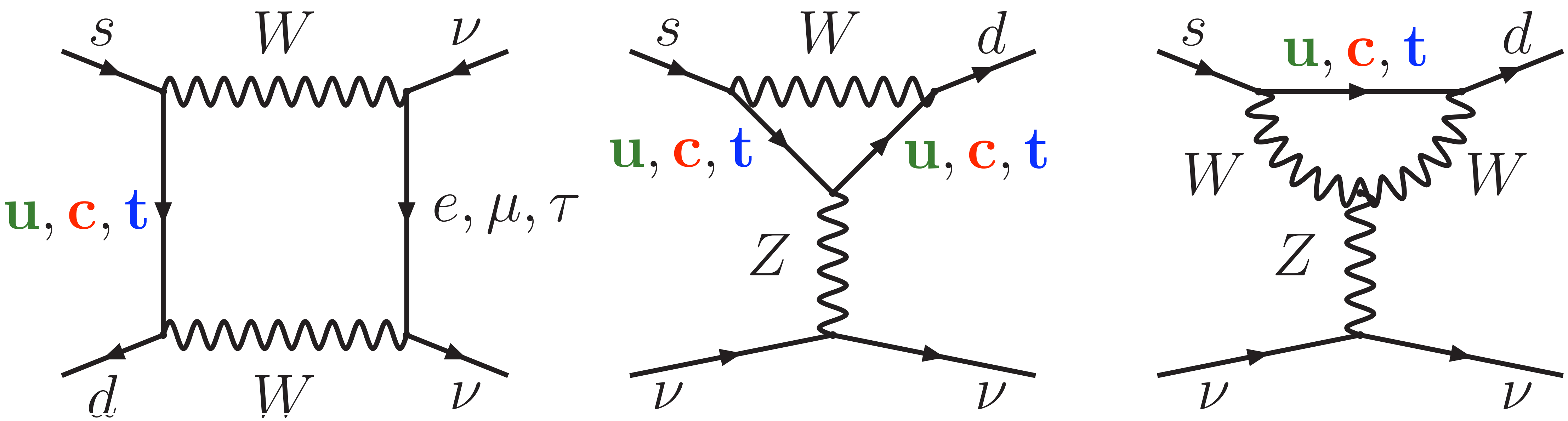


This talk:

- Perturbative contributions to $K \to \pi\nu\bar{\nu}$ & ϵ_K
- Will not discuss $K_L \to \pi^0 l^+ l^- \ldots$

Introduction: $K \to \pi \nu \bar{\nu}$



- **Dominant Operator:** $Q_\nu = (\bar{s}_L \gamma_\mu d_L)(\bar{\nu}_L \gamma^\mu \nu_L)$

$$\sum_i V_{is}^* V_{id} F(x_i) = V_{ts}^* V_{td}(F(x_t) - F(x_u)) + V_{cs}^* V_{cd}(F(x_c) - F(x_u))$$

Quadratic GIM: $\lambda^5 \frac{m_t^2}{M_W^2}$ $\quad \lambda \frac{m_c^2}{M_W^2} \ln \frac{M_W}{m_c}$ $\quad \lambda \frac{\Lambda^2}{M_W^2}$

Use isospin symmetry and normalise to: $K^+ \to \pi^0 e^+ \nu$

Rare K decays and New Physics:

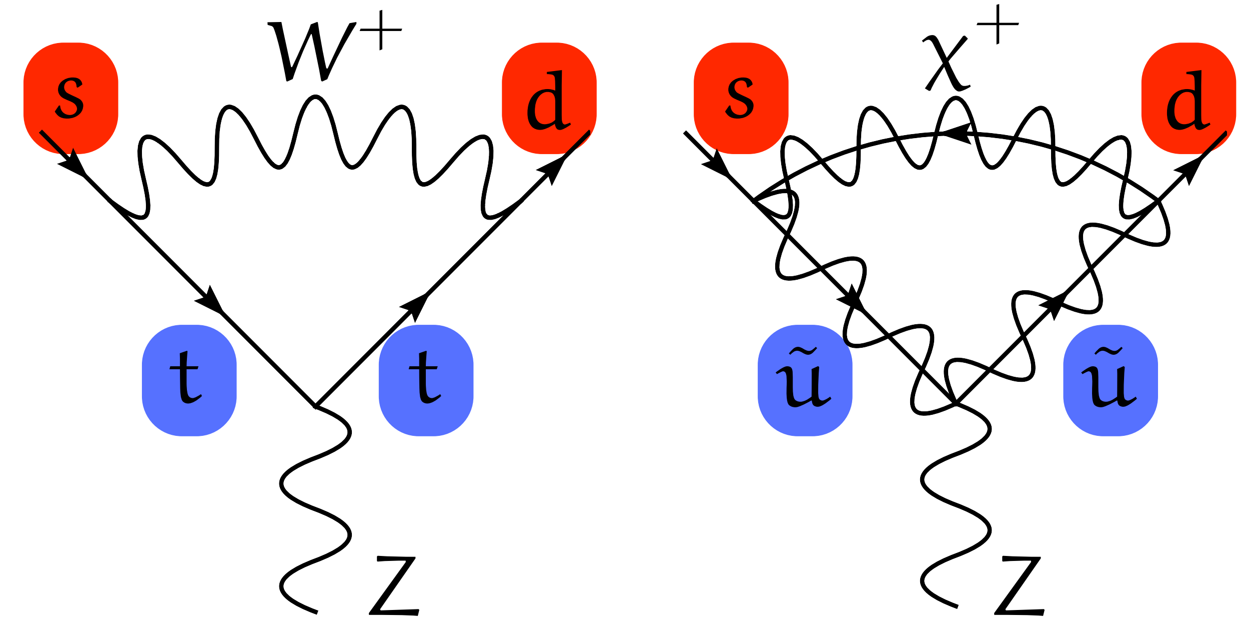
$$\mathcal{L}_{\text{eff}} = \frac{C(s \to d)}{\Lambda_{NP}^2} (\bar{s}\Gamma d)(\bar{\nu}\Gamma\nu)$$

Low NP scale $\Lambda_{NP} \simeq 1$ TeV

NP Flavour Sector $C(s \to d) < \lambda^5$

For Generic NP $C(s \to d) \simeq 1$

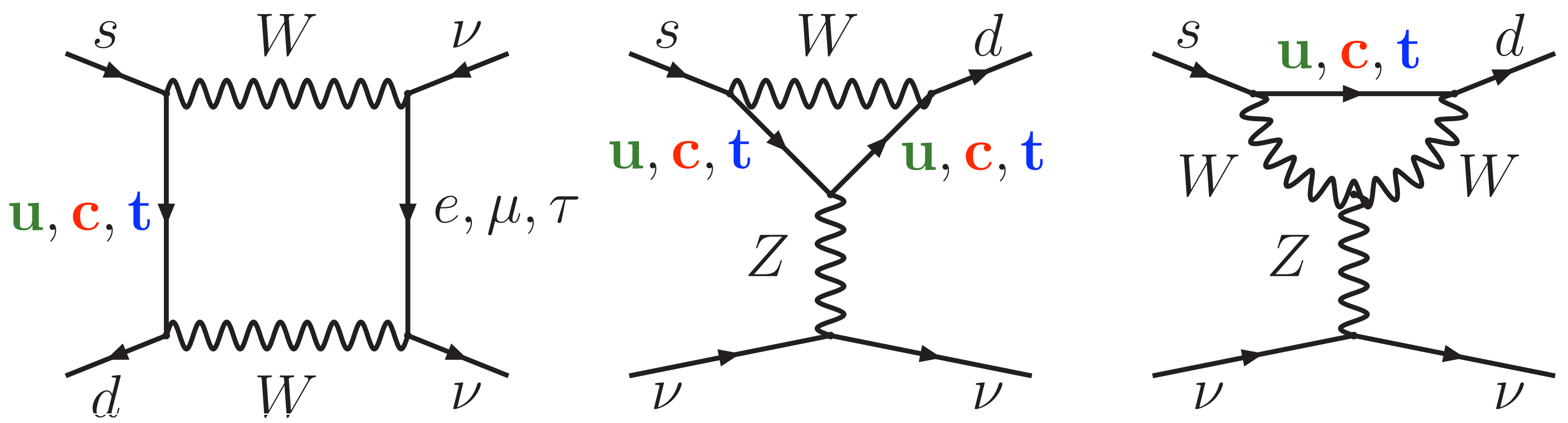
New Physics scale $\Lambda_{NP} > 50$ TeV



- Test deviation from flavour alignment
- Precise theory prediction
- Sensitive to small deviations from MFV

$$K_L \to \pi^0 \bar{\nu}\nu \qquad K^+ \to \pi^+ \bar{\nu}\nu$$

$K_L \to \pi^0 \bar{\nu}\nu$: Effective Hamiltonian



CP violating: DCPV : ICPV : CPC = 1 : 10^{-2} : < 10^{-4}

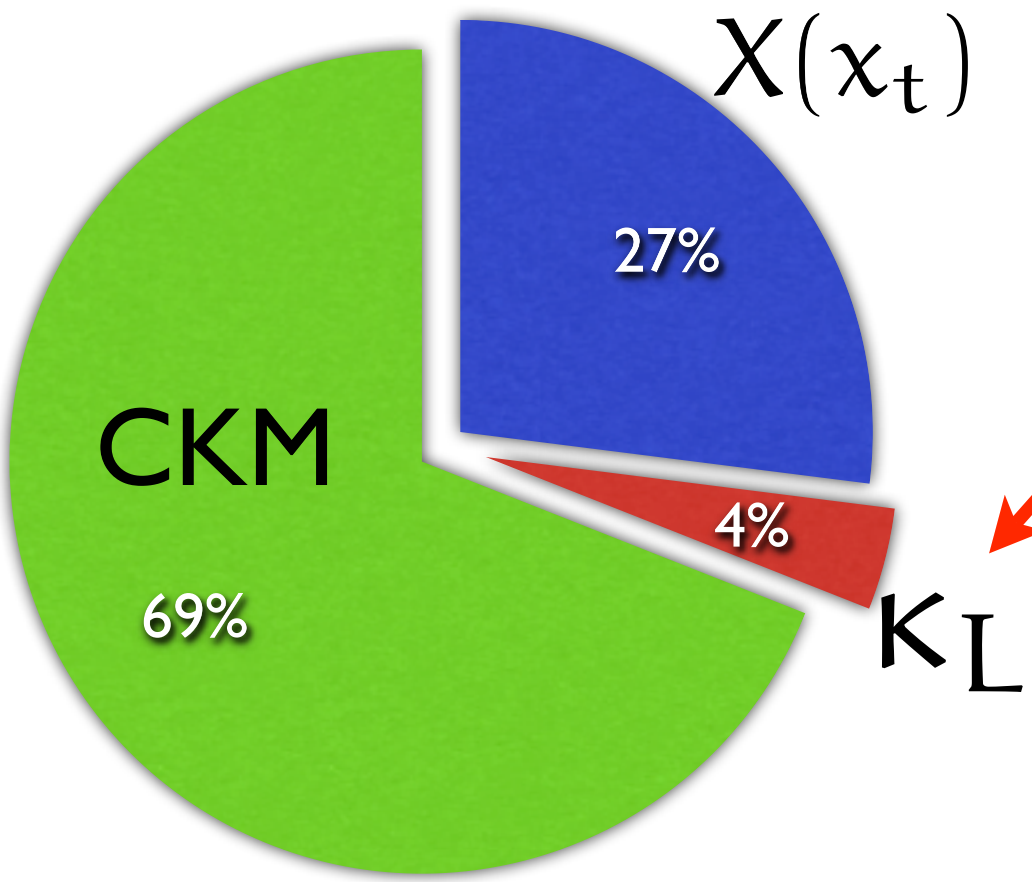
[Buchalla, Isidori ´98]

Only top quark contributes: $H_{\mathrm{eff}} = \frac{4 G_F}{\sqrt{2}} \frac{\alpha V_{ts}^* V_{td}}{2\pi \sin^2 \Theta_W} \boxed{X(x_t)} Q_\nu$

Use isospin symmetry and normalise to: $K^+ \to \pi^0 e^+ \nu$

$$\mathcal{B}r(K_L \to \pi^0 \bar{\nu}\nu) = \kappa_L \left(\frac{\mathrm{Im}(V_{ts}^* V_{td})}{\lambda^5} \boxed{X(x_t)} \right)^2$$

$K_L \to \pi^0 \bar{\nu} \nu$: Theoretical Status



Matrix element extracted from K_{l3} decays. $N^{\frac{3}{2}}LO\ \chi PT$

[Mescia, Smith ´07; Bijnens, Ghorbani ´07]

No further long distance uncertainty

$$\mathcal{B}r_{K_L} = (2.6 \pm 0.4) \times 10^{-11}$$

$X(x_t)$: NLO QCD calculation: ±1% error

[Misiak, Urban ´99; Buchalla, Buras ´99]

$X(x_t)$: Electroweak (EW) corrections: ±2% error

[Buchalla, Buras ´99]

Reduce error with 2 loop electroweak calculation

$X(x_t)$: Electroweak Corrections

- $X(x_t)$: Dominant theoretical uncertainty for $K_L \to \pi^0 \bar{\nu} \nu$
- For example a change $\sin^{OS} \theta_W \leftrightarrow \sin^{\overline{MS}} \theta_W$ results in 5% uncertainty

$$H_{eff} = \frac{4G_F}{\sqrt{2}} \frac{\alpha \, V_{ts}^* V_{td}}{2\pi \sin^2 \Theta_W} X(x_t) Q_\nu$$

