SUSY effects in Kaon physics: Lepton Universality tests and rare decays

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KAON '09

TSUKUBA, JAPAN  12 June 2009
General Considerations

Flavor Physics in the LHC era

- **High energy experiments** are the key tool to determine the energy scale $\Lambda$ by direct production of NP particles.

- **Low energy experiments** are a fundamental ingredient to determine the symmetry properties of the new d.o.f. via their virtual effects in precision observables.
General Considerations

Flavour physics in the LHC era

LHC [high $p_T$]

A unique effort toward the high-energy frontier

[to determine the energy scale of NP]

Flavour physics

- Improved CKM fits
- Rare B decays
- CPV in the $B_s$ system
- Universality tests in $B$ & $K$
- Rare $K$ decays
- LFV in $\mu$ & $\tau$ decays
- EDMs
- $g-2$

A collective effort toward the high-intensity frontier

[to determine the flavour structure of NP]
Where to look for New Physics?

- Processes very **suppressed** or even forbidden in the SM
  - FCNC processes ($\mu \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$, $B_{s,d}^0 \rightarrow \mu^+\mu^-$, $K \rightarrow \pi\nu\bar{\nu}$)
  - CPV effects (electron/neutron EDMs, $d_{e,n}$....)
  - CPV in $B_{s,d}$ decay/mixing amplitudes

- Processes predicted with **high precision** in the SM
  - EWPO as $\Delta\rho$, $(g - 2)_\mu$....
  - LU tests in $R_M^{e/\mu} = \frac{\Gamma(K(\pi) \rightarrow e\nu)}{\Gamma(K(\pi) \rightarrow \mu\nu)}$
Conclusion and outlook

• The charged current analyses using $K_{l3}$ and $K_{l2}$ data have entered an era of very high precision
  – Improvements on the theoretical side: EM, isospin breaking corrections, dedicated dispersive parametrizations to analyse the FFs with the best precision.
  – On the experimental side, very precise data on $K_{l3}$ and $K_{l2}$ decays

  Flavianet Kaon WG

• This allows for very precise tests of the SM (test of unitarity of the 1$^{\text{st}}$ line of CKM matrix, universality, quark mass ratios...) and New Physics scenarios (Charged right-handed currents, scalar couplings, Lepton flavour violation...)

• But still on the experimental side, need $K^+$ measurements (FFs...). Experimental puzzle on $f_0(t)$ (NA48 doesn’t agree with the other experiments).

• On theoretical side, $f_+(0)$ determination should be improved

  disagreement between analytical and lattice determinations. Lattice improvements are promising.

E. Passemard

Kaon’09, Tsukuba
### General Considerations

**NP search strategies**

μ − e universality in $R_K = \Gamma(K \to e\nu_e)/\Gamma(K \to \mu\nu_\mu)$

<table>
<thead>
<tr>
<th></th>
<th>$(R_K^{e/\mu})_{\text{exp.}} [10^{-5}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDG 2006</td>
<td>2.45 ± 0.11</td>
</tr>
<tr>
<td>NA48/2 ’03 prel.</td>
<td>2.416 ± 0.043 ± 0.024</td>
</tr>
<tr>
<td>NA48/2 ’04 prel.</td>
<td>2.455 ± 0.045 ± 0.041</td>
</tr>
<tr>
<td>KLOE prel.</td>
<td>2.55 ± 0.05 ± 0.05</td>
</tr>
<tr>
<td>SM prediction</td>
<td>2.477 ± 0.001</td>
</tr>
</tbody>
</table>

\[
(R_K^{e/\mu})_{\text{exp.}} = (2.457 ± 0.032) \times 10^{-5}
\]

- A dedicated run (of 4 month) for $R_K$ by P326/NA62 (former NA48) has been performed @ the CERN. Goal: the error @ 0.3%!

