

Precision tests of the Standard Model with kaon decays

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Outline

- Introduction
- V_{ud} from nuclear β decays
Hardy-Towner, Marciano-Sirlin
- V_{us} , V_{us}/V_{ud} from kaon (and pion) decays
FlaviaNet working group
- CKM unitarity and G_F universality

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CKM unitarity: G_F universality

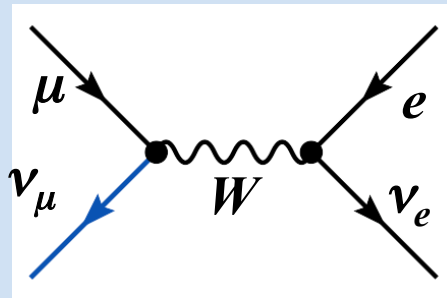
SM coupling to W : $\frac{g}{\sqrt{2}} W_\alpha^+ \left(\bar{U}_L V_{CKM} \gamma^\alpha D_L + \bar{e}_L \gamma^\alpha \nu_{eL} + \bar{\mu}_L \gamma^\alpha \nu_{\mu L} + \bar{\tau}_L \gamma^\alpha \nu_{\tau L} \right)$

one gauge coupling *unitary matrix*

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1$$

negligible **Most precise test of CKM unitarity from 1st row**

μ decay

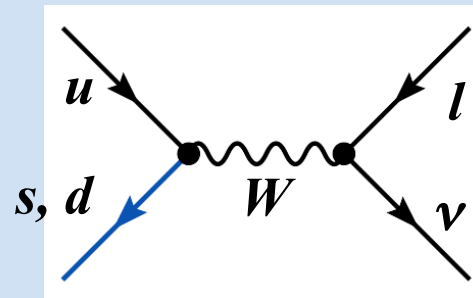


$$(g_\mu g_e)^2 / M_W^4 = G_F^2$$

?

=

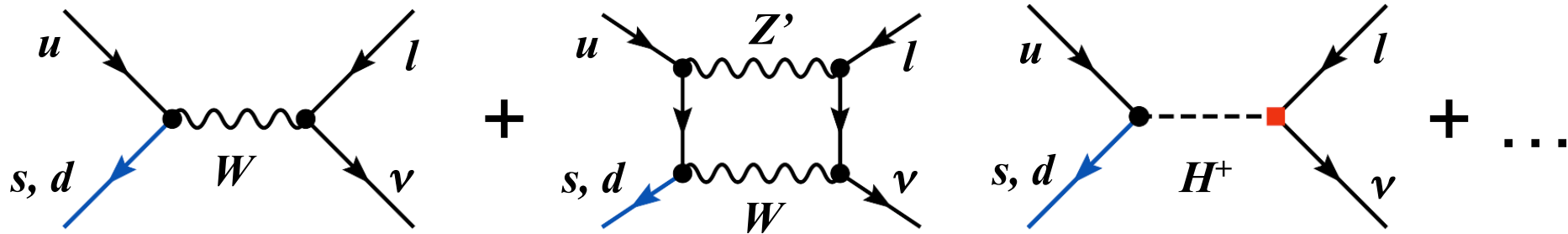
K, π and nuclear β decays



$$(g_q g_l)^2 (|V_{ud}|^2 + |V_{us}|^2) / M_W^4 = G_{CKM}^2$$

CKM unitarity: G_F universality

New Physics extensions of the SM can break gauge universality



$$SM + NP \propto G_F^2 |V_{uq}|^2 \left(1 + a(M_W/M_X)^2\right)^2 \quad \text{naively } a_{tree} \approx 1, a_{loop} \approx g^2$$

μ lifetime:

$$G_F = 1.166371(6) \times 10^{-5} \text{ GeV}^{-2}$$

τ decay:

$$G_\tau = 1.1678(26)$$

ew precision test:

$$G_{ew} = 1.1655(12)$$

V_{us} at 0.5%:

$$G_{CKM} = 1.16xx(4)$$

$$\Rightarrow M_{tree} \approx 10 \text{ TeV}, M_{loop} \approx 1 \text{ TeV}$$

V_{ud} from nuclear β decays

Experimental survey + radiative corrections (nucleus-dep.)

Towner, Hardy arXiv:0812.1202

Radiative corrections (nucleus-indep.)

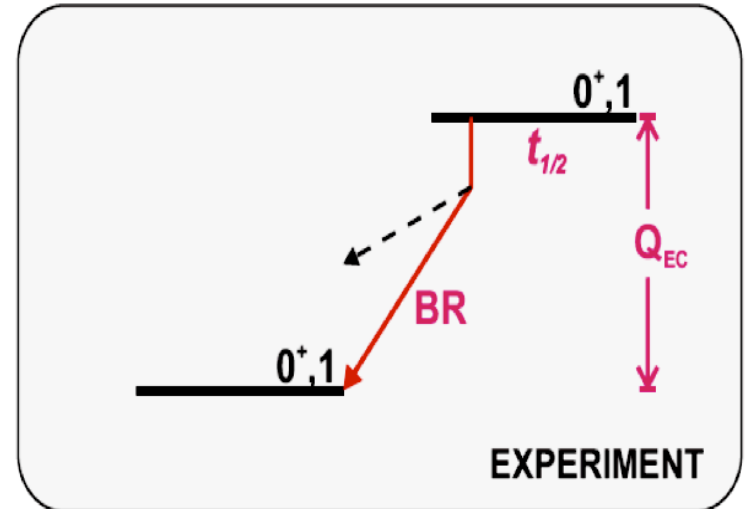
Marciano, Sirlin PRL 96 (2006)

V_{ud} from Fermi transitions ($0^+ \rightarrow 0^+$)

$$V_{ud}^2 = \frac{K}{2G_F^2 \mathcal{F}t(1 + \Delta_R)}$$

$$\mathcal{F}t = ft(1 + \delta'_R)(1 - (\delta_C - \delta_{NS}))$$

nucleus independent



- Measured on 13 nuclei:

$$t = t_{1/2}/BR \text{ partial half life} \quad f = \text{statistical rate function } f(Z, Q_{EC})$$

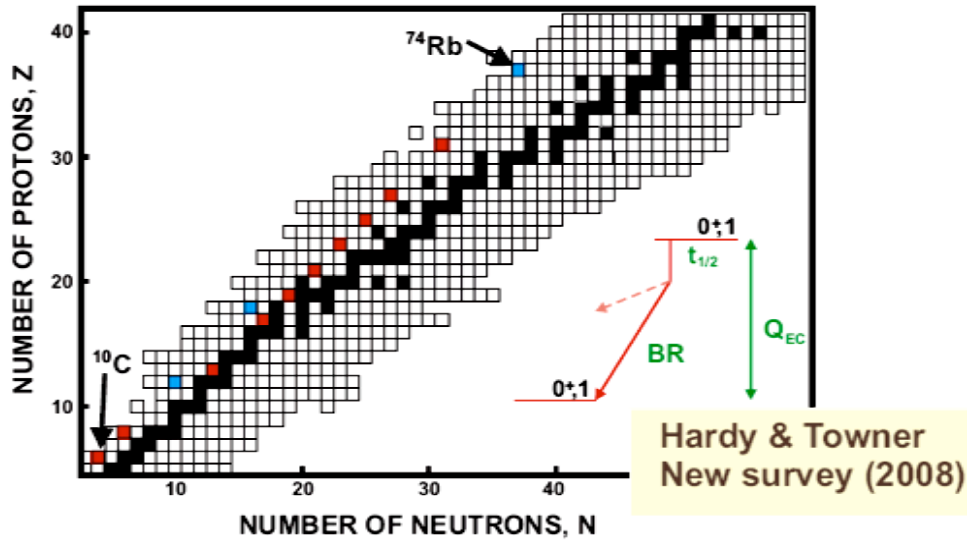
- Radiative corrections:

$$\Delta_R = 2.361(38)\%, \text{ nucleus-independent (Marciano-Sirlin 2006)}$$

$$\delta'_R, \delta_{NS} \text{ nucleus-dependent}$$

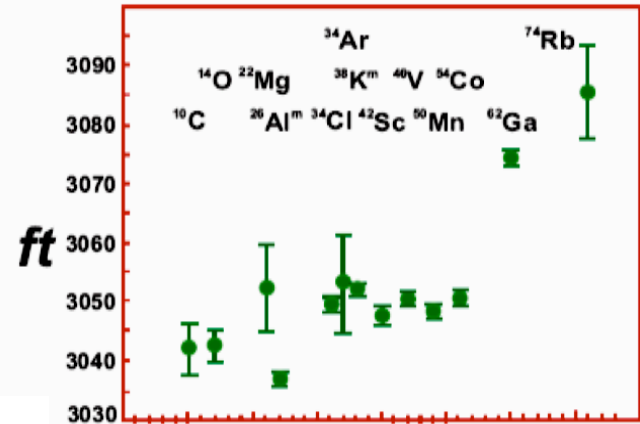
- δ_C nucleus dependent isospin breaking

World data for $0^+ \rightarrow 0^+$ decay, 2008

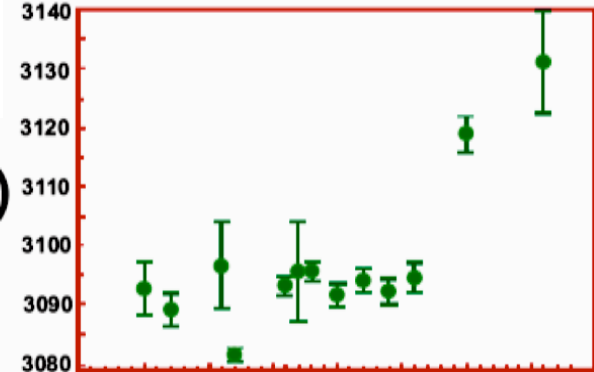


10 cases with ft -values measured to **~0.1% precision**; 3 more cases with **<0.3% precision**