- Uncertainty estimated in the large m_t limit $\sim 2\%$
  [Buchalla, Buras '99]
- Dominant uncertainty: Do the calculation!
  [Brod, MG, Stamou]

$X(x_t)$: Electroweak Corrections

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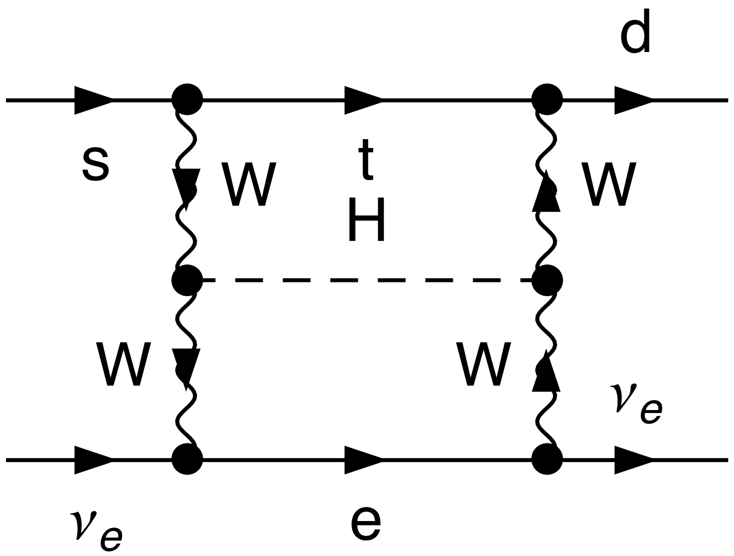
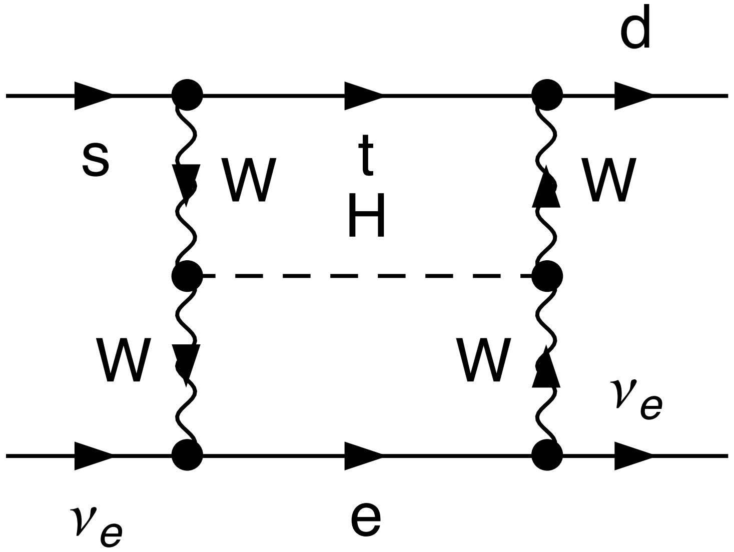
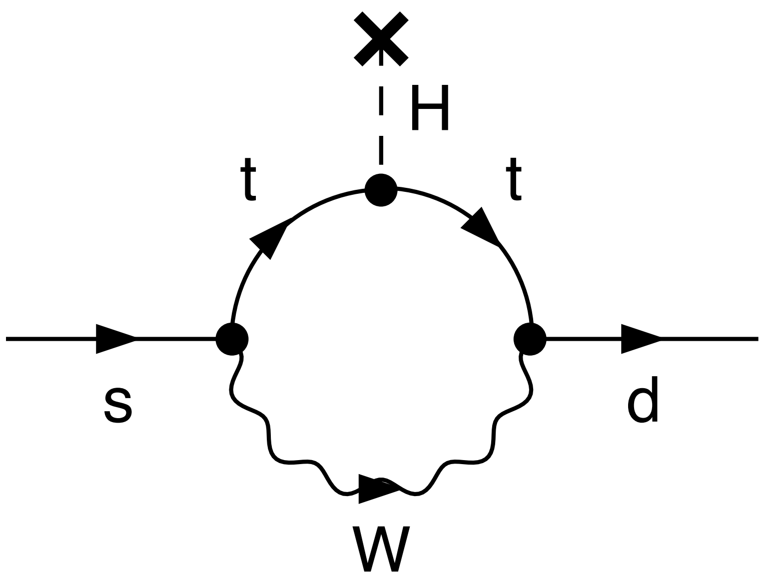
- Use the $\overline{\text{MS}}$ scheme



$X(x_t)$: Electroweak Corrections

- Use the $\overline{\text{MS}}$ scheme
- VEV minimises renormalised potential: include tadpoles





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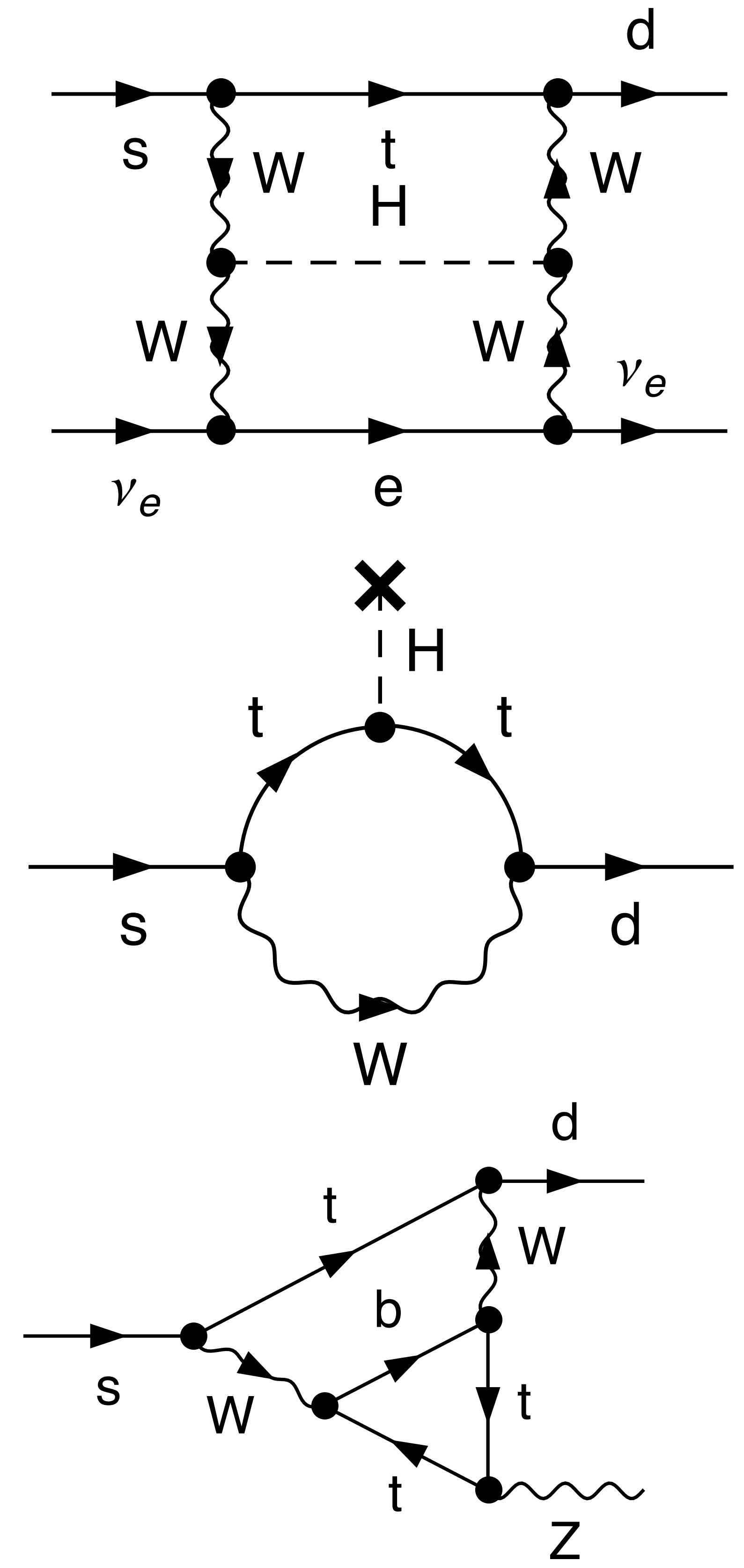
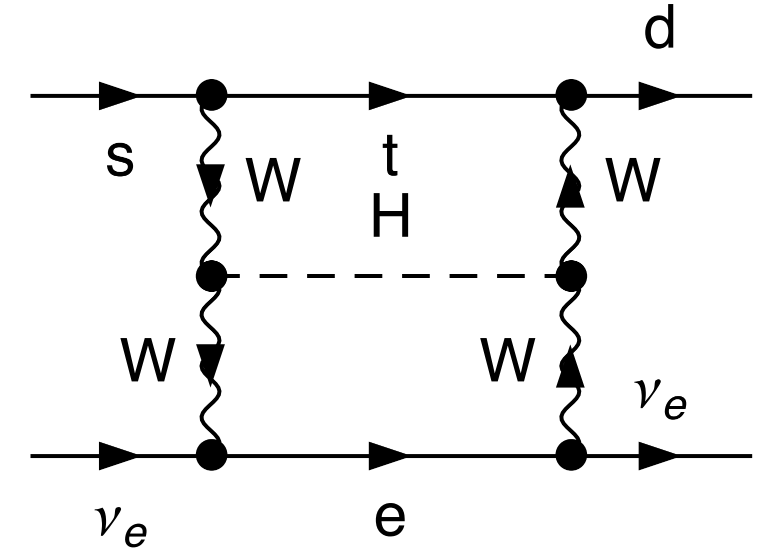
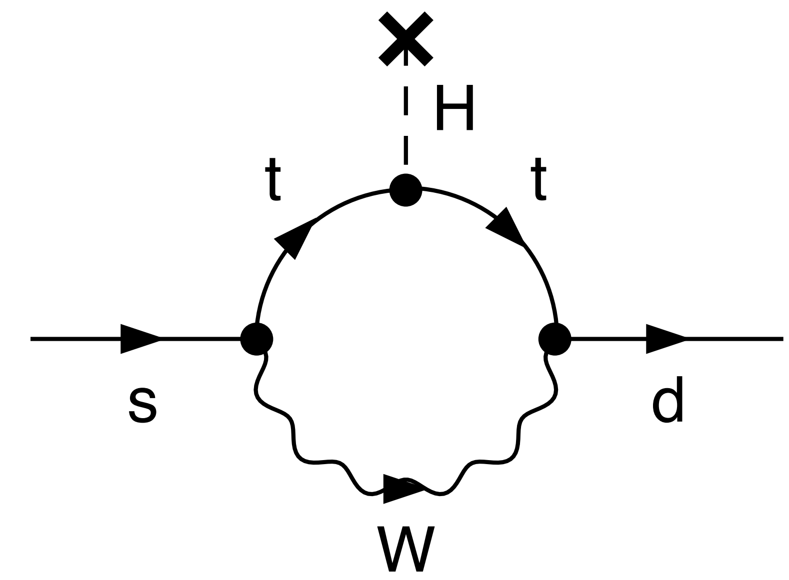
- Traces with γ_5: use HV scheme

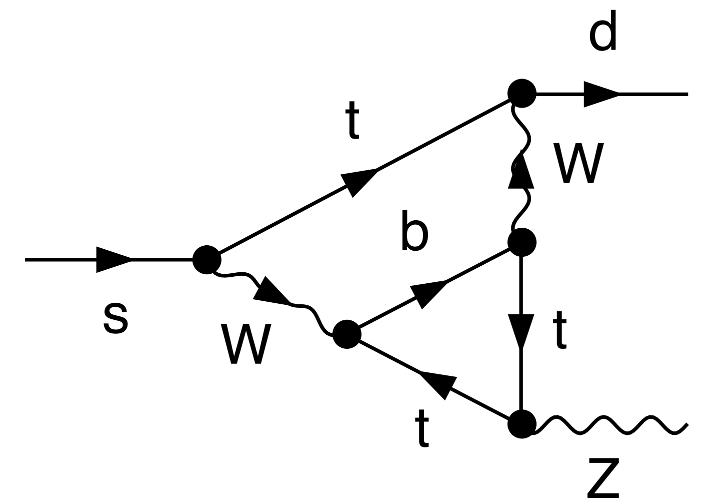


$X(x_t)$: Electroweak Corrections

- Use the $\overline{\text{MS}}$ scheme
- VEV minimises renormalised potential: include tadpoles
- Traces with γ_5: use HV scheme
- Preliminary numerics (without proper definition of G_F): Give small effects







$K^+ \to \pi^+ \bar{\nu}\nu$ and $K_L \to \pi^0 \bar{\nu}\nu$

Different from $K_L \to \pi^0 \bar{\nu}\nu$

- CP conserving: Top & charm contribute

$$\mathcal{B}r\left(K^+ \to \pi^+\nu\bar{\nu}(\gamma)\right) = \kappa_+(1+\Delta_{EM})$$

$$\times \left| \frac{V^*_{ts}V_{td}X_t(m_t^2) + \lambda^4 \mathrm{Re} V^*_{cs}V_{cd}\left(P_c(m_c^2) + \delta P_{c,u}\right)}{\lambda^5} \right|^2 .$$

$\frac{m_c^2}{M_W^2}$ suppression lifted by $\log(\frac{m_c}{M_W})\frac{1}{\lambda^4}$

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$\frac{m_c^2}{M_W^2}$ suppression lifted by $\log(\frac{m_c}{M_W})\frac{1}{\lambda^4}$