- $R_\pi^{\text{exp.}} = (1.230 ± 0.004) \cdot 10^{-4}$ PDG
SM prediction for $R_{K,\pi}$

- $R_{K}^{SM} = (2.472 \pm 0.001) \cdot 10^{-5}$ \text{ SM}
- $R_{\pi}^{SM} = (1.2354 \pm 0.0002) \cdot 10^{-4}$ \text{ SM}

Marciano Sirlin '93, Finkemeyer '96

- $R_{K}^{SM} = (2.477 \pm 0.001) \cdot 10^{-5}$ \text{ SM}
- $R_{\pi}^{SM} = (1.2352 \pm 0.0001) \cdot 10^{-4}$ \text{ SM}

Cirigliano Rossell '07

The total errors in $R_{K,\pi}$ are dominated by the EXP. ERRORS!!!
**R_K @ NA62**

**Preliminary result (40% data set)**

\[ R_K = (2.500 \pm 0.012_{\text{stat}} \pm 0.011_{\text{syst}}) \times 10^{-5} \]

\[ = (2.500 \pm 0.016) \times 10^{-5} \]  

(New, June 09)

**Uncertainties**

<table>
<thead>
<tr>
<th>Source</th>
<th>( \delta R_K \times 10^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistical</td>
<td>0.012</td>
</tr>
<tr>
<td>( K_\mu^2 )</td>
<td>0.004</td>
</tr>
<tr>
<td>Beam halo</td>
<td>0.001</td>
</tr>
<tr>
<td>( K_{e2\gamma} (SD^+) )</td>
<td>0.004</td>
</tr>
<tr>
<td>Electron ID</td>
<td>0.001</td>
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<tr>
<td>IB simulation</td>
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</tr>
<tr>
<td>Acceptance</td>
<td>0.002</td>
</tr>
<tr>
<td>Trigger timing</td>
<td>0.007</td>
</tr>
<tr>
<td>Total</td>
<td>0.016</td>
</tr>
</tbody>
</table>

(0.64% precision)

The whole sample will allow a statistical uncertainty \( \sim 0.3\% \), and total uncertainty of 0.4--0.5\%.
$R_K: KLOE$ result

- The result does not depend upon the kaon charge: $K^+: 2.496(37) vs K^-: 2.490(38)$ (uncorrelated errors only)
- Agrees with SM prediction

**Total error:**

- $1.3\% = 1.0\%_{\text{stat}} + 0.8\%_{\text{syst}}$
- $0.9\%$ from $14k$ Ke2 dominated
- $+ bkg$ subtraction by statistics

**PDG 2008:**

$R_K = (2.45\pm0.11) \times 10^{-5}$

4.5\% accuracy

**New world average:**

$R_K = (2.468\pm0.025) \times 10^{-5}$

1\% accuracy

$R_K^{SM} = 2.477(1) \times 10^{-5}$
Any deviation from the SM expectation for $R_{K,\pi}$ due to NP can be written as

$$R_{K,\pi} = R_{K,\pi}^{SM} \left( 1 + \Delta r_{K,\pi}^{e-\mu_{NP}} \right),$$

Violations of LU in CCI can be classified as

i) **Corrections** to $(V-A) \times (V-A)$ interaction through $W\ell\nu_{\ell}$ vertex correction induced by a loop of NP particles

$$\Delta r_{\text{SUSY}}^{e-\mu} \sim \frac{\alpha_2}{4\pi} \left( \frac{\tilde{m}_\mu^2 - \tilde{m}_e^2}{\tilde{m}_\mu^2 + \tilde{m}_e^2} \right) \frac{m_W^2}{M_{\text{SUSY}}^2} \leq 10^{-4}$$

ii) **New Lorentz Structures**, i.e. scalar CCI with

$$H\ell\nu \sim m_\ell \tan \beta$$
μ – e universality in $M \to \ell \nu$

- Four-Fermi interaction for $M \to \ell \nu$ induced by $W^\pm$, $H^\pm$

$$\frac{4G_F}{\sqrt{2}} V_{ud} \left[ (\bar{u}\gamma_\mu P_L d)(\bar{\ell}\gamma^\mu P_L \nu_\ell) - t_\beta^2 \left( \frac{m_d m_\ell}{m_{H^\pm}^2} \right)(\bar{u}P_R d)(\bar{\ell}P_L \nu_\ell) \right]$$