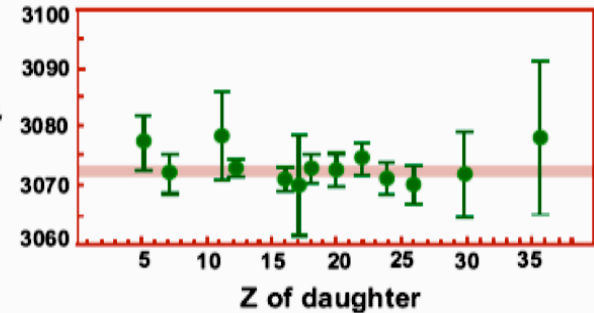
$$\mathcal{F}t = ft(1 + \delta'_R)(1 - (\delta_C - \delta_{NS}))$$



$(1 + \delta'_R)$



$\mathcal{F}t$



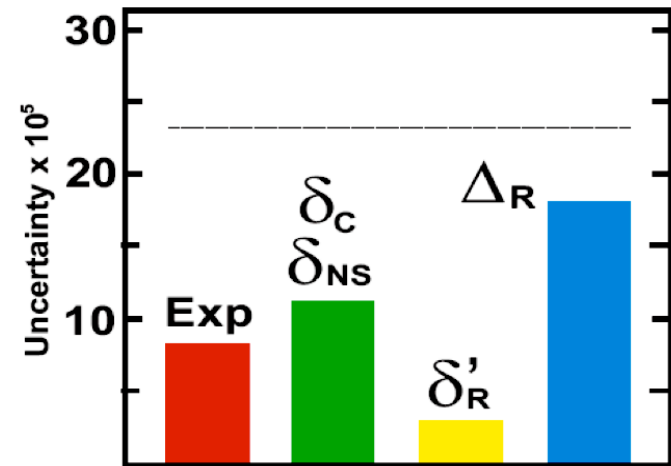
V_{ud} from $0^+ \rightarrow 0^+$ decay in 2008

$$V_{ud} = 0.97424 \pm 0.00022$$

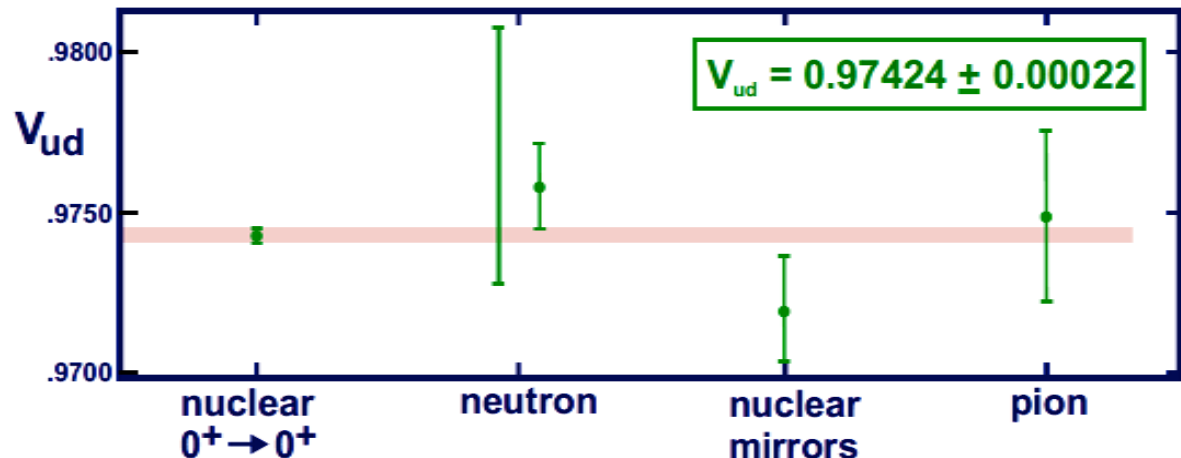
Hardy-Towner, CIPANP '09

- Nuclear-structure dependent corrections, δ_C and δ_{NS} , can be tested by experiment
- Add new transistions

error budget



nuclear $0^+ \rightarrow 0^+$
still the most
precise
(by a factor ~10)



V_{us} and V_{us}/V_{ud} from kaon decays

by FlaviaNet working group

- “official” analysis: arXiv:0801.1817
- partial update presented today

V_{us} from K_{l3} decay rates

$$\Gamma(K_{l3}(\gamma)) = \frac{C_K^2 G_F^2 M_K^5}{192\pi^3} S_{EW} |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 I_{Kl}(\lambda) (1 + 2\Delta_K^{SU(2)} + 2\Delta_{Kl}^{EM})$$

with $K = K^+, K^0$; $l = e, \mu$ and $C_K^2 = 1/2$ for K^+ , 1 for K^0

Inputs from theory:

- S_{EW} Universal short distance EW correction (1.0232)
- $f_+^{K^0\pi^-}(0)$ Hadronic matrix element at zero momentum transfer ($t=0$)
- $\Delta_K^{SU(2)}$ Form factor correction for SU(2) breaking
- Δ_{Kl}^{EM} Long distance EM effects

Inputs from experiment:

- $\Gamma(K_{l3}(\gamma))$ **Branching ratios** with well determined treatment of radiative decays; **lifetimes**
- $I_{Kl}(\lambda)$ Phase space integral: λ s parameterize form factor dependence on t :
 - K_{e3} : **only λ_+ (or $\lambda_+' \lambda_+''$)**
 - $K_{\mu 3}$: **need λ_+ and λ_0**

V_{us} from kaon decays: recent history

→ **2002**
(2004 PDG)

Old KI3 data give $1 - V_{ud}^2 - V_{us}^2 = 0.0035(15)$

A 2.3σ hint of unitarity violation?

2003

BNL 865 measures higher $BR(K^+e3) = 5.13(10)\%$

V_{us} consistent with unitarity

**2004-
present**

Many new measurements from KTeV, KLOE, ISTRA+, NA48

- **BRs, lifetimes, form-factor slopes**
- **Much higher statistics** than older measurements
- Importance of **radiative corrections**
- Proper reporting of correlations between measurements

**2008-
beyond**

Value of V_{us} used in precision tests of Standard Model

Extraction of V_{us} from K_{l3} data

1) BR and lifetime averages: K_L and K_S

2) BR and lifetime averages: K^\pm

3) Form factor slopes and phase space

4) Extraction of $|V_{us}| f_+(0)$ (and V_{us})

Fit to K_L BR and lifetime measurements

21 input measurements:

5 **KTeV** ratios

NA48 BR(K_{e3} /2 track)

4 **KLOE** BRs

KLOE, **NA48** BR($\pi^+\pi^-/K_{l3}$)

KLOE, **NA48** BR($\gamma\gamma/3\pi^0$)

PDG ETAFIT BR($2\pi^0/\pi^+\pi^-$)

KLOE τ_L from $3\pi^0$

Vosburgh '72 τ_L

KTeV BR($\pi^+\pi^-\gamma/\pi^+\pi^-(\gamma)$)

E731, **2KTeV** BR($\pi^+\pi^-\gamma_{DE}/\pi^+\pi^-\gamma$)

*all published
results*

1 constraint: $\Sigma BR=1$

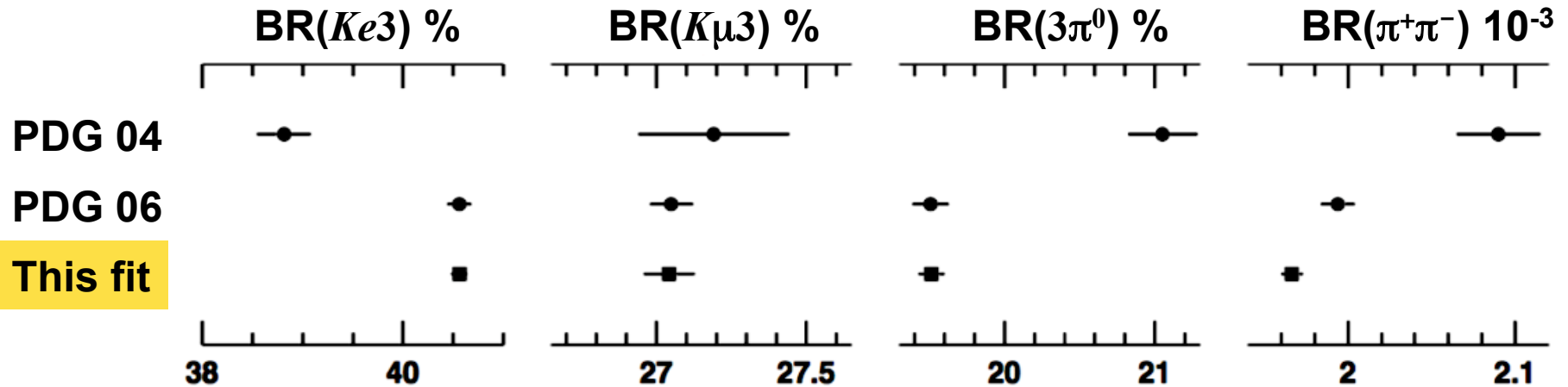
FlaviaNet fit

Parameter	Value	S
$Ke3$	0.4056(9)	1.3
$K\mu3$	0.2704(10)	1.5
$3\pi^0$	0.1952(9)	1.2
$\pi^+\pi^-\pi^0$	0.1254(6)	1.1
$\pi^+\pi^-(\gamma_{IB})$	$1.967(7)\times 10^{-3}$	1.6
$\pi^+\pi^-\gamma$	$4.15(9)\times 10^{-5}$	1.3
$\pi^+\pi^-\gamma_{DE}$	$2.84(8)\times 10^{-5}$	1.1
$2\pi^0$	$8.65(4)\times 10^{-4}$	1.4
$\gamma\gamma$	$5.47(4)\times 10^{-4}$	1.1
τ_L	51.16(21) ns	1.1