Like in $K_L \to \pi^0 \bar{\nu}\nu$

- Only Q_ν: Quadratic GIM & Isospin symmetry
- Top quark contribution like in $K_L \to \pi^0 \bar{\nu}\nu$

$K^+ \to \pi^+ \bar{\nu}\nu$ Long distance

- Matrix element extracted from K_{l3} decays
  [Mescia, Smith ´07]

- $K^+ \to \pi^+ \nu\bar{\nu}$ is $K^+ \to \pi^+ \nu\bar{\nu}(\gamma)$
  QED radiative corrections included:

$$\Delta_{EM}(E_\gamma < 20\text{MeV}) = -0.003$$

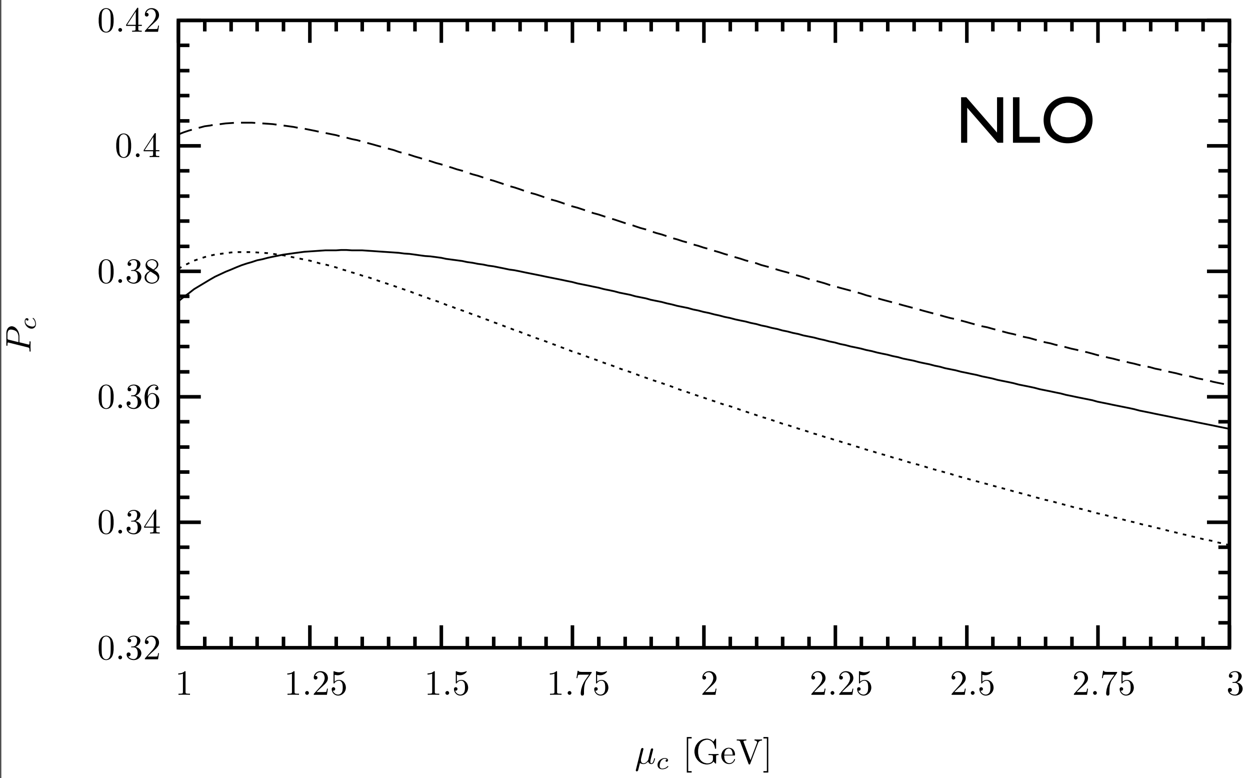
- Uncertainty in $\kappa_+(1-\Delta_{EM})$ reduced by $\frac{1}{7}$

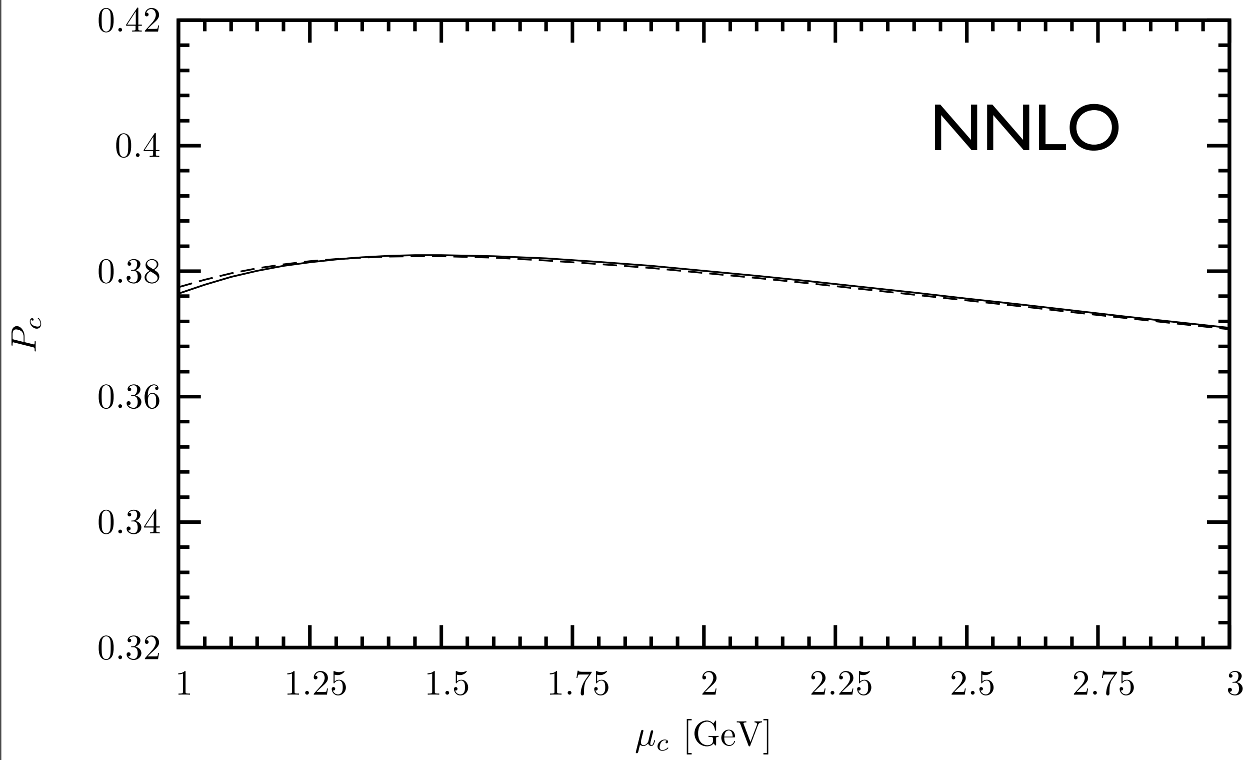
- Below charm scale: Dimension 8 operators
  [Falk et. al. ´01]

- Together with light quarks: $\delta P_{c,u} = 0.04 \pm 0.02$
  [Isidori, Mescia, Smith ´05]

- Could be Improved by Lattice [Isidori et. al. ´05]

$K^+ \to \pi^+ \bar{\nu}\nu$ charm contr. (QCD)



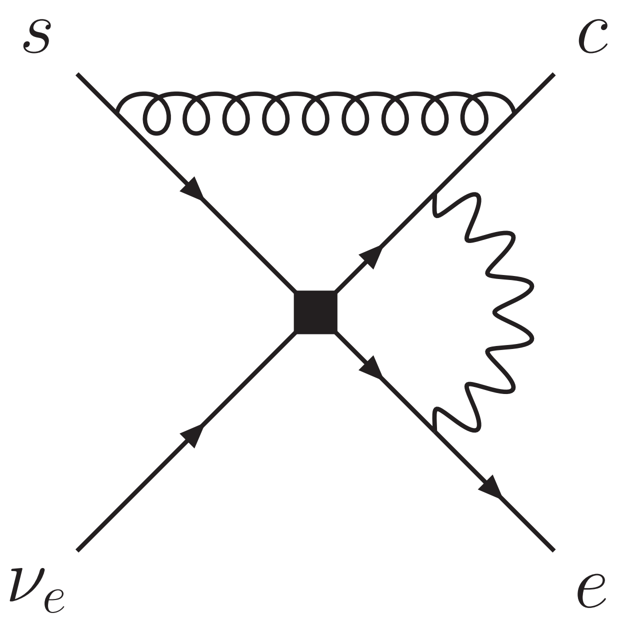


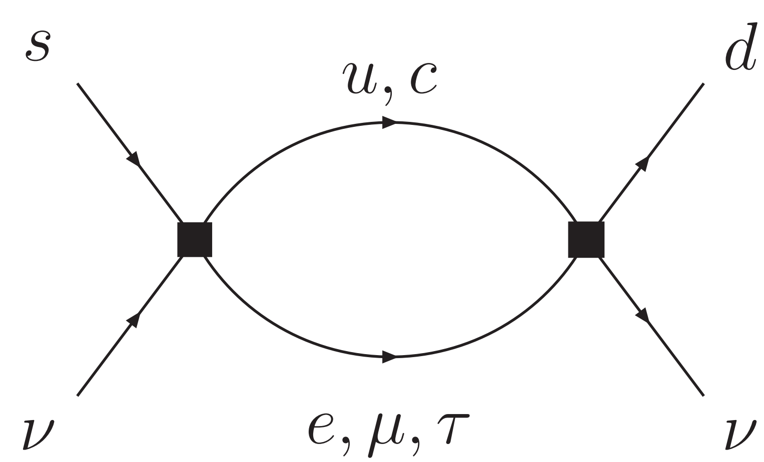
- Resum $\log \frac{m_c}{M_W}$ in P_c

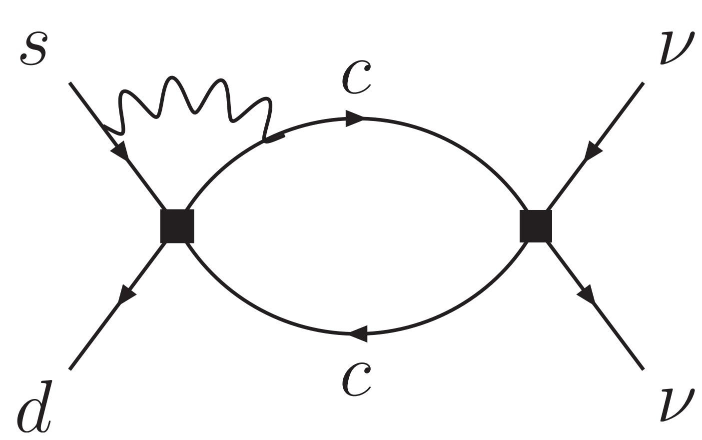
P_c at NLO: ±10% (theory)
[Buras, MG, Haisch, Nierste ´06]

P_c at NNLO: ±2.5% (theory)
[Buras, MG, Haisch, Nierste ´06]

$K^+ \to \pi^+ \bar{\nu}\nu$ charm contr. (EW)

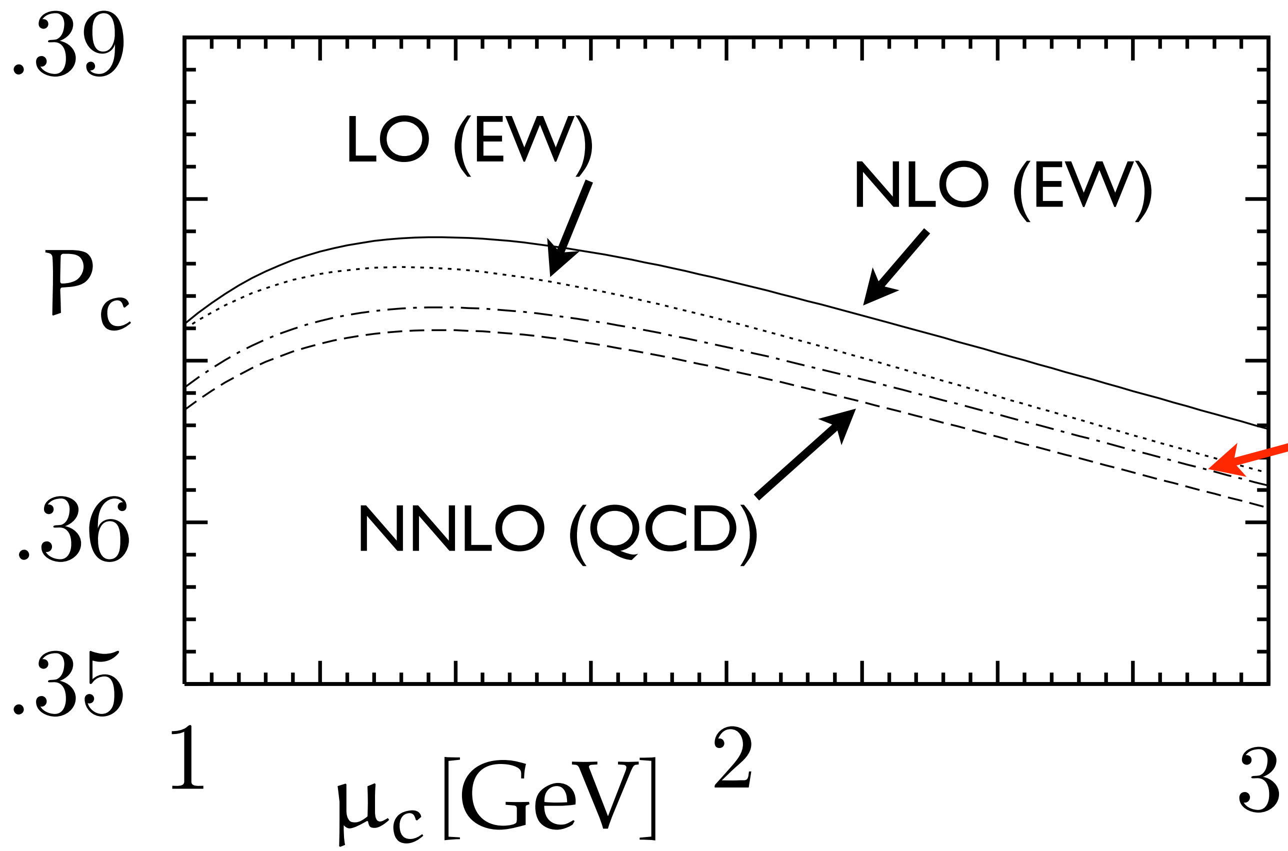






- Large QED logs? Does Q_ν run?
- Semileptonic operator has QED running and mixes into Q_ν.
- No $\mathcal{O}(\alpha/\alpha_s)$ but $\mathcal{O}(\alpha)$ corrections: NLO QEDxQCD calculation
- Bilocal mixing is $\mathcal{O}(G_F^2)$
- What is the parameter $x_c = \frac{m_c^2}{M_W^2}$
- EW corrections define M_W