- PCAC’s
  - $<0|\bar{u}\gamma_\mu \gamma_5 d|M> = i f_M p_\mu^M$  
  - $<0|\bar{u}\gamma_5 d|M> = -i f_M \frac{m_M^2}{m_d + m_u}$

- $H^\pm$ ($W^\pm$) amplitude is proportional to $m_\ell$ because of the Yukawa coupling (helicity suppression)

$$\frac{\Gamma^{H^\pm + W^\pm}(M \to \ell \nu)}{\Gamma^{W^\pm}(M \to \ell \nu)} = r_M = \left[ 1 - t_\beta^2 \left( \frac{m_d}{m_u + m_d} \right) \frac{m_M^2}{m_{H^\pm}^2} \right]^2.$$

Tree level $H^\pm$ effects ($r_M$) are lepton flavour blind
$\mu - e$ universality in $M \rightarrow l\nu$

WHAT ARE WE MISSING?............

$$R^\text{EXP.}_K = \frac{\Gamma(K \rightarrow e\nu_e) + \Gamma(K \rightarrow e\nu_\mu) + \Gamma(K \rightarrow e\nu_\tau)}{\Gamma(K \rightarrow \mu\nu_\mu) + \Gamma(K \rightarrow \mu\nu_e) + \Gamma(K \rightarrow \mu\nu_\tau)}$$

............EXPERIMENTALLY THE NEUTRINO FLAVOUR IS UNDETERMINED!!

Masiero, Paradisi, Petronzio, ’06
**μ − e universality in \( M \to l\nu \)**

**WHAT ARE WE MISSING?.........**

\[
R_K^{\text{EXP.}} = \frac{\Gamma(K \to e\nu_e) + \Gamma(K \to e\nu_\mu) + \Gamma(K \to e\nu_\tau)}{\Gamma(K \to \mu\nu_\mu) + \Gamma(K \to \mu\nu_e) + \Gamma(K \to \mu\nu_\tau)}
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R_K^{EXP.} = \frac{\Gamma(K \to e\nu_e) + \Gamma(K \to e\nu_\mu) + \Gamma(K \to e\nu_\tau)}{\Gamma(K \to \mu\nu_\mu) + \Gamma(K \to \mu\nu_e) + \Gamma(K \to \mu\nu_\tau)}
\]

..........EXPERIMENTALLY THE NEUTRINO FLAVOUR IS UNDETERMINED !!

Masiero, Paradisi, Petronzio, ’06
$R_{K}^{LFV} = \frac{\sum_i K \to e\nu_i}{\sum_i K \to \mu\nu_i} \approx \frac{\Gamma_{SM}(K \to e\nu_{e}) + \Gamma(K \to e\nu_{\tau})}{\Gamma_{SM}(K \to \mu\nu_{\mu})}, \ i = e, \mu, \tau$

\[ eH^{\pm} \nu_{\tau} \rightarrow \frac{g_2}{\sqrt{2}} \frac{m_{\tau}}{M_{W}} \Delta_{R}^{31} \tan^{2}\beta \]

\[ \Delta_{R}^{31} \sim \frac{\alpha_2}{4\pi} \delta_{RR}^{31} \]

\[ \Delta_{R}^{31} \sim 5 \cdot 10^{-4} \quad t_{\beta} = 40 \quad M_{H^{\pm}} = 500 \text{GeV} \]

\[ \Delta r_{K}^{e-\mu} \approx 10^{-2} \quad \Rightarrow \quad Br_{(exp.)}^{th.}(\tau \rightarrow eX) \leq 10^{-10}(-7) \]
Which is the sign of $\Delta r_{NP}^{e-\mu}$?