$\chi^2/ndf = 19.8/12$ (Prob = 7.1%)

compare PDG'08: 35.7/17 (0.5%)

Evolution of K_L BRs



Differences between FlaviaNet and PDG are minor

Since PDG '04:

- Proper radiative corrections, especially for $Ke3$
- Exclusion of old measurements

from K_L BRs:	PDG '04	This fit	
$K_{\mu3}/K_{e3}$	0.701(9)	0.6666(29)	ok with lepton universality
$ \eta_{+-} \times 10^3$	2.284(14)	2.223(6)	$>3\sigma$ difference

BR($K_S \rightarrow \pi e \nu$) and K_S lifetime

BRs from KLOE tagged K_S beam, 1.2×10^8 events

KLOE 2006

$$\text{BR}(K_S \rightarrow \pi e \nu) / \text{BR}(K_S \rightarrow \pi^+ \pi^-) = 10.19(13) \times 10^{-4} \quad \sigma \sim 1.3\%$$

$$\text{BR}(K_S \rightarrow \pi^+ \pi^-) / \text{BR}(K_S \rightarrow \pi^0 \pi^0) = 2.2459(54)$$

Add NA48 result (fit K_S and K_L components in decay-length distribution)

NA48 2007

$$\Gamma(K_S \rightarrow \pi e \nu) / \Gamma(K_L \rightarrow \pi e \nu) = 0.993(26)(22) \quad \sigma \sim 3.4\%$$

we obtain $\text{BR}(K_S \rightarrow \pi e \nu) = 7.05(8) \times 10^{-4}$

PDG

$$\tau_S = 0.08958(5) \text{ ns}$$

From fit to CP parameters, does not assume CPT
Dominated by **NA48 '02** and **KTeV '03** values

Extraction of V_{us} from K_{l3} data

1) BR and lifetime averages: K_L and K_S

2) BR and lifetime averages: K^\pm

3) Form factor slopes and phase space

4) Extraction of $|V_{us}| f_+(0)$ (and V_{us})

Recent results on K^\pm BRs

BNL 865
2003

$$\text{BR}(K^+ \rightarrow \pi^0_{\text{D}} e^+ \nu) / \text{BR}(K^+ \rightarrow \pi^0_{\text{D}} X^+) = 0.1962(8)(35)$$

the start of the V_{us} revolution

NA48/2 2007

$$\text{BR}(K^\pm_{e3}) / \text{BR}(\pi^\pm \pi^0) = 0.2470(9)(4)$$

$$\text{BR}(K^\pm_{\mu3}) / \text{BR}(\pi^\pm \pi^0) = 0.1637(6)(3)$$

$K^+ K^-$ simultaneous beams

ISTRA+ 2007

$$\text{BR}(K^-_{e3}) / \text{BR}(\pi^- \pi^0) = 0.2449(4)(14)$$

still preliminary, not used in the updated fit

KLOE
2004-2008

$$\text{BR}(K^+ \rightarrow \mu^+ \nu) = 63.66(9)(15)\%$$

$$\text{BR}(K^+ \rightarrow \pi^+ \pi^0) = 20.65(5)(8)\%$$

$$\text{BR}(K^\pm_{e3}) = 4.965(53)(38)\%$$

$$\text{BR}(K^\pm_{\mu3}) = 3.233(29)(26)\%$$

$$\text{BR}(K^+ \rightarrow \pi^+ \pi^0 \pi^0) = 1.763(13)(22)\%$$

absolute BR measurements with tagged K^\pm beams

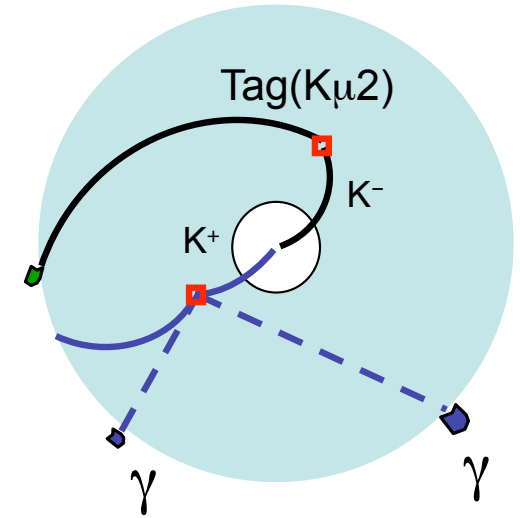
KLOE: measurement of K^\pm lifetime

Poor consistency (CL=0.2%) between old meas., need confirmation

Tag events with $K^\pm \rightarrow \mu^\pm \nu$ decay

Identify a kaon decay vertex on the opposite side

- **1st method: obtain τ_\pm from the K decay length**
take into account the energy loss: $\tau_K = \sum_i \Delta L_i / (\beta_i \gamma_i c)$
- **2nd method: obtain τ_\pm from the K decay time**
use photons from $K^\pm \rightarrow \pi^\pm \pi^0$ decay



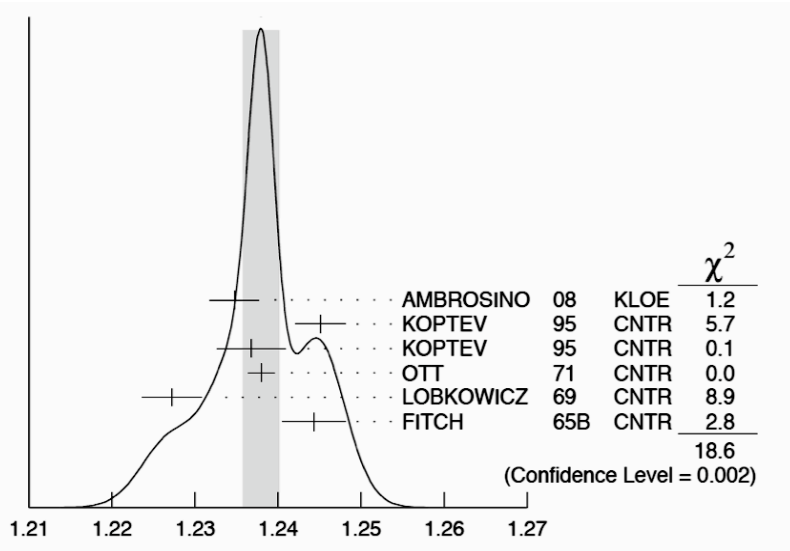
**Compatible results,
average with correlations:**

$$\tau_\pm = 12.347(30) \text{ ns}$$

$$\tau_+/\tau_- = 1.004(4)$$

**JHEP (01)
2008**

K^\pm lifetime: older data



PDG
2008 average

$\tau_{\pm} = 12.379(21)$ ns
 $S = 1.9$

Serious doubts about accuracy of error estimates for older experiments

Ott & Pritchard '71

Statistical errors incorrect by x2.3 ($\sim 0.56/\sqrt{N}$)
Uncorrelated combination of systematic errors

Koptev et al. '95

$\sim 2\sigma$ difference between results for same experiment, different stoppers → **use average with scaled errors**

Lobkowicz et al. '69

Focused on τ_+/τ_- , precision absolute scale not emphasized, fit not compatible with exponential behaviour → **not used in our fit**


Fit to K^\pm BR and lifetime measurements

Compared to PDG, our fit:

We exclude all six Chiang '72 measurements

- No radiative corrections
- 6 BRs constrained by $\Sigma\text{BR} = 1$, correlations unavailable

After a careful review, we exclude also

- 2 $\text{Ke}_3/2$ -body (Eckshtruth '68, Cester '66)
- 2 $\text{K}\mu_3/\text{Ke}_3$ (Botterill '68, Heintze '77)
- $\Gamma(\text{K}^+ \rightarrow \pi^+\pi^+\pi^-)$ from Ford '67  *new measurement needed!*

Lifetime measurements

- No Lobkowitz '69
- Koptev '95 measurements combined with scaled errors

Compared to previous Flavianet fit (KAON 07):

- $\text{Ke}_3/\pi\pi^0$ from ISTRA+ excluded (never published)
- new KLOE $\text{BR}(\text{K}^+ \rightarrow \pi^+\pi^0)$, important as normalization channel

Fit results to K^\pm BR and lifetime

17 input measurements:

3 older τ values in PDG

1 KLOE τ

KLOE BR($\mu\nu$), BR($\pi\pi^0$)