$K^+ \to \pi^+ \bar{\nu}\nu$ charm contr. (EW)



[Brod, MG ´08]

- Use $\overline{MS}$ scheme
- Normalise to G_F
- use

$$x_c = \sqrt{2}\,\frac{\sin^2\theta_W}{\pi\alpha}\,G_F m_c^2(\mu_c)$$

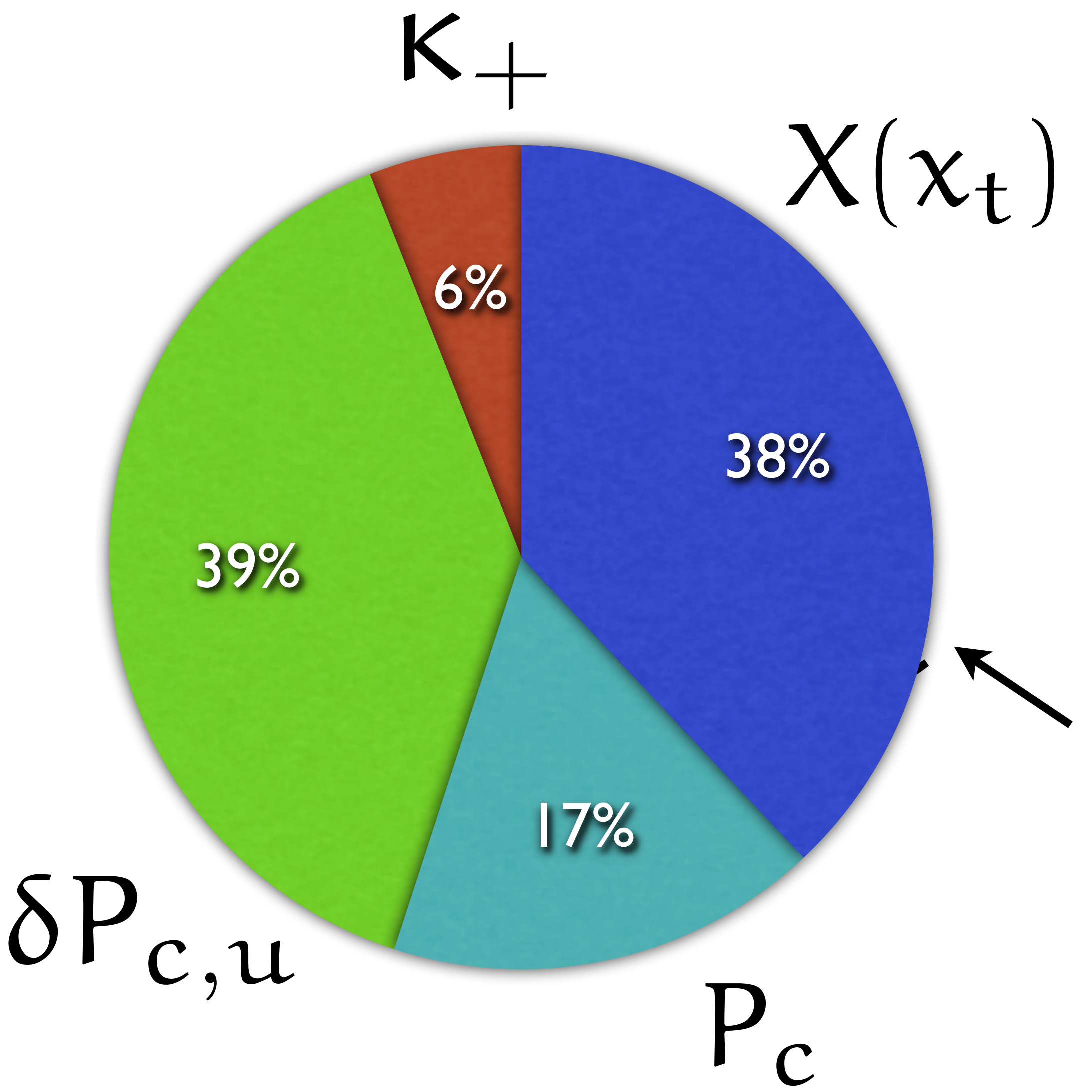
- instead of

$$x_c = \frac{m_c(\mu)^2}{M_W^2}$$

- P_c enhanced by up to 2% for all EW

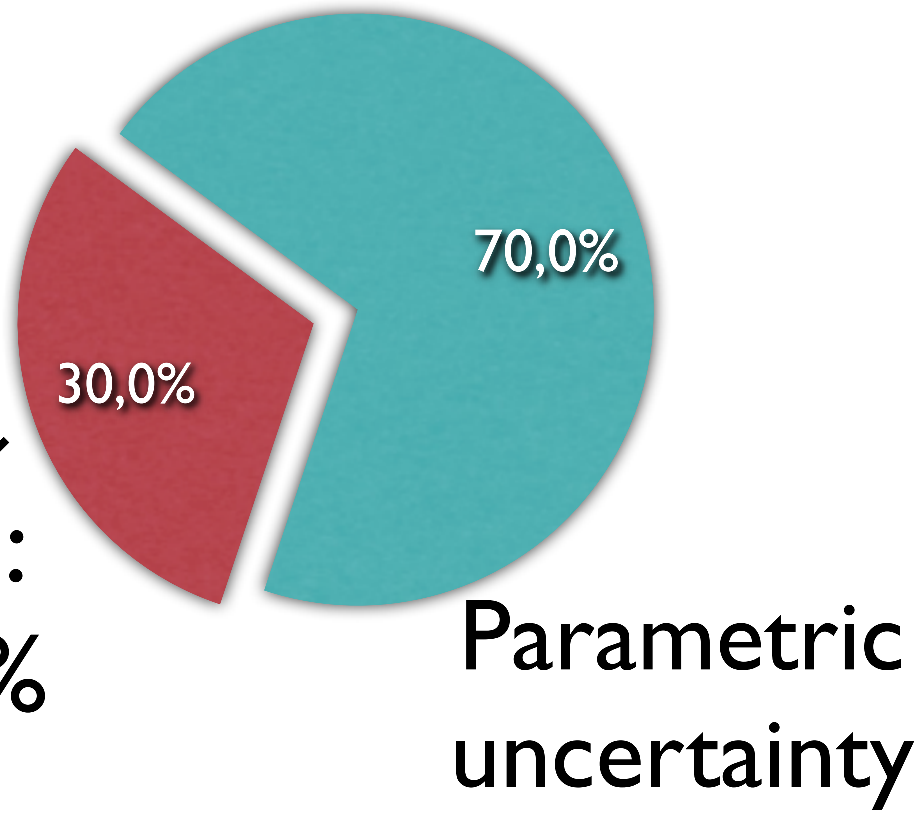
$K^+ \to \pi^+ \bar{\nu}\nu$ Error budget

Theory error budget



for $m_c(m_c) = (1286 \pm 13)MeV$
[Kühn et. al. ´07]

$$\mathcal{B}r_{K^+} = (0.85 \pm 0.07) \times 10^{-10}$$

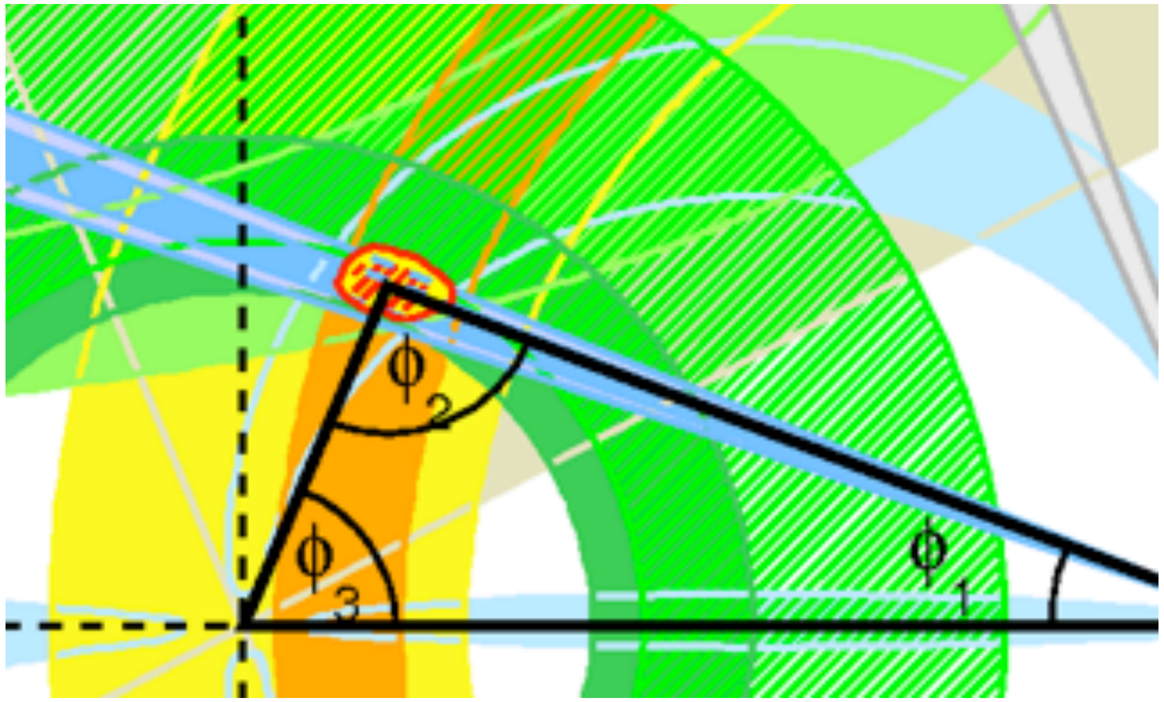


Theory error:
10%x30% = 3%

Experiment [E787, E949 ´08]

$$\mathcal{B}r_{K^+} = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$$

ϵ_K :Indirect CP violation



$$\epsilon_K \simeq \frac{\langle(\pi\pi)_{I=0}|K_L\rangle}{\langle(\pi\pi)_{I=0}|K_S\rangle}$$

$$\epsilon_K = e^{i\phi_\epsilon}\sin\phi_\epsilon\left(\frac{\mathrm{Im}(M_{12}^K)}{\Delta M_K} + \xi\right)$$

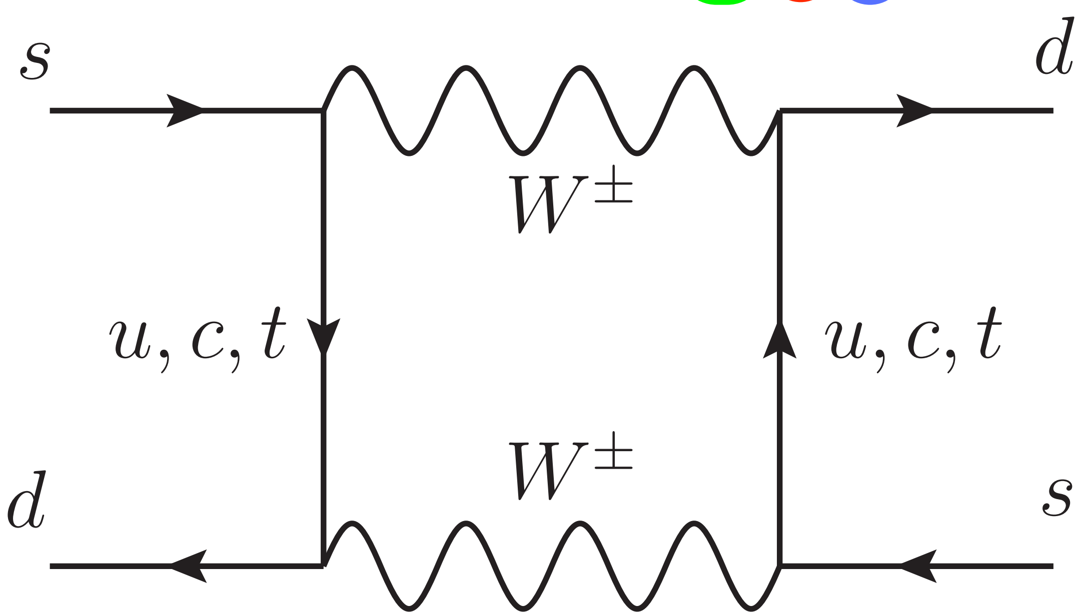
- In almost all old analysis: $\phi_\epsilon = 45^\circ$ and $\xi = 0$
- In reality: $\xi \neq 0$ $\phi_\epsilon \neq 45^\circ$ [Andriyash et. al.'04; Buras et.al. ,08]

$$|\epsilon_K^{SM}| = \kappa_\epsilon |\epsilon_K|(\phi_\epsilon = 45^\circ, \xi = 0)$$

$$\kappa_\epsilon = 0.92 \pm 0.02$$ [Buras Guadagnoli ,08]