- LFV effects to LFC channels in $R_M$

$$\ell H^{\pm} \nu_\ell \rightarrow \frac{g_2}{\sqrt{2}} \frac{m_\ell}{M_W} \tan \beta \left(1 + \frac{m_\tau}{m_\ell} \Delta_{RL}^{\ell \ell} \tan \beta \right) \quad (\ell = e, \mu)$$

$$\Delta_{RL}^{\ell \ell} \sim \frac{\alpha_1}{4\pi} \delta_{RR}^{\ell 3} \delta_{LL}^{3 \ell} f_{\text{loop}} \leq 10^{-4}$$

- Deviations from $\mu - e$ universality in $K_{l2}$ and $\pi_{l2}$

$$\frac{R_{K,\pi}^{L_{\ell}}}{R_{K,\pi}^{SM}} \sim \left[ \left(1 - \frac{m_\tau}{m_e} \frac{m_{K,\pi}^2}{M_{H^\pm}^2} \Delta_{RL}^{11} \tan^3 \beta \right)^2 + \frac{m_\tau^2}{m_e^2} \frac{m_{K,\pi}^4}{M_{H^\pm}^4} \left| \Delta_{R}^{31} \right|^2 \tan^6 \beta \right]$$

$$R_{K}^{L_{\ell}} \simeq R_{K}^{SM} (1 - 0.032), \quad R_{\pi}^{L_{\ell}} \simeq R_{\pi}^{SM} (1 - 0.0021)$$
Black points explain the $(g - 2)_\mu$ anomaly

Masiero, P.P., Petronzio, ’08
General Considerations NP search strategies $\mu - e$ universality in $R_K = \Gamma(K \to e\nu) / \Gamma(K \to \mu\nu)$

SM prediction for $R_K^{\mu\nu}$, $\pi K \to \pi\nu\bar{\nu}$ in the SM $K \to \pi\nu\bar{\nu}$ and NP Gaugino mediated $K \to \pi\nu\bar{\nu}$

Conclusion

$R_{LFV}^{K}$ in SUSY

$R_{LFV}^{K}$ in SUSY at a (Super)B factories

SUSY MFV scenario

$\tan\beta$ Lightest Higgs boson mass WMAP & $g - 2$ Constraints

B-physics & $g - 2$ under WMAP constraints

Black points explain the $(g - 2)_\mu$ anomaly

Masiero, P.P., Petronzio, ’08

$R_{LFV}^{K}$. SUSY effects in Kaon physics: Lepton Universality tests and rare decays
General Considerations

NP search strategies \( \mu - e \) universality in 

\[ R_K^{\mu/e} = \frac{\Gamma(\tau \to \mu \nu \bar{\nu})}{\Gamma(\tau \to e \nu \bar{\nu})} \]

\[ R_K^{\mu/e} \approx 1 - 10^{-3} \left( \frac{t_\beta}{50} \right)^2 \left( \frac{200 \text{GeV}}{M_{H^\pm}} \right)^2 \]

Mursula et al. '83

\[ R_{B \to D}^{\tau/\ell} = \frac{\Gamma(B \to D \tau \nu)}{\Gamma(B \to D \ell \nu)} \]

Hou '92, Tanaka '95, Kiers & Soni '97

\[ \frac{R_{B \to D}^{\tau/\mu}}{R_{B \to D}^{\tau/\mu}|_{SM}} \approx 1 - 0.3 \left( \frac{t_\beta}{50} \right)^2 \left( \frac{200 \text{GeV}}{M_{H^\pm}} \right)^2 \]

Nierste et al.'08, Kamenik & Mescia '08
How natural is the MFV SUSY scenario @ large $\tan \beta$?

- Top-Bottom Yukawa unification in GUT $\Rightarrow \tan \beta = (m_t/m_b)$
- $m_h > 114$GeV constraint better satisfied
- $\Delta a_\mu = (g - 2)_\mu/2 = (3 \pm 1) \times 10^{-9}$ naturally explained
- WMAP constraints "naturally" satisfied
- Correlations between $\text{BR}(B \to \tau \nu)$ and $\text{BR}(B \to X_s \gamma)$, $\Delta M_{B_s}$, $\text{BR}(B_{s,d} \to \ell^+ \ell^-)$, $(g - 2)_\mu$ and $m_{h^0}$

Ellis et al.