KLOE $Ke3$, $K\mu3$ BRs

NA48/2 $Ke3/\pi\pi^0$, $K\mu3/\pi\pi^0$

E865 $Ke3/K_{dal}$

KEK246 $K\mu3/Ke3$

3 old $\pi\pi^0/\mu\nu$

1 old + 1 KLOE results on 3π

*all published
results*

1 constraint: $\Sigma BR=1$

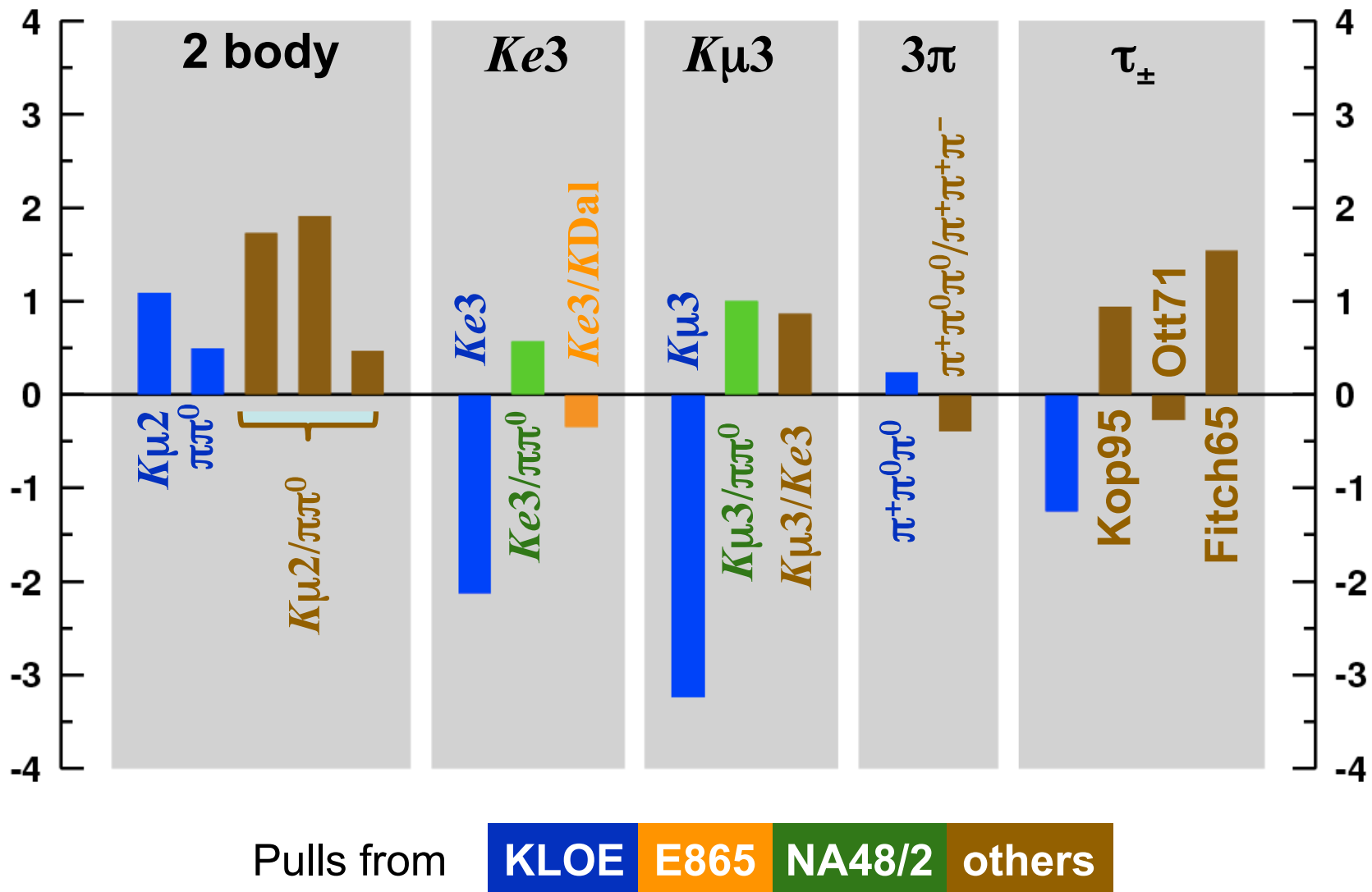
FlaviaNet fit

Parameter	Value	S
BR($\mu\nu$)	63.47(18)%	1.3
BR($\pi\pi^0$)	20.61(8)%	1.1
BR($\pi\pi\pi$)	5.73(16)%	1.2
BR($Ke3$)	5.078(31)%	1.3
BR($K\mu3$)	3.359(32)%	1.9
BR($\pi\pi^0\pi^0$)	1.757(24)%	1.0
τ_\pm	12.384(15) ns	1.2

$\chi^2/ndf = 25.8/11$ (Prob = 0.7%)

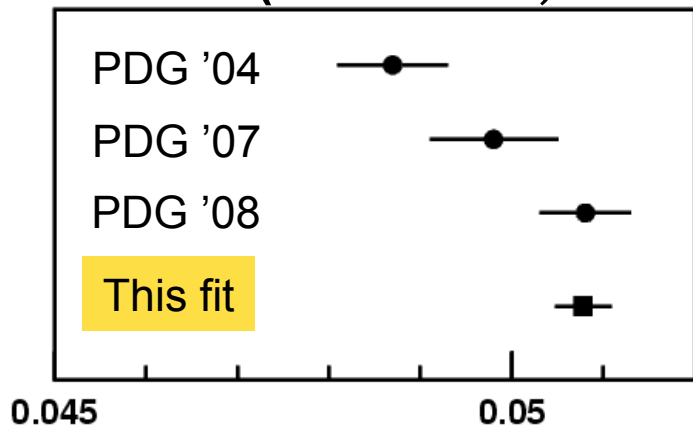
compare PDG'08: 52/24 (0.1%)

K^\pm BR fit vs. data

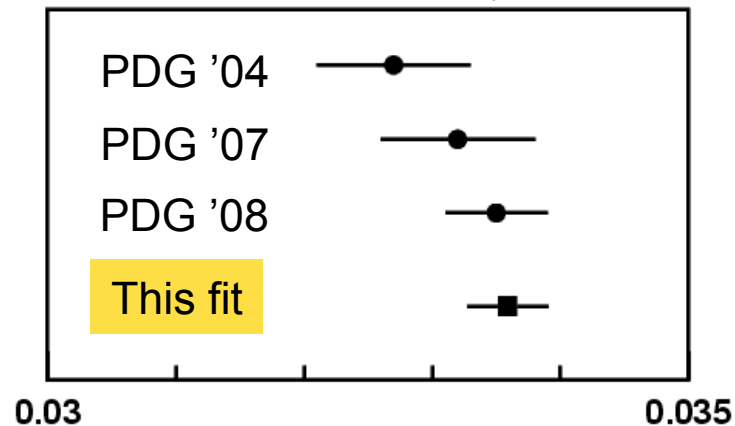


Evolution of K^\pm BRs

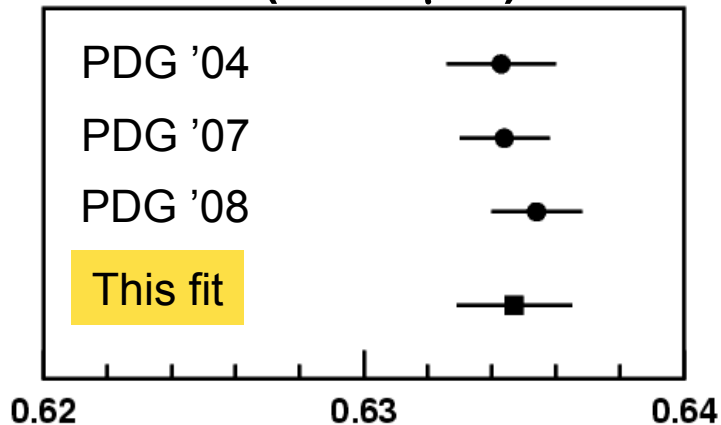
BR($K^\pm \rightarrow \pi^0 e^\pm \nu$)



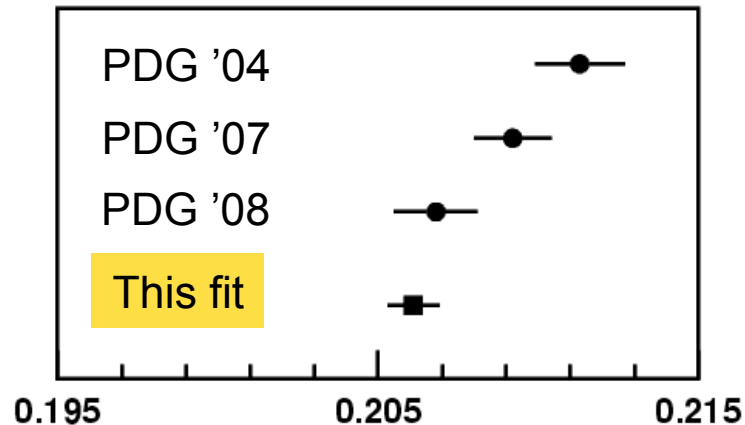
BR($K^\pm \rightarrow \pi^0 \mu^\pm \nu$)



BR($K^\pm \rightarrow \mu^\pm \nu$)



BR($K^\pm \rightarrow \pi^\pm \pi^0$)