Calculation of $M_{12}^{K} = \langle K^0 | \mathcal{H}_{eff}^{\Delta S=2} | \bar{K}^0 \rangle$

Box diagram
with internal u,c,t



$$\lambda_i \lambda_j A(x_i, x_j)$$

$$\lambda_i = V_{is}^* V_{id}$$

plus GIM:

$$\lambda_c + \lambda_t = -\lambda_u$$

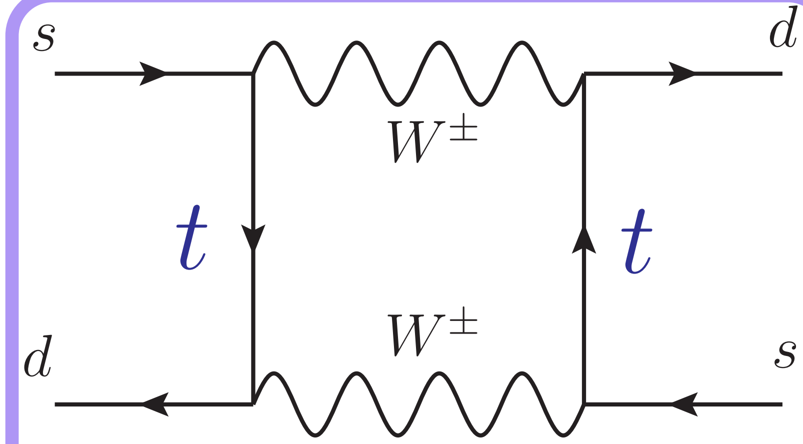
Gives three different contributions for

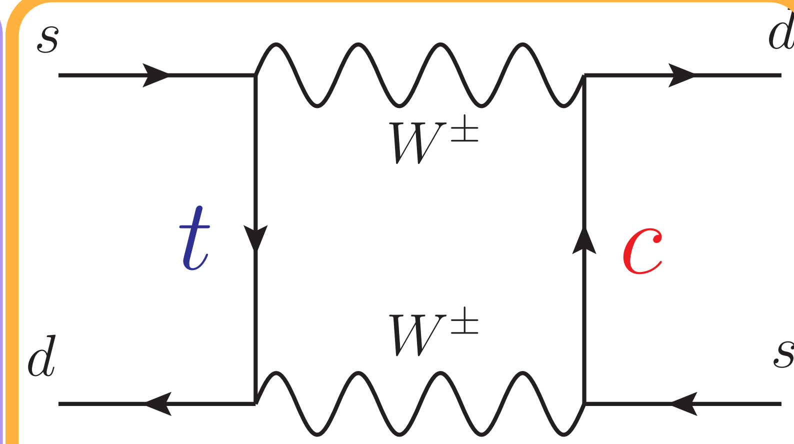
$$M_{12}^{K} = \langle K^0 | \mathcal{H}_{eff}^{\Delta S=2} | \bar{K}^0 \rangle$$

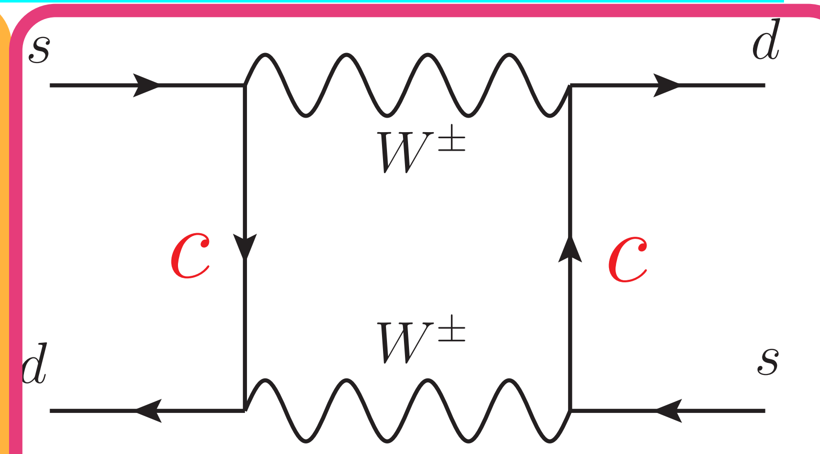
$$\mathcal{H} \propto \Big[\lambda_t^2 \eta_t S(x_t) \quad \text{top}$$
$$+2\lambda_c \lambda_t \eta_{ct} S(x_c, x_t) \quad \text{charm top}$$
$$+\lambda_c^2 \eta_c S(x_c)\Big] b(\mu) \tilde{Q} \quad \text{charm}$$

$$\tilde{Q} = (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L)$$

Calculation of $M_{12}^{K} = \langle K^0 | \mathcal{H}_{eff}^{\Delta S=2} | \bar{K}^0 \rangle$







	top	**charm top**	**charm**
	$\log x_t$	$\log x_c$	$(\log x_c)^0$ hard GIM
LO	$(\alpha_s \log x_c)^n$	$(\alpha_s \log x_c)^n \log x_c$	$(\alpha_s \log x_c)^n$
NLO	$\alpha_s(\alpha_s \log x_c)^n$	$(\alpha_s \log x_c)^n$	$\alpha_s(\alpha_s \log x_c)^n$
ϵ_K	75%	37%	12%
scale	1.8%	7.5%	17.7%

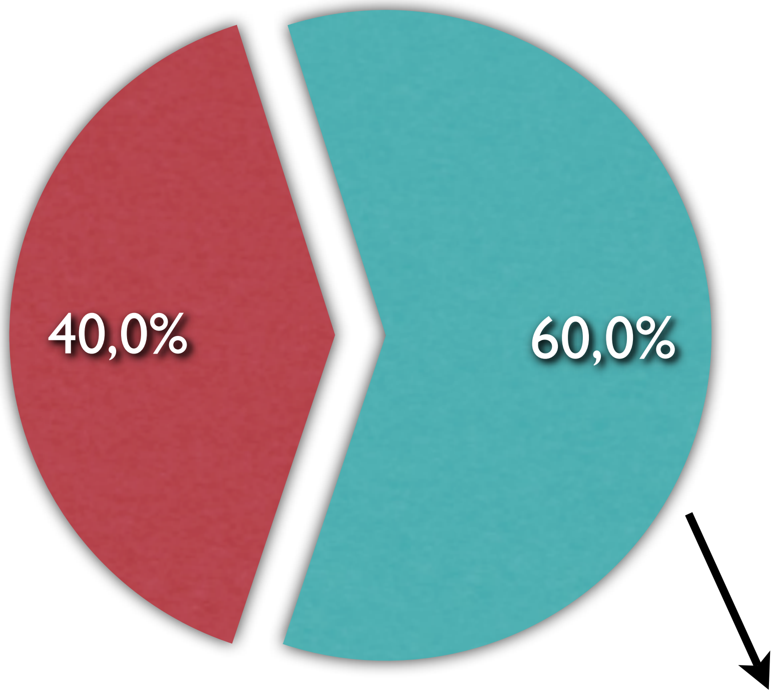
Error Budget for ϵ_K @ NLO

Error Budget for ϵ_K @ NLO

$$\epsilon_K = \kappa_\epsilon C_\epsilon B_K |V_{cb}|^2 \lambda^2 \bar{\eta} (|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt} S_0(x_t) + \eta_{ct} S_0(x_c, x_t) - \eta_{cc} x_c)$$

$$\epsilon_K = (1.78 \pm 0.25) \cdot 10^{-3}$$

[Buras, Guadagnoli'09]



Parametric uncertainty

$$|V_{cb}| = 41.2(1.1) \cdot 10^{-3}$$

$$\bar{\eta}, \bar{\rho}, \ldots$$

Error Budget for ϵ_K @ NLO

$$\epsilon_K = \boxed{\kappa_\epsilon} C_\epsilon \boxed{B_K} |V_{cb}|^2 \lambda^2 \bar{\eta} (|V_{cb}|^2 (1-\bar{\rho}) \eta_{tt} S_0(x_t)$$
$$+ \boxed{\eta_{ct}} S_0(x_c, x_t) - \boxed{\eta_{cc}} x_c)$$

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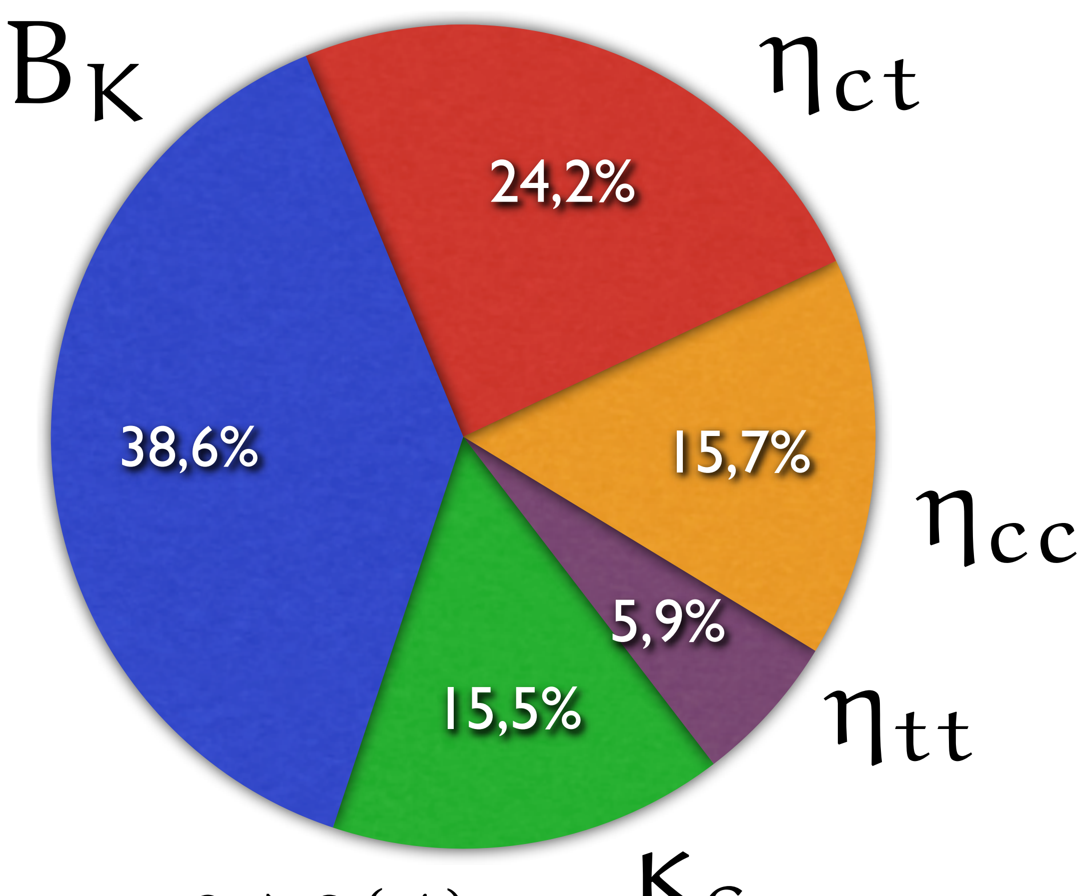
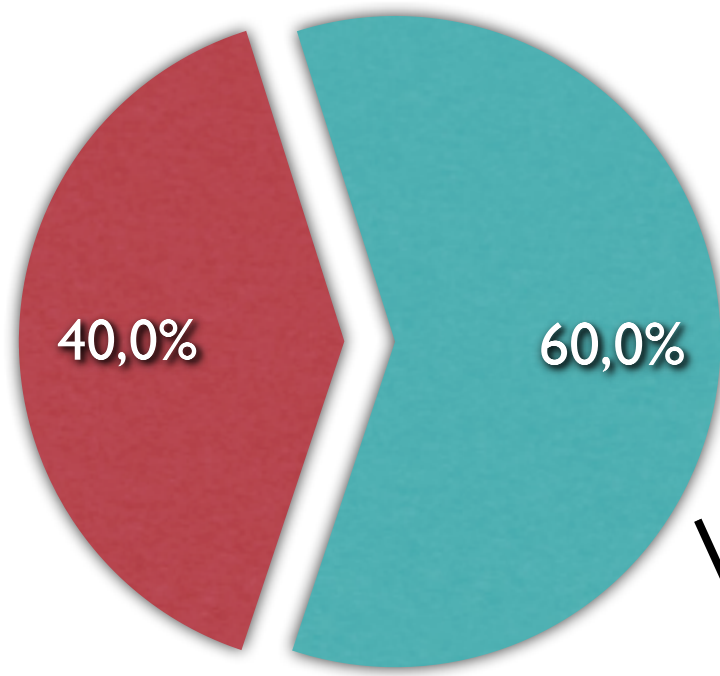
[Buras, Guadagnoli'09]