Isidori, P.P., '06
Lightest Higgs boson mass WMAP & $(g-2)_\mu$

\[ \Delta a_\mu \simeq 3 \times 10^{-9} \left( \frac{400 \text{GeV}}{\tilde{m}} \right)^2 \left( \frac{t_\beta}{50} \right) \text{sign} \mu \]

\[ \mu = 500 \text{ GeV} \]

\[ t_\beta = 20 \, (\text{green}), \, 30 \, (\text{red}), \, 50 \, (\text{black}) \]

\[ M_{1}, M_{H}, m_h \]

Isidori et al., 06', 07'
Constraints

- $B \to X_s \gamma$: $[1.01 < R_{Bs\gamma} < 1.24]$

- $a_\mu$: $[2 < 10^{-9} (a_\mu^{\text{exp}} - a_\mu^{\text{SM}}) < 4]$

- $B \to \mu^+ \mu^-$: $[B^{\text{exp}} < 8.0 \times 10^{-8}]$

- $\Delta M_{B_s}$: $[\Delta M_{B_s} = 17.35 \pm 0.25 \text{ ps}^{-1}]$

- $B \to \tau \nu$: $[0.8 < R_{B\tau\nu} < 0.9]$
B-physics & \((g-2)_\mu\) under WMAP constraints

\(\Delta a_\mu = (30\pm10)\times10^{-10}\)

\(\delta(B\rightarrow\tau\nu) \geq 20\%\)

\(\delta(B\rightarrow\tau\nu) \leq 10\%\)

\(M_{H} \sim 2M_1\)

Isidori, Mescia, P.P., Temes, 07
**K → πν̄ν in the SM**

- **K → πν̄ν** processes offer a unique possibility in probing the underlying **flavour mixing mechanism**:
  - No SM tree-level contributions (**FCNC decays**);
  - One-loop SM contributions CKM-suppressed \((V^*_{ts}V_{td} \sim \lambda^5)\);
  - High precision of the SM prediction thanks to short distance (e.w.) dynamics dominance:

\[
\mathcal{H}_{\text{eff}}^{(s.d.)} = \sum_{l=e,\mu,\tau} V^*_{ts} V_{td} \left[ X_L (\bar{s}d)_{V-A} + X_R (\bar{s}d)_{V+A} \right] (\bar{\nu}_l \nu_l)_{V-A}
\]

\[
Br(K \rightarrow \pi\nu\bar{\nu}) \sim (X = X_L + X_R)^2
\]

\[
X \sim c_{SM} \frac{y^2_t V^*_{ts} V_{td}}{16\pi^2 M_W^2}
\]

\[
X_{L}^{SM} = 1.464 \pm 0.041, \quad X_{R}^{SM} = 0
\]
$K \rightarrow \pi \nu \bar{\nu}$ and NP

- $K \rightarrow \pi \nu \bar{\nu}$ has a high sensitivity to NP effects of many theories as SUSY, LHT, Z’ models.....

$$X(s \rightarrow d)_{\text{FCNC}} \sim c_{\text{SM}} \frac{y_t^2 V_{ts}^* V_{td}}{16 \pi^2 M_W^2} + c_{\text{NP}} \frac{\delta_{21}}{16 \pi^2 \Lambda_{NP}^2}$$

$$Br(K \rightarrow \pi \nu \bar{\nu}) \sim (X = X_L + X_R)^2$$

$$X_L = X_{L}^{\text{SM}} + X_{L}^{\text{NP}}, \quad X_R = X_{R}^{\text{NP}}$$

- Large NP effects only if $\delta_{21} \sim V_{ts}^* V_{td}$ (beyond MFV)

see the talk by Buras
Rare Kaon decays beyond the SM [general properties]

Two basic scenarios:

**Minimal Flavour Violation**
flavour symmetry broken only by the (SM) Yukawa couplings

- Small deviations (10-20%) from SM
- Stringent correlations with other rare decays in B physics \([B_d \rightarrow X_{s,d} \nu \nu, B_d \rightarrow X_{s,d} l^+ l^-, B_{s,d} \rightarrow l^+ l^-]\)

Recent (almost) model-indep. analysis:

Consistent with results of specific models:
- Constrained MSSM [Buras et al. '01]
- One universal extra dim. [Buras et al. '03]
- Littlest-Higgs [Buras et al. '05]
General Considerations

NP search strategies

$\mu - e$ universality in $R_K = \Gamma(K \rightarrow e\nu e) / \Gamma(K \rightarrow \mu\nu\mu)$

The SM prediction for $R_K, \pi K \rightarrow \pi\nu\bar{\nu}$ in the SM

$K \rightarrow \pi\nu\bar{\nu}$

Conclusion

Gaugino mediated $K \rightarrow \pi\nu\bar{\nu}$

- The dominant effects to $K \rightarrow \pi\nu\bar{\nu}$ arise from $\tilde{\chi}/\tilde{u}$ diagrams with double-MIA [Colangelo, Isidori '98].