Extraction of V_{us} from K_{l3} data

1) BR and lifetime averages: K_L and K_S

2) BR and lifetime averages: K^\pm

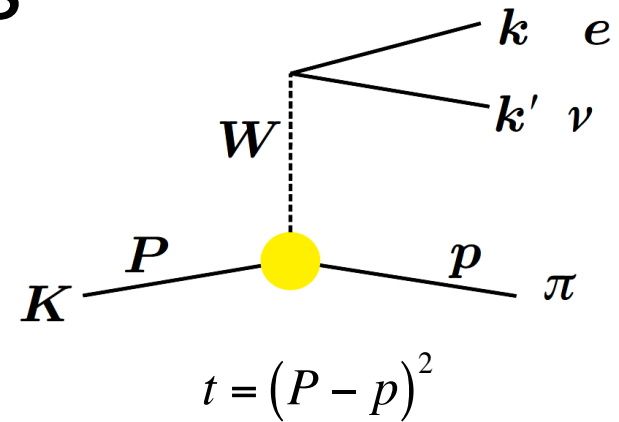
3) Form factor slopes and phase space

4) Extraction of $|V_{us}| f_+(0)$ (and V_{us})

K_{l3} form-factors

Hadronic matrix element:

$$\langle \pi | J_\alpha^V | K \rangle = f_+(0) \times \left(\tilde{f}_+(t)(P + p)_\alpha + \tilde{f}_-(t)(P - p)_\alpha \right)$$



$f_-(t)$ term only important for $K_{\mu 3} \rightarrow$ scalar ff

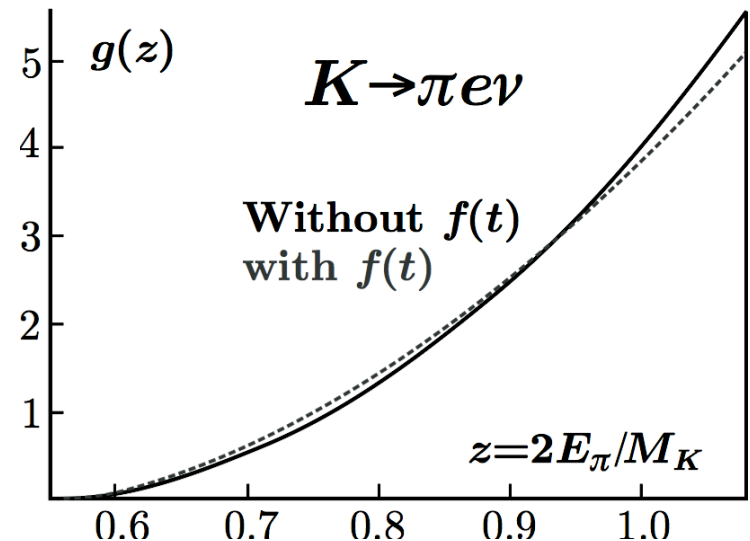
$$\tilde{f}_0(t) = \tilde{f}_+(t) + \tilde{f}_-(t) \frac{t}{m_K^2 - m_\pi^2}$$

To evaluate phase space integrals (for V_{us}) need to parameterize (and measure) ff -dependence on t

Quadratic:
$$\tilde{f}_{+,0}(t) = 1 + \lambda'_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right) + \lambda''_{+,0} \left(\frac{t}{m_{\pi^+}^2} \right)^2$$

Fits exhibit high correlation for λ', λ''

Polar:
$$\tilde{f}_{+,0}(t) = \frac{M_{V,S}^2}{M_{V,S}^2 - t} \begin{cases} \lambda' = (m_{\pi^+} / M)^2 \\ \lambda'' = 2(\lambda')^2 \end{cases}$$



Current data on $Ke3$ form-factor slopes

	$\lambda'_+ \times 10^3$	$\lambda''_+ \times 10^3$	analysis
ISTRA+ PLB 589 (2004)	24.9 ± 1.7	1.9 ± 0.9	$0.9 \times 10^6 K^-e3$ $\rho(E_e, E_\pi)$
KTeV PRD 70 (2004)	21.7 ± 2.0	2.9 ± 0.8	$1.9 \times 10^6 K_L e3$ t_\perp^π
NA48 PLB 604 (2004)	28.0 ± 2.4	0.4 ± 0.9	$5.6 \times 10^6 K_L e3$ $\rho(t_{\text{low}}, t_{\text{high}})$
KLOE PLB 632 (2006)	25.5 ± 1.8	1.4 ± 0.8	$2.0 \times 10^6 K_L e3$ t from K_S