Theory uncertainty



$$B_K = 0.72(4)$$



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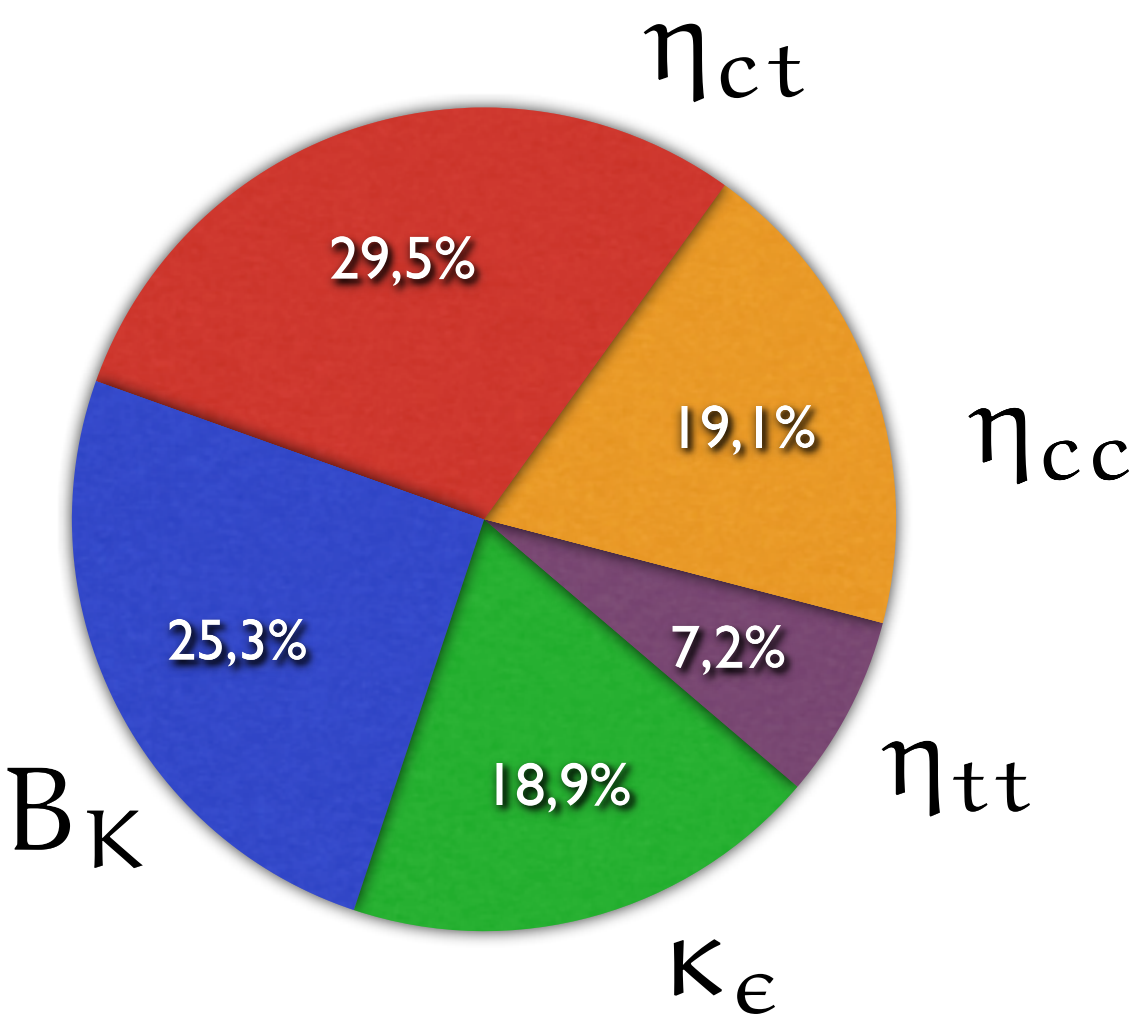
$$\bar{\eta}, \bar{\rho}, \ldots$$

Error Budget for ϵ_K @ NLO

For a 3% uncertainty in B_K
the perturbative uncertainties
become dominant



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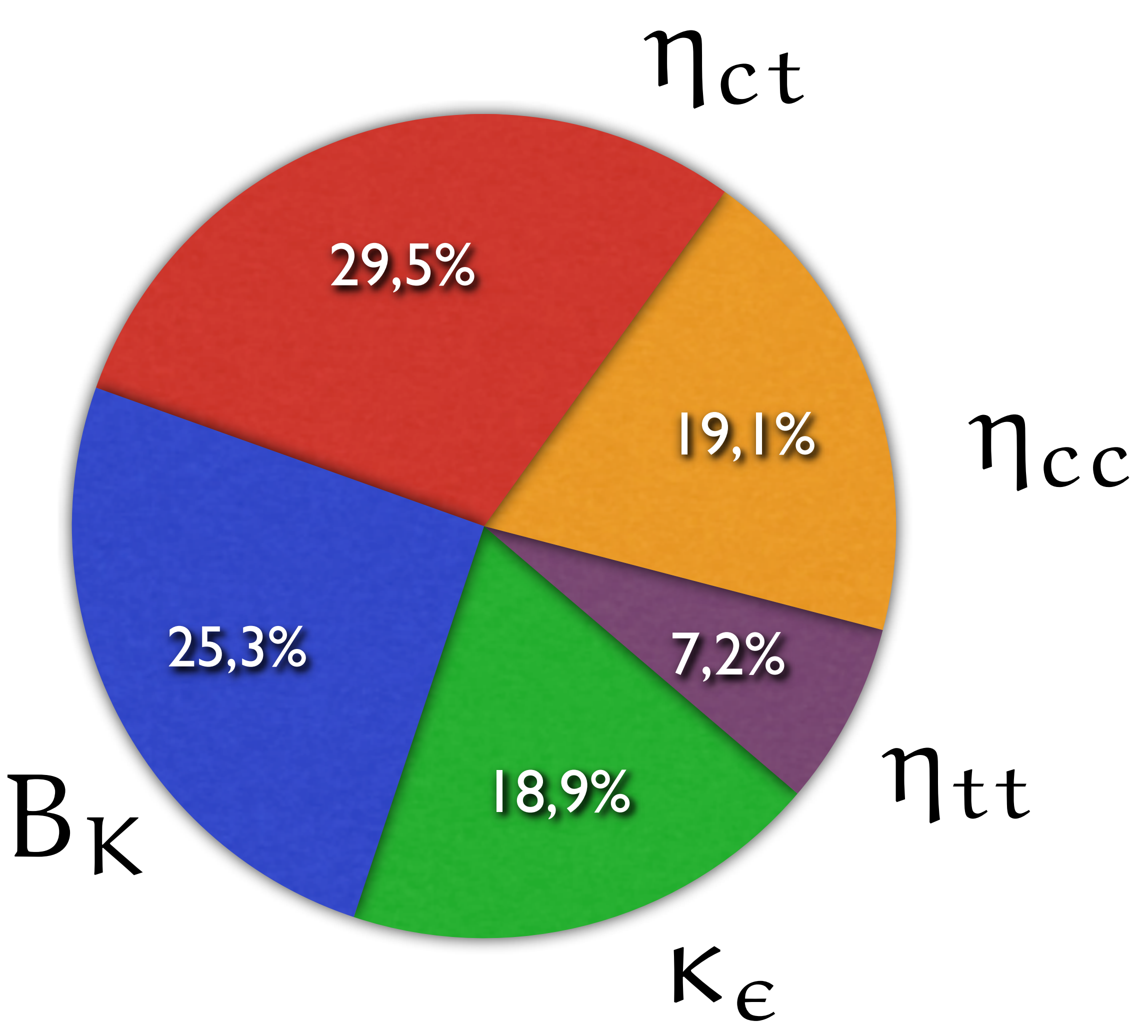


η_{ct} : largest uncertainty

needs a 3 loop RGE analysis

[Brod, MG in progress]

η_{cc} : second largest perturbative uncertainty

needs a 3 loop matching calculation

[Brod, MG in progress]