- Gluino-type amplitudes (LL, RR and LR-down squarks type mixings) essentially negligible contrary to $\epsilon_K$, $b \rightarrow s\gamma$, $B^0 - \bar{B}^0$

- Minor effects within pure MFV.

- The maximal sensitivity to the up-type trilinear terms is obtained for

  - Light stop and charginos
  - small $\tan \beta$
**Rare Kaon decays beyond the SM** [general properties]

Two basic scenarios:

- E.g.: II. Generic MSSM
  - New sources of Flavour Symmetry breaking around the TeV scale
  - Potentially large effects, especially in the three CPV $K_L$ decays (no $\lambda^5$ suppression)
  - Correlations with observables in B physics not obvious

Grossman-Nir bound:

$$\Gamma(K_L \to \pi^0\nu\bar{\nu}) < \Gamma(K^+ \to \pi^+\nu\bar{\nu})$$
Chargino mediated $K \rightarrow \pi \nu \bar{\nu}$

G. Isidori, F. Mescia, P. P., C. Smith, S. Trine, '06
Chargino mediated $K \rightarrow \pi \nu \bar{\nu}$

G. Isidori, F. Mescia, P. P., C. Smith, S. Trine, ’06
Chargino mediated $K \rightarrow \pi \nu \bar{\nu}$

G. Isidori, F. Mescia, P. P., C. Smith, S. Trine, ’06
Higgs mediated $K \rightarrow \pi \nu \bar{\nu}$

$K \rightarrow \pi \bar{\nu} \nu$ and large $\tan \beta$

1-loop: $H_u$ couples to down quarks

$$\mathcal{L}^Y_{\text{eff}} = \delta Y_{ij} d_i^d H_u^* \cdot Q^j_L$$

$$\rightarrow Y_{ij}^d \propto M_{ij}$$

$\rightarrow \tan \beta$ Enhanced effects

$B_s \rightarrow \mu^+ \mu^-$ in MFV: $(\tan \beta)^6$ [Babu, Kolda '00]

$K \rightarrow \pi \bar{\nu} \nu$ Beyond MFV:

$$(\tan \beta)^4 (\delta_{RRt_s}^d \delta_{RRt_d}^d)^2$$ [Isidori, Paradisi '06]

$B \rightarrow \mu^+ \mu^-$: improved bound

$K \rightarrow \pi \nu \bar{\nu}$ decouples slower

Complementary Information
Conclusion

Where to look for New Physics?

- LU breaking @ % in $R_{K}^{e/\mu}$ from SUSY LFV effects

- LU breaking @ % in $R_{K}^{e/\mu}$ implies $M_{H^{\pm}} < 1\text{TeV}$ and it can be compatible with the $(g-2)_{\mu}$ anomaly

- LU breaking @ % in $R_{K}^{e/\mu}$ $\Rightarrow$ $\text{BR}(\tau \rightarrow e\gamma) > 10^{-9}$

- The relevant SUSY parameter space for $R_{K}^{e/\mu}$ @ % is allowed by the constraints of rare LFV decays, $B$-physics observables and Dark Matter

$\Downarrow$

$R_{K}^{e/\mu}$ offers a great chance to probe SUSY LFV.
Where to look for New Physics?

- $K \rightarrow \pi \nu \bar{\nu}$ is a golden channel where to look for NP because:
  - $K \rightarrow \pi \nu \bar{\nu}$ is predicted with high resolution in the SM
  - $K \rightarrow \pi \nu \bar{\nu}$ has a high sensitivity to NP effects of many theories as SUSY, LHT, Z’ models.....

\[ \downarrow \]

Kaon Physics will play a major role in the LHC ERA to unveil and to understand NP