* ISTRA+ results are rescaled by $(m_{\pi^+}/m_{\pi^0})^2$

$Ke3$ slopes: quadratic fits

Slopes from

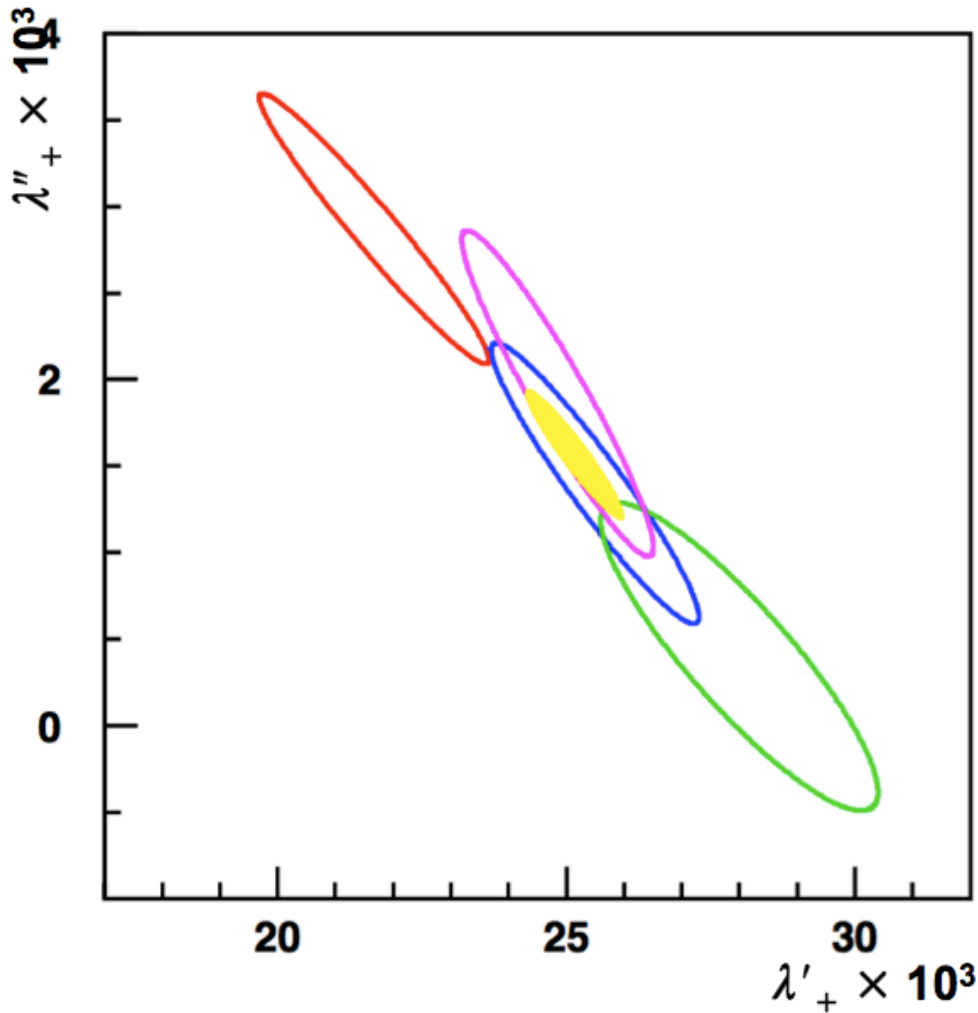
KTeV

KLOE

ISTRA+

NA48

FlaviaNet fit



Slope parameters $\times 10^3$

$$\lambda'_+ = 25.15 \pm 0.87$$

$$\lambda''_+ = 1.57 \pm 0.38$$

$$\rho(\lambda'_+, \lambda''_+) = -0.941$$

$$\chi^2/\text{ndf} = 5.3/6 \quad (51\%)$$

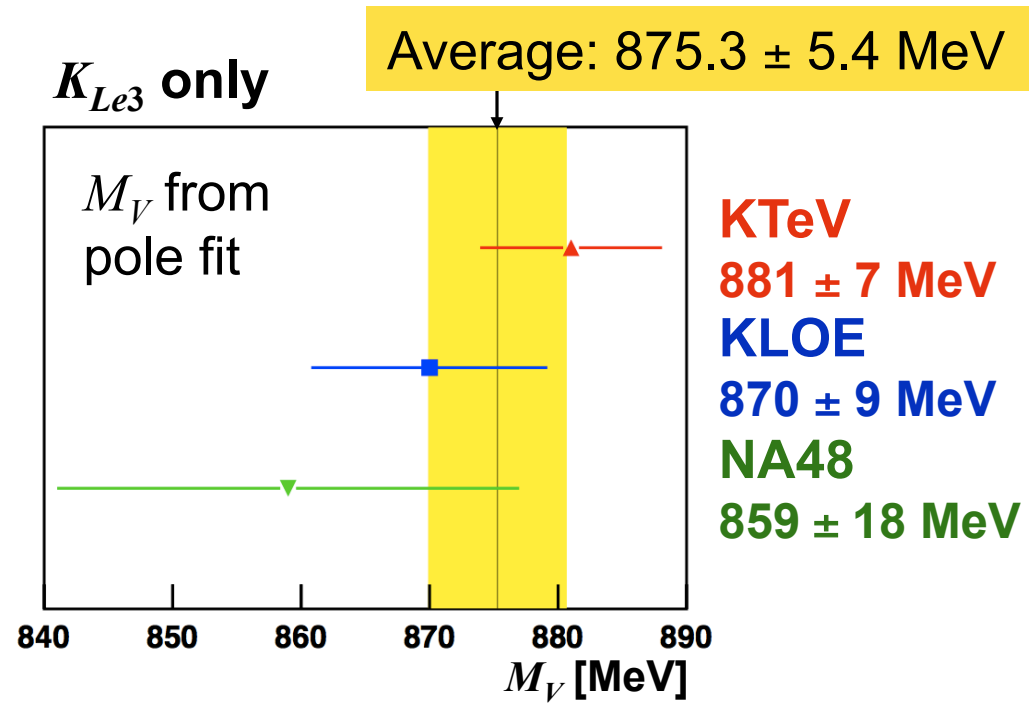
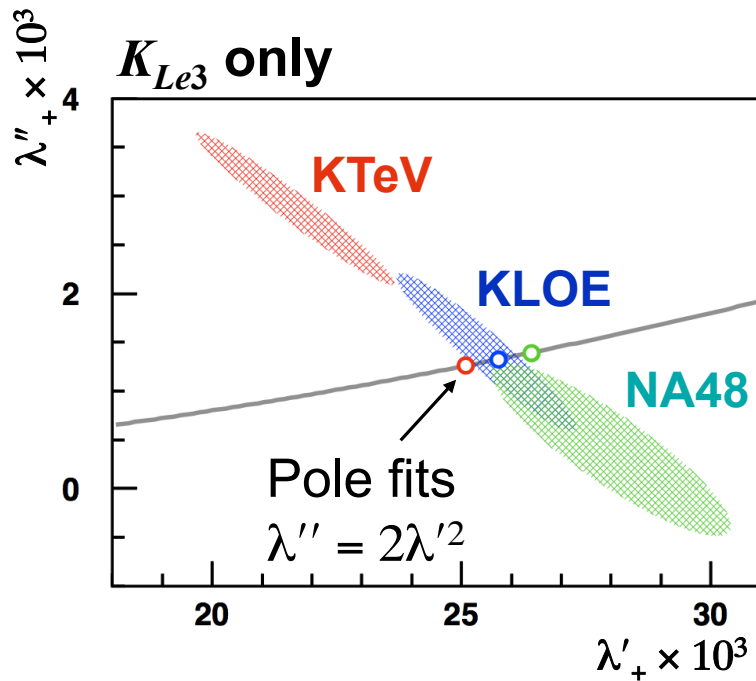
Excellent compatibility

Significance of $\lambda''_+ \sim 4\sigma$

$$I(K^0e3) = 0.15463(21)$$

$$I(K^+e3) = 0.15900(22)$$

K_{e3} slopes: Quadratic vs. pole fits



K_{Le3} data	$P(\chi^2)$ fit Quad	K_{Le3} integral Quad	$P(\chi^2)$ fit Pole	K_{Le3} integral Pole	Difference
KTeV	54%	0.15378(51)	43%	0.15449(25)	+0.46%
KLOE	92%	0.15472(42)	92%	0.15489(33)	+0.11%
K_{Le3} avg		0.15463(21)		0.15481(18)	+0.12%

Current data on $K\mu 3$ form-factor slopes

	$\lambda'_+ \times 10^3$	$\lambda''_+ \times 10^3$	$\lambda_0 \times 10^3$	analysis
ISTRA+ PLB 581 (2004)	23.0 ± 6.4	2.3 ± 2.3	17.1 ± 2.2	$0.9 \times 10^6 K^- \mu 3$ $\rho(E_\mu, E_\pi)$
KTeV PRD 70 (2004)	17.0 ± 3.7	4.4 ± 1.5	12.8 ± 1.8	$1.5 \times 10^6 K_L \mu 3$ $\rho(t^\mu_\perp, M_{\pi\mu})$
NA48 PLB 647 (2007)	20.5 ± 3.3	2.6 ± 1.4	9.5 ± 1.4	$2.3 \times 10^6 K_L \mu 3$ $\rho(E_\mu, E_\pi)_{\text{low}}$
KLOE JHEP 12 (2007)	25.6 ± 1.8	1.5 ± 0.8	15.4 ± 2.2	$1.8 \times 10^6 K_L \mu 3$ E_ν spectrum + $K_L e 3$

* KLOE quotes $K e 3 + K \mu 3$ average

* ISTRA+ don't quote systematic error for quadratic fit: *we exclude from global fit*

Fit to $Kl3$ form factor slopes

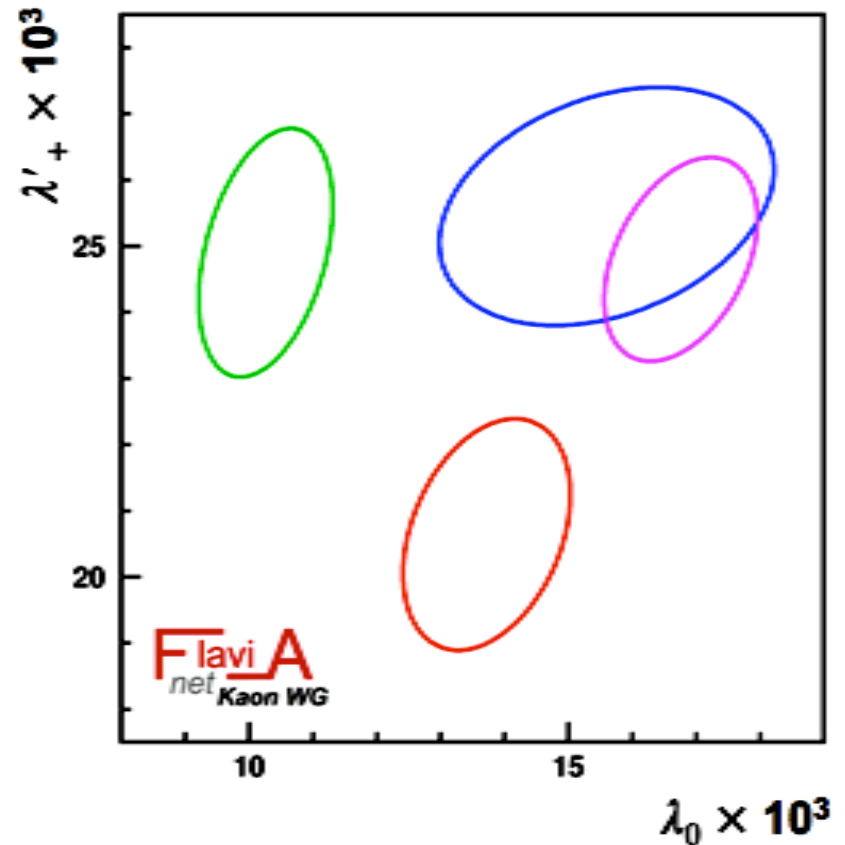
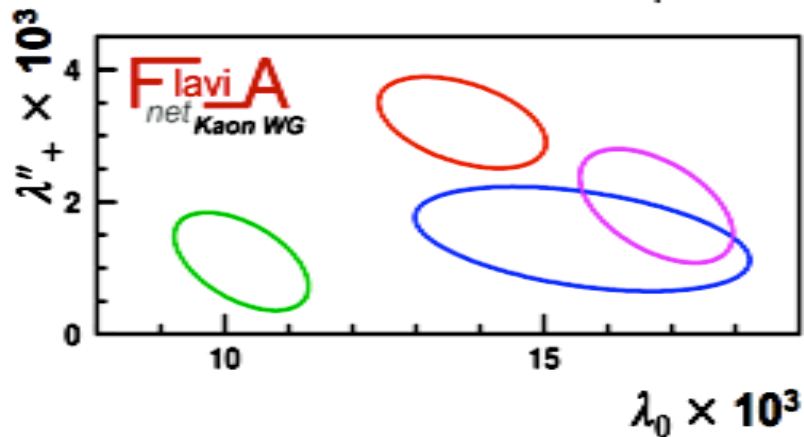
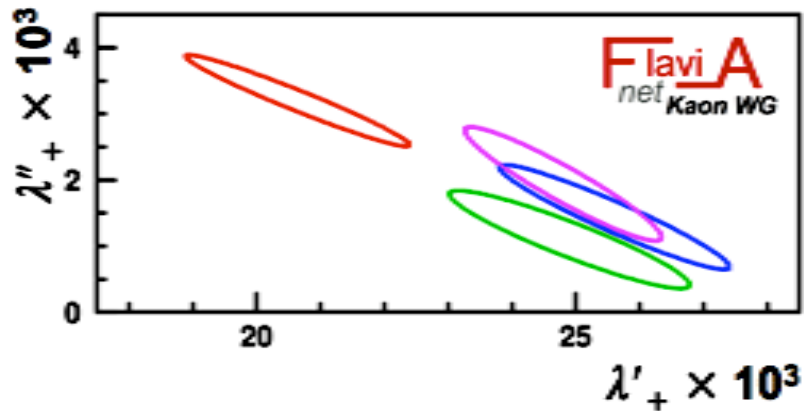
$e3-\mu3$ averages from

KTeV

KLOE

ISTRA+

NA48



$Kl3$ form factor slopes: fit results

Compatibility poor, inconsistency parameterized by scale factors for fit results

Slopes $\times 10^3$:

$$\lambda'_+ = 24.54 \pm 0.87 \quad \mathbf{S = 1.1}$$

$$\lambda''_+ = 1.81 \pm 0.41 \quad \mathbf{S = 1.2}$$

$$\lambda_0 = 11.67 \pm 1.38 \quad \mathbf{S = 1.9}$$

$$\chi^2/\text{ndf} = 29/8 \quad (3 \times 10^{-4})$$

Phase space integrals

$$I(K^0 e3) \quad 0.15449(20)$$

$$I(K^+ e3) \quad 0.15885(21)$$

$$I(K^0 \mu3) \quad 0.10171(32)$$

$$I(K^+ \mu3) \quad 0.10467(33)$$

Correlation coefficients:

	λ'_+	λ_0
λ''_+	-0.94	-0.52
λ'_+		+0.44

Without NA48 $K\mu3$:

- $\chi^2/\text{ndf} = 9.5/7$ (22%)
- $I(K\mu3)$: +0.7%
- $e3\text{-}\mu3$ average for $|V_{us}|f_+(0)$: -0.09%

Extraction of V_{us} from K_{l3} data


- 1) BR and lifetime averages: K_L and K_S
- 2) BR and lifetime averages: K^\pm
- 3) Form factor slopes and phase space
- 4) Extraction of $|V_{us}|f_+(0)$ (and V_{us})**

$SU(2)$ correction

$$\Delta^{SU(2)} = \frac{f_+(0)^{K^+\pi^0}}{f_+(0)^{K^0\pi^-}} - 1$$

Strong isospin breaking
Quark mass differences
 $\eta - \pi^0$ mixing in $K^+\pi^0$ channel

+2.36(22)% Cirigliano '07 (updating Cirigliano et al. '02)
 $O(p^4)$ chiral correction
Quark mass-ratio analysis based on Leutwyler '96

 **+2.9(4)%** **Kastner & Neufeld '08**
Quark mass-ratio from Ananthanarayan, Moussallam '04

+2.31% Bijnens, Ghorbani '07
 $O(p^6)$ chiral correction

Long-distance EM corrections

Δ^{EM} = mode-dependent integrated correction to the decay rate from Dalitz plot modifications from EM effects; values depend on acceptance for events with additional real photon(s)

All recent measurements assumed fully inclusive

Δ^{EM}	Cirigliano et al. '04 ChPT	Andre '04 Had. model	Cirigliano et al. '08 ChPT
K^0_{e3}	+0.52(10)%	+0.65(15)%	+0.50(11)%
K^+_{e3}	+0.03(10)%		+0.05(13)%
$K^0_{\mu3}$		+0.95(15)%	+0.70(11)%
$K^+_{\mu3}$			+0.01(13)%

Cirigliano et al. '08 perform comprehensive analysis at fixed order e^2p^2

Fully inclusive prescription to treat real photon emission

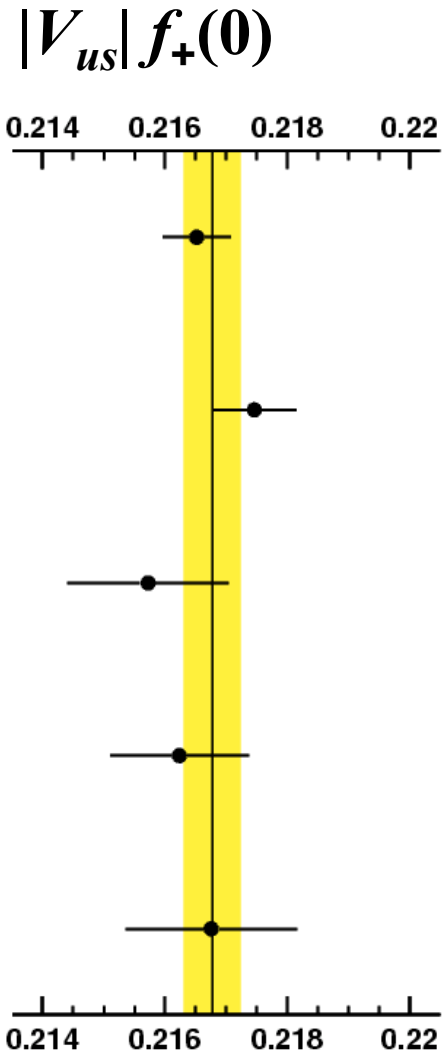
Updated values of LECs for structure-dependent terms

Correlations evaluated, e.g.: $\rho(K^0e3, K^0\mu3) = +0.69$, $\rho(K^0e3, K^+e3) = +0.08$

$|V_{us}|f_+(0)$ from K_{l3} data

Approx. contrib. to % err from:

% err **BR** τ Δ **Int**



$K_L e3$	0.21652(56)	0.25	0.11	0.20	0.11	0.10
$K_L \mu3$	0.21746(69)	0.32	0.17	0.19	0.11	0.15
$K_S e3$	0.21572(132)	0.61	0.60	0.03	0.11	0.10
$K^\pm e3$	0.21624(113)	0.52	0.31	0.06	0.41	0.09
$K^\pm \mu3$	0.21676(141)	0.65	0.48	0.06	0.41	0.15

Average: $|V_{us}|f_+(0) = 0.21660(47)$ $\chi^2/\text{ndf} = 3.03/4$ (55%)

$|V_{us}|f_+(0): K^\pm$ vs $K_{L,S}$

Fit 5 modes with separate values of $|V_{us}|f_+(0)$ for K^\pm and $K_{L,S}$ modes

- Using results of overall fit to form-factor slopes
- With $SU(2)$ corrections for K^\pm modes $[\Delta^{SU(2)}]_{\text{theory}} = 2.9(4)\%$

K^\pm modes

$$|V_{us}|f_+(0) = 0.2163(11)$$



$K_{L,S}$ modes

$$|V_{us}|f_+(0) = 0.2167(5)$$

0.3 σ difference

$$\chi^2/\text{ndf} = 2.9/3 \text{ (41\%)} \quad \rho = 0.04$$

When fit performed without $SU(2)$ corrections for K^\pm modes, obtain an experimental value for $\Delta^{SU(2)}$

K^\pm modes, no $SU(2)$

$$|V_{us}|f_+(0) = 0.2226(7)$$

$$\Delta^{SU(2)}_{\text{exp}} = 2.7(4)\%$$

$K_{\ell 3}$ data and lepton universality

For each state of kaon charge, we evaluate:

$$r_{\mu e} = \frac{(R_{\mu e})_{\text{obs}}}{(R_{\mu e})_{\text{SM}}} = \frac{\Gamma_{\mu 3}}{\Gamma_{e 3}} \cdot \frac{I_{e 3} (1 + \delta_{e 3})}{I_{\mu 3} (1 + \delta_{\mu 3})} = \frac{[|V_{us}| f_+(0)]_{\mu 3, \text{obs}}^2}{[|V_{us}| f_+(0)]_{e 3, \text{obs}}^2} = \frac{g_{\mu}^2}{g_e^2}$$

Modes	2004 BRs*	This fit
K^{\pm}	1.016(12)	1.005(9)
$K_{L,S}$	1.056(15)	1.009(6)

*Assuming current values for form-factor parameters and Δ^{EM} ; K_S not included

Average $K/3$
 $r_{\mu e} = 1.008(5)$

Compare with:

$$\pi \rightarrow \ell \nu \quad (r_{\mu e}) = 1.0042(33)$$

Ramsey-Musolf, Su, Tulin '07

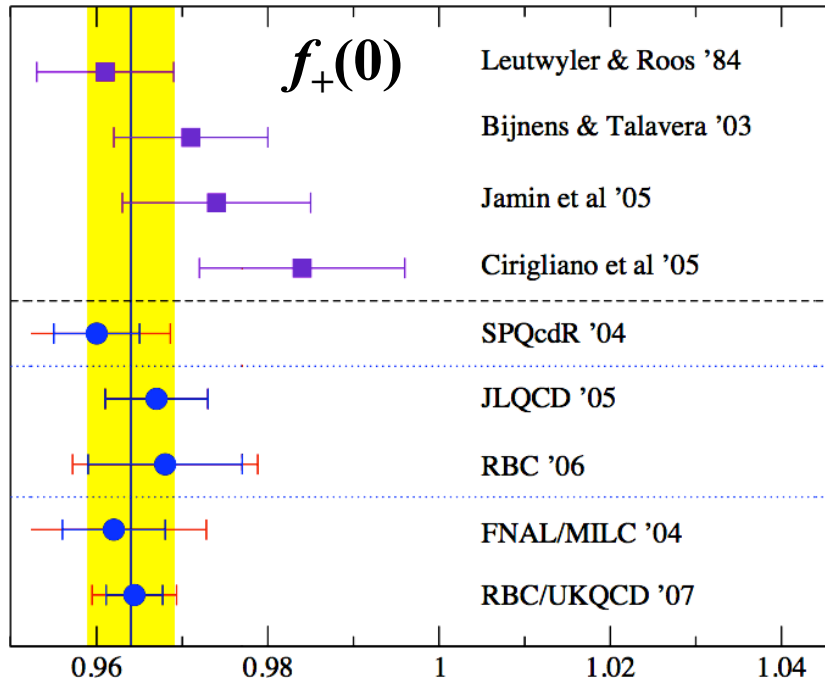
$$\tau \rightarrow \ell \nu \nu \quad (r_{\mu e}) = 1.000(4)$$

Davier, Hoecker, Zhang '06

Without NA48 $K_{\mu 3}$: $r_{\mu e} = 1.002(5)$

Evaluations of $f_+(0)$

Compilations: L. Lellouch, Lattice '08



Quark model

ChPT + QM

ChPT + disp

ChPT + $1/N_c$

$N_f = 0$

2 Wilson

2 DWF

2 Stag+Wilson

(2+1) DWF

Many evaluations available, analytic and lattice-based

ChPT results tend to give higher values for $f_+(0)$

Trend is to use lattice results, but which ones?

FlaviaNet Lattice Averaging Group (FLAG) has promised to make a recommendation

For now use:

$$f_+(0) = 0.9644(49)$$

RBC/UKQCD '07

Provisional coding (Lellouch), similar to FLAG scheme

ref.	publication	N_f , action, etc	mass extrap	$a \rightarrow 0$	finite volume
JLQCD '05	●	●	●	●	●
RBC '06	●	●	●	●	●
FNAL/MILC '04	●	●	●	●	●
RBC/UKQCD '08	●	●	●	●	●

V_{us} from K_{l3} data and CKM unitarity

$$K_{l3} \text{ average: } |V_{us}| f_+(0) = 0.21660(47)$$

was 0.2167 at Kaon'07

With $f_+(0) = 0.9644(49)$ from lattice QCD:

$$K_{l3} \text{ average: } |V_{us}| = 0.2246(12)$$

Using $|V_{ud}| = 0.97424(22)$ from average $0^+ \rightarrow 0^+$ β decays (Towner-Hardy '09)

$$V_{ud}^2 + V_{us}^2 - 1 = -0.0004(7)$$

Compatibility with unitarity -0.6σ

was 0031(15) in PDG04

$$\sigma(V_{ud}^2) \sim 0.0004 \quad \sigma(V_{us}^2) \sim 0.0005$$

V_{us}/V_{ud} from $K, \pi \rightarrow \mu\nu$ decays

$$\frac{\Gamma(K_{\mu 2(\gamma)})}{\Gamma(\pi_{\mu 2(\gamma)})} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_K^2}{f_\pi^2} \frac{m_K (1 - m_\mu^2/m_K^2)^2}{m_\pi (1 - m_\mu^2/m_\pi^2)^2} [1 + \alpha(C_K - C_\pi)]$$

Inputs from theory:

$f_K/f_\pi = 1.189(7)$ from lattice QCD

(HPQCD/UKQCD '07)

Cancellation of lattice-scale uncertainties

$1 + \alpha(C_K - C_\pi) = 0.9930(35)$

(Marciano '04)

Uncertainty from SD virtual corrections (Finkemeier '96)

Inputs from experiment:

K^\pm BR fit:

$\text{BR}(K^\pm_{\mu 2(\gamma)}) = 0.6347(18)$

$\tau_{K^\pm} = 12.384(15)$ ns

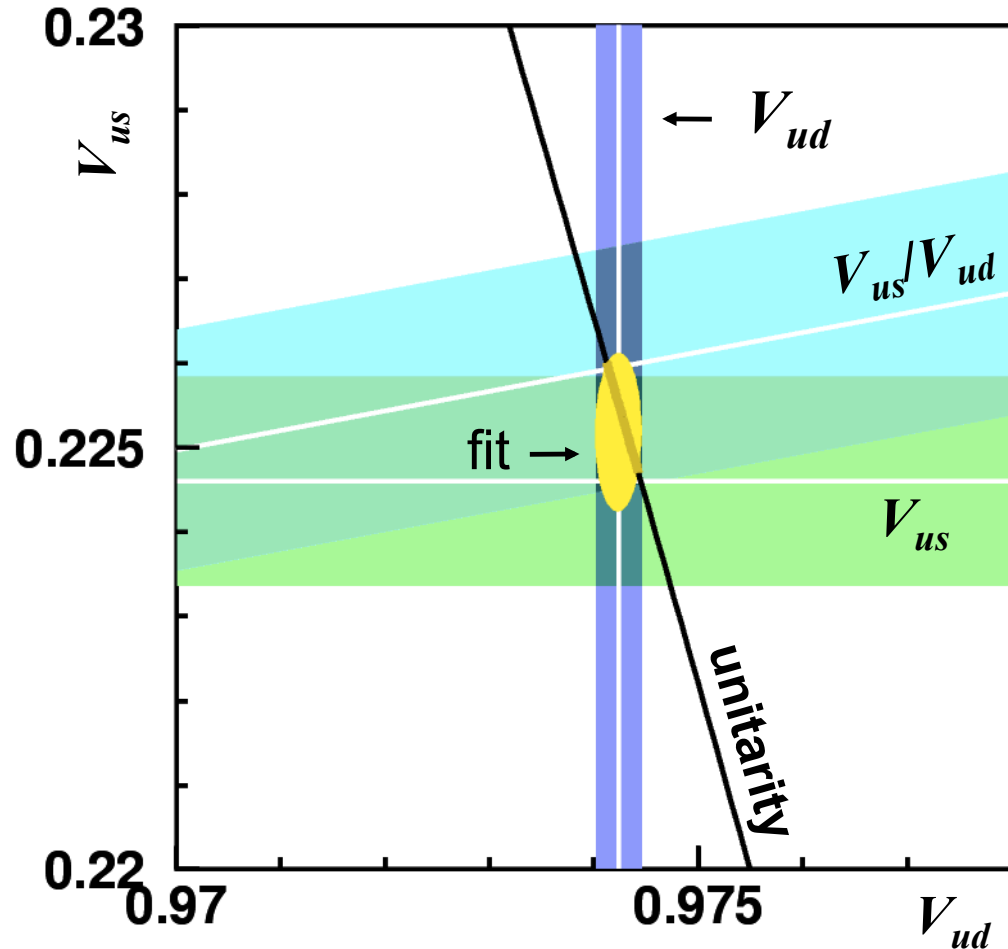
PDG:

$\text{BR}(\pi^\pm_{\mu 2(\gamma)}) = 0.9999$

$\tau_{\pi^\pm} = 26.033(5)$ ns

$$|V_{us}|/|V_{ud}| = 0.2319(15)$$

CKM unitarity



Now can fit:

- V_{us} from $Kl3$
- V_{us}/V_{ud} from K_{u2}/π_{u2}
- V_{ud} from β decay

$$V_{ud} = 0.97424(22)$$

$$V_{us} = 0.2252(9)$$

$$\chi^2/\text{ndf} = 0.52/1 \text{ (47\%)}$$

$$V_{ud}^2 + V_{us}^2 - 1 = -0.0001(6)$$

We use $f_+(0) = 0.9644(49)$, $f_K/f_\pi = 1.189(7)$

1st-row unitarity and gauge universality

MuLan '07

$$G_\mu = 1.166371(6) \times 10^{-5} \text{ GeV}^{-2}$$

From new physics:

$$G_F \rightarrow G_F [1 + a(M_W/\Lambda_{\text{NP}})^2]$$

CKM 1st row

$$1.16626(30) \times 10^{-5} \text{ GeV}^{-2}$$

Precision EW tests

$$1.1655(12) \times 10^{-5} \text{ GeV}^{-2}$$

τ decay

$$1.1678(26) \times 10^{-5} \text{ GeV}^{-2}$$

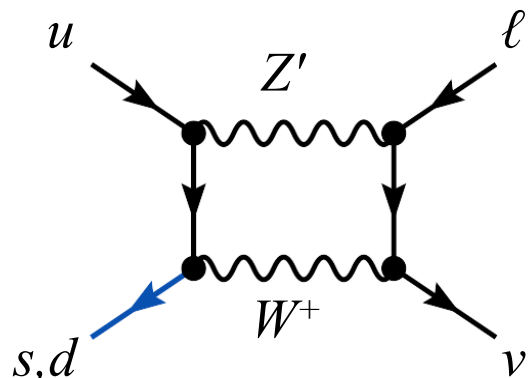
Probe mass scales

$$a_{\text{loop}} \sim g^2/16\pi^2 \sim 1 \text{ TeV}$$

$$a_{\text{tree}} \sim 1 \quad \sim 10 \text{ TeV}$$

Example: Z' bosons

$$G_\mu = G_{\text{CKM}} \left[1 - 0.007 Q_{eL} (Q_{\mu L} - Q_{dL}) \frac{2 \ln(m_{Z'}/m_W)}{m_{Z'}^2/m_W^2 - 1} \right]$$



Z' boson from $SO(10)$ GUT:

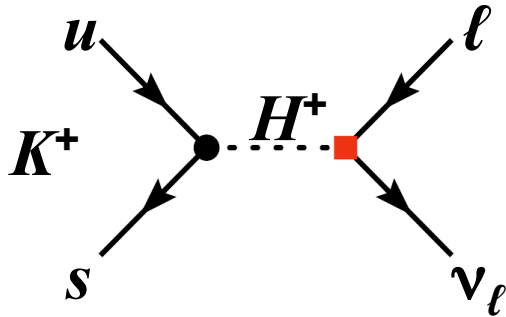
$$Q_{eL} = Q_{uL} = -3Q_{dL} = 1$$

$m_{Z'} > 700 \text{ GeV}$ 95% CL

About same sensitivity as direct searches

(Marciano)

$K_{\mu 2}$: sensitivity to new physics



Scalar currents, e.g. due to Higgs exchange, could affect $K \rightarrow \mu \nu$ width

$$\frac{\Gamma(K^+ \rightarrow \ell \nu)}{\Gamma_{\text{SM}}(K^+ \rightarrow \ell \nu)} = \left[1 - \left(\frac{m_{K^+}^2}{m_{H^+}^2} \right) \frac{\tan^2 \beta}