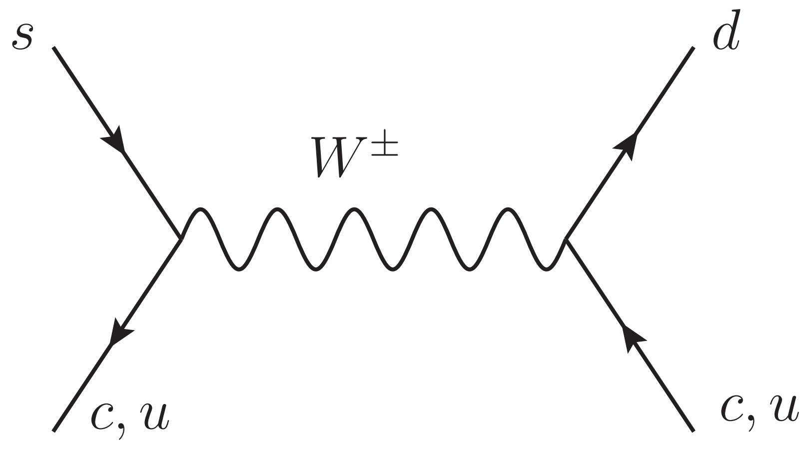
η_{ct} :Charm Top at LO

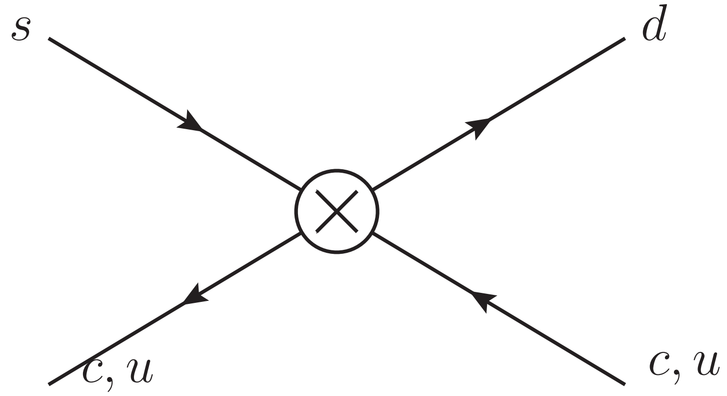
- The Leading Order result

  $(\alpha_s \log x_c)^n \log x_c$

  starts with a $\log x_c$

η_{ct} :Charm Top at LO



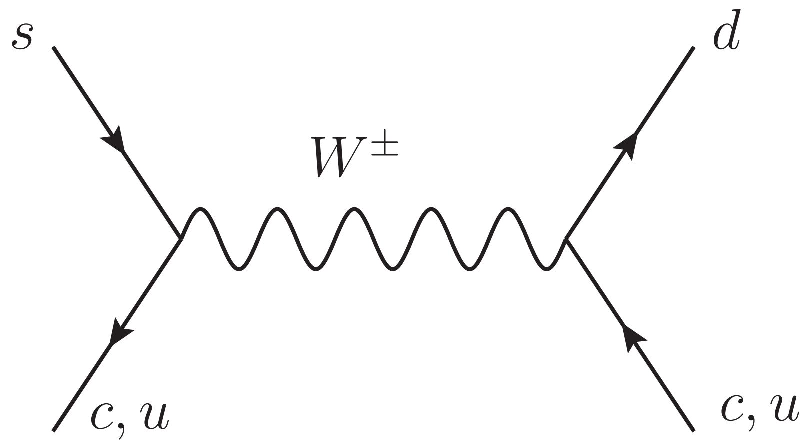


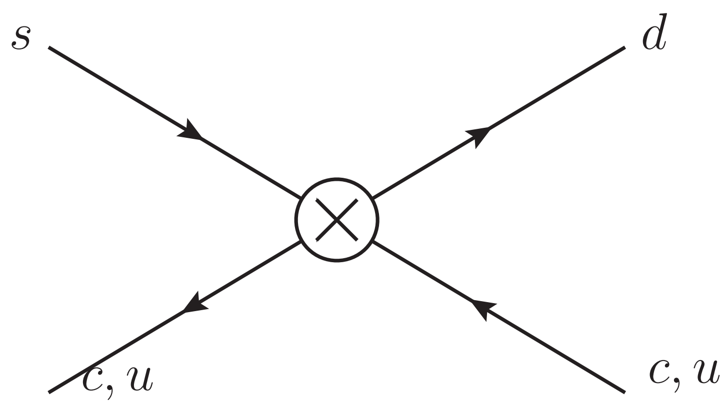
- The Leading Order result

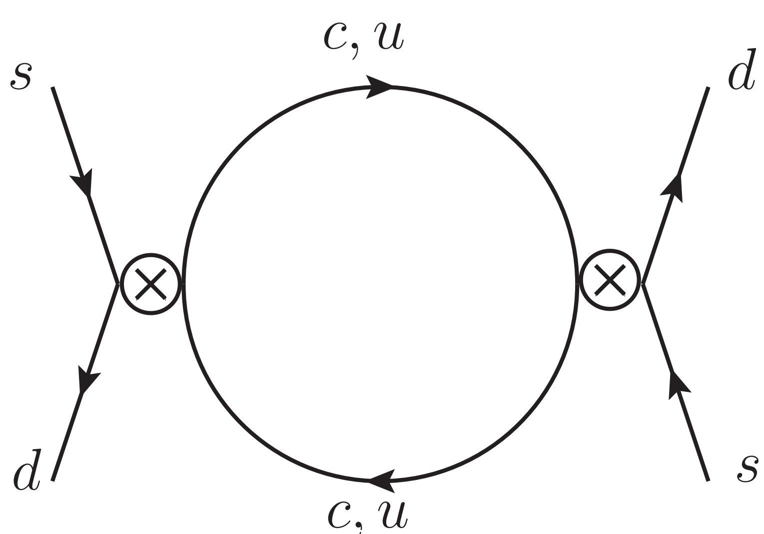
  $$(\alpha_s \log x_c)^n \log x_c$$

  starts with a $\log x_c$
- Tree level matching

η_{ct} :Charm Top at LO





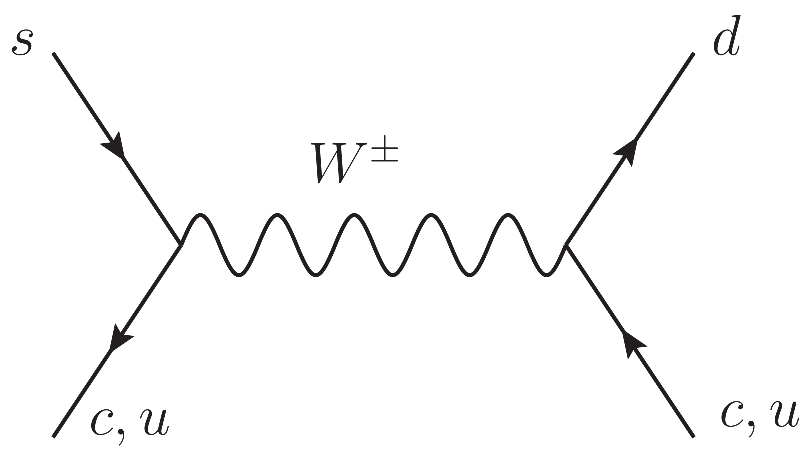


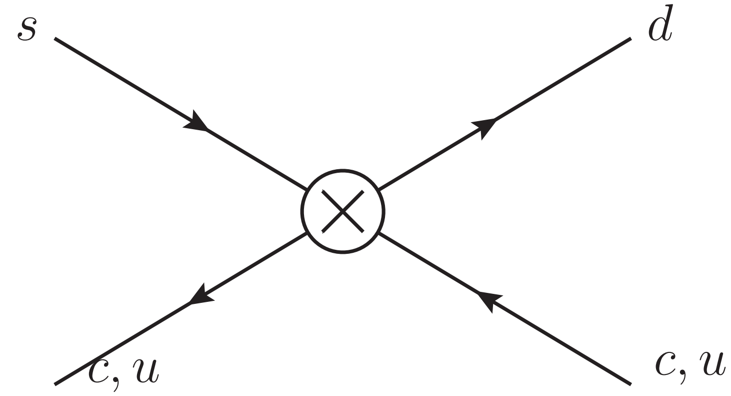
- The Leading Order result

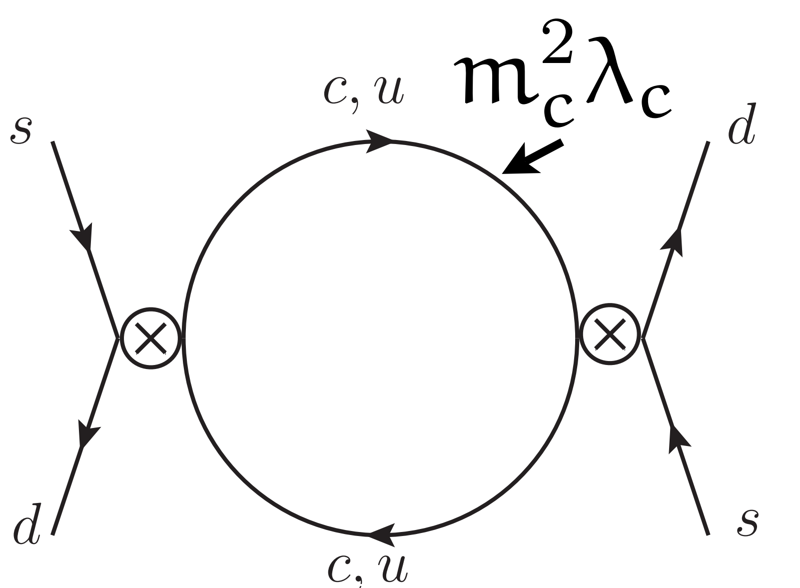
  $$(\alpha_s \log x_c)^n \log x_c$$

  starts with a $\log x_c$
- Tree level matching
- One-loop Renormalistion Group Equation

η_{ct} :Charm Top at LO







- The Leading Order result

  $$(\alpha_s \log x_c)^n \log x_c$$

  starts with a $\log x_c$
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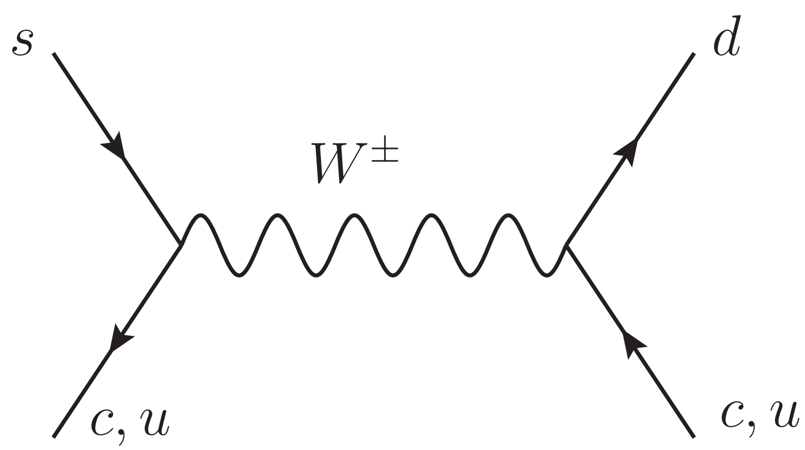
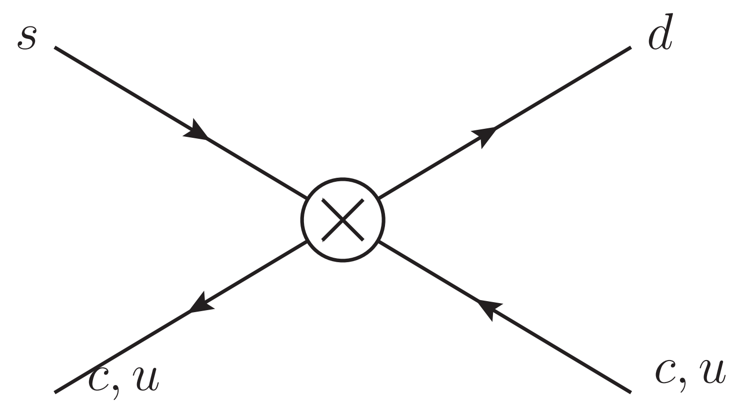
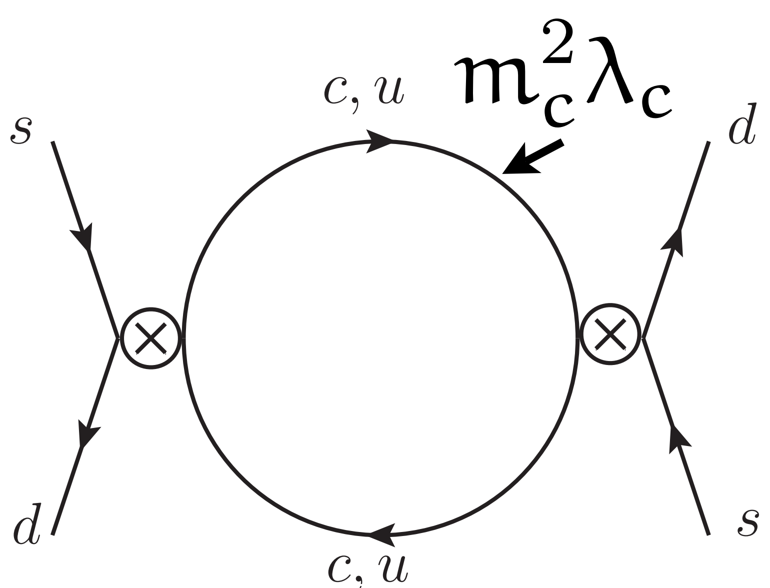
  $m_c^2 \lambda_c$

η_{ct} :Charm Top at LO







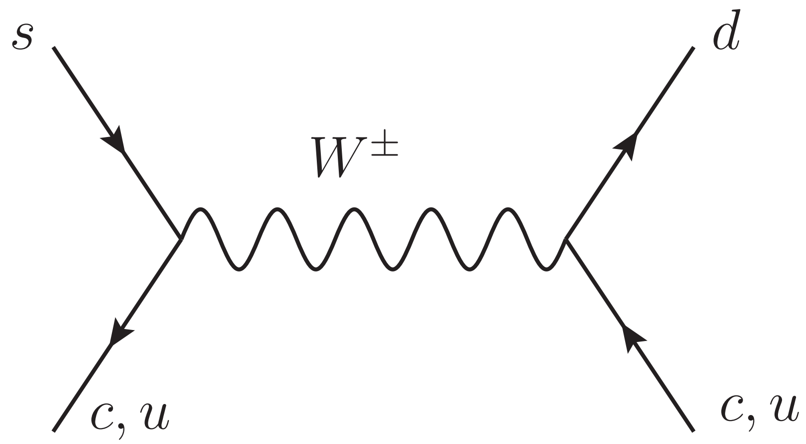
- The Leading Order result

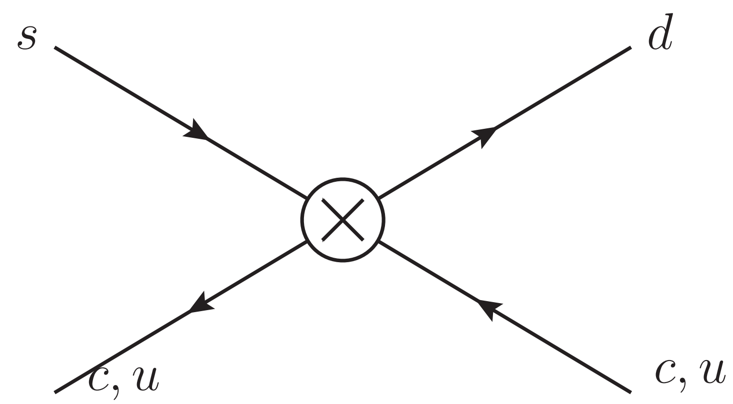
  $$(\alpha_s \log x_c)^n \log x_c$$

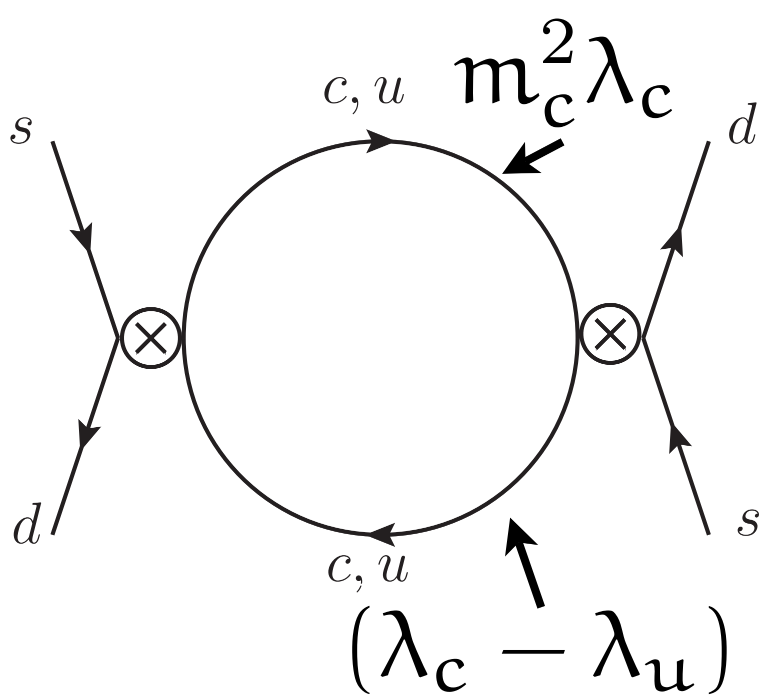
  starts with a $\log x_c$
- Tree level matching
- One-loop Renormalistion Group Equation

  $$m_c^2 \lambda_c (\lambda_c - \lambda_u)$$

η_{ct} :Charm Top at LO







- The Leading Order result

  $$(\alpha_s \log x_c)^n \log x_c$$

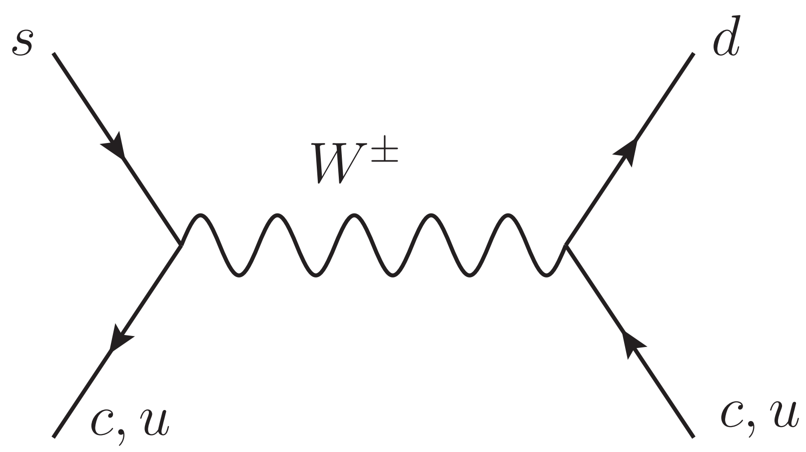
  starts with a $\log x_c$

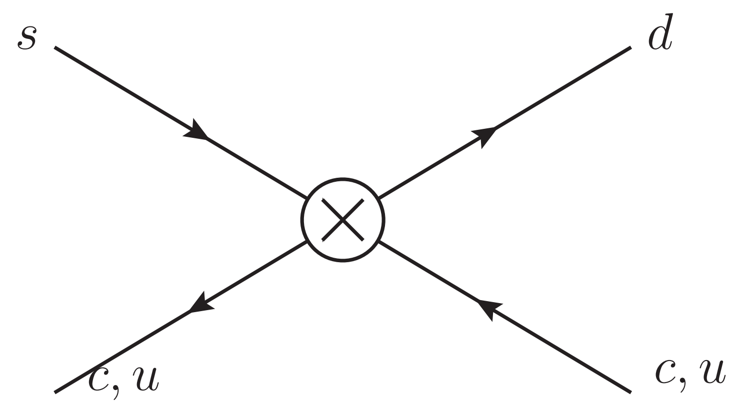
- Tree level matching

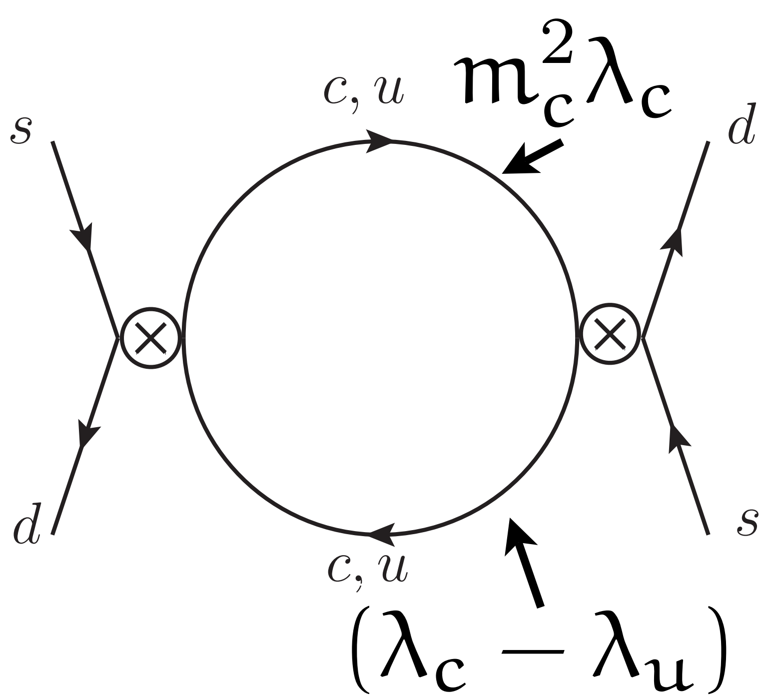
- One-loop Renormalistion Group Equation

  $$m_c^2 \lambda_c (\lambda_c - \lambda_u)$$

η_{ct} :Charm Top at LO







- The Leading Order result

  $$(\alpha_s \log x_c)^n \log x_c$$

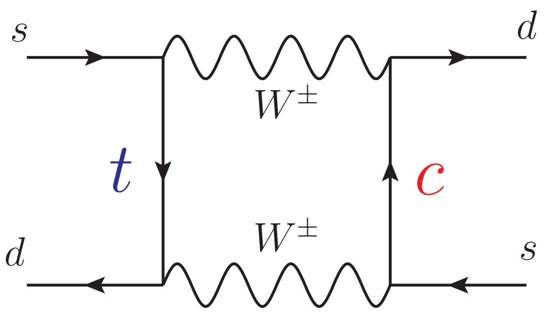
  starts with a $\log x_c$
- Tree level matching
- One-loop Renormalistion Group Equation

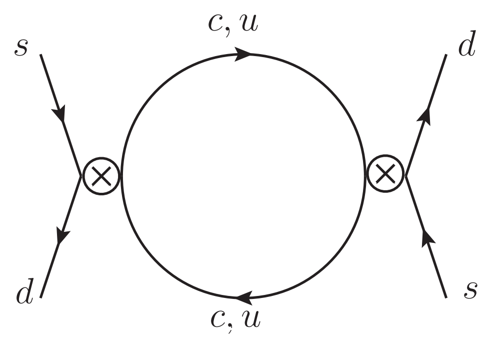
  $$m_c^2 \lambda_c (\lambda_c - \lambda_u) \log \frac{m_c}{M_W}$$

  $$\rightarrow m_c^2 \lambda_c \lambda_t \tilde{Q} \log x_c$$

η_{ct} :Charm Top beyond LO

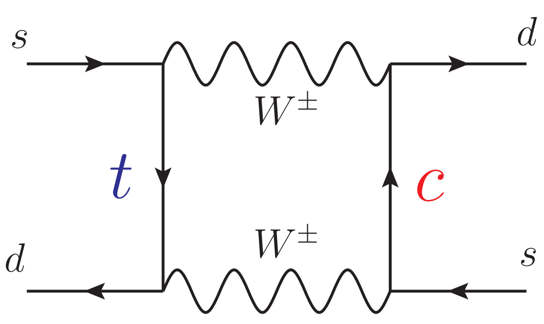
η_{ct} :Charm Top beyond LO

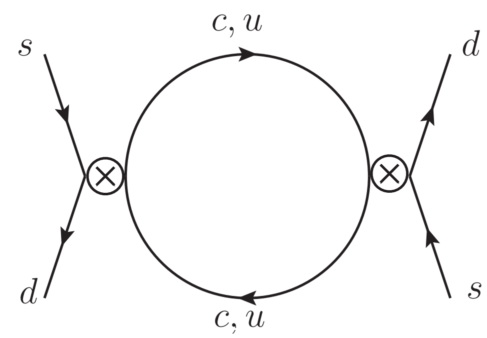


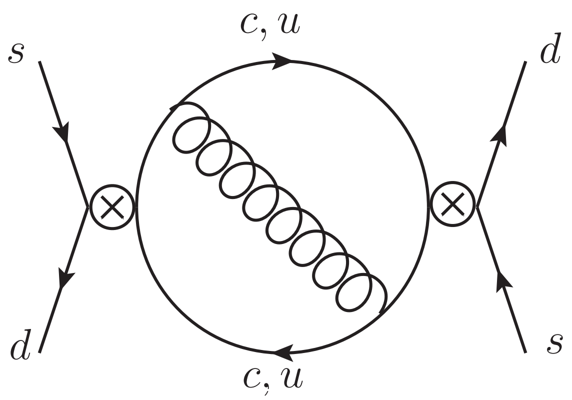


- One-loop matching at μ_t
- One-loop matching at μ_c

η_{ct} :Charm Top beyond LO

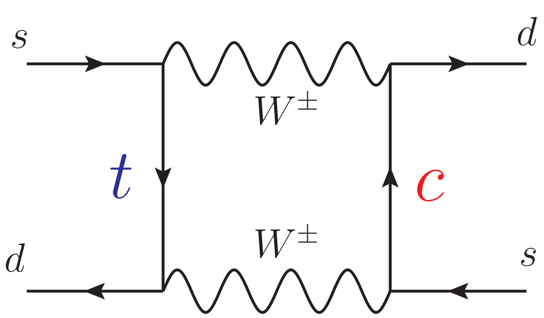


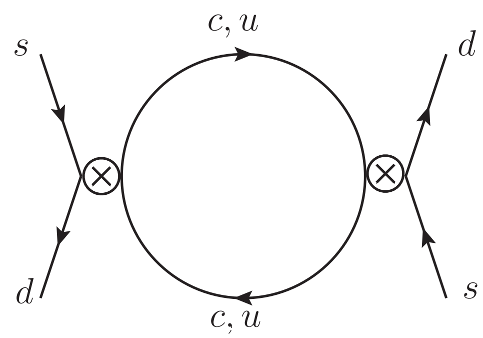


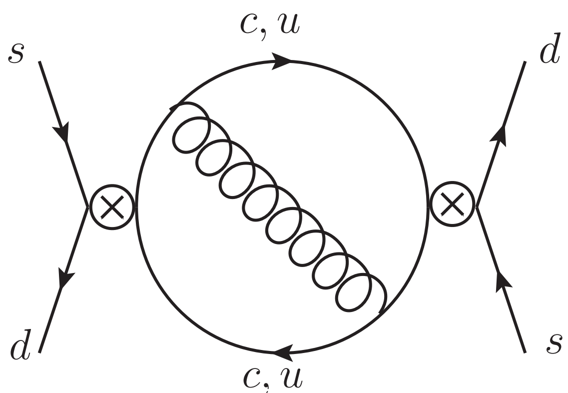


- One-loop matching at μ_t
- One-loop matching at μ_c
- Two-loop RG running
- Plus d=6 operators NLO
  [Herrlich, Nierste]

η_{ct} :Charm Top beyond LO

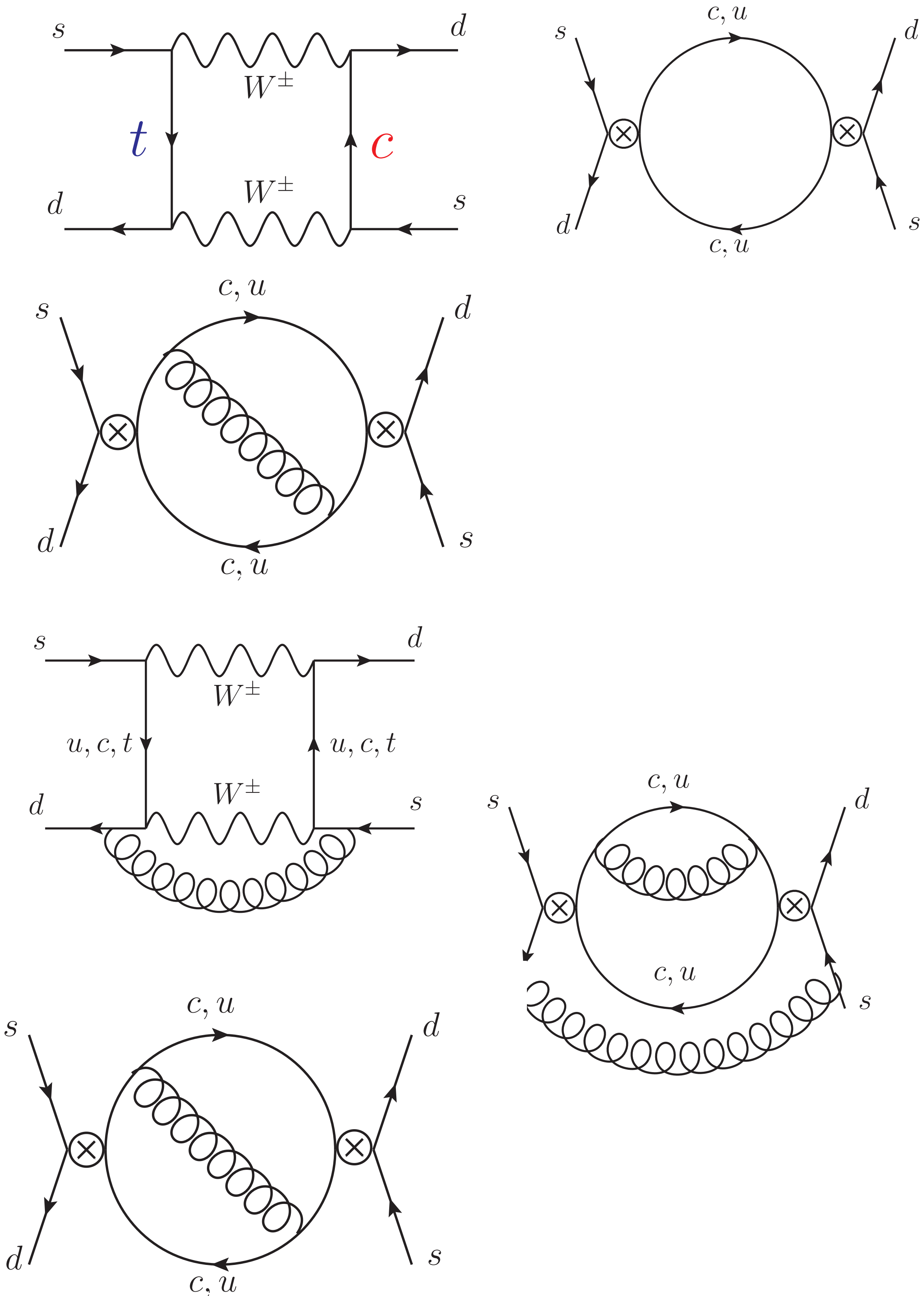






- One-loop matching at μ_t
- One-loop matching at μ_c
- Two-loop RG running
- Plus d=6 operators NLO [Herrlich, Nierste]
- NNLO: RGE and matching for d=6 operators RGE: [MG, Haisch `04], Matching: [Bobeth, et. al. `00]

η_{ct} :Charm Top beyond LO



- One-loop matching at μ_t
- One-loop matching at μ_c
- Two-loop RG running
- Plus d=6 operators NLO
  [Herrlich, Nierste]
  $$\eta_{ct} = 0.47 \pm 0.04$$
- NNLO: RGE and matching for d=6 operators RGE: [MG, Haisch `04], Matching: [Bobeth, et. al. `00]
- Still O(10000) diagrams calculated

Conclusions

$K \to \pi \bar{\nu} \nu$ decays are theoretically very clean

charm quark contribution known at NNLO

two-loop electroweak corrections calculated

ϵ_K Improvements in the non-perturbative parameter B_K make a NNLO calculation mandatory.

Calculation of the dominant charm-top contribution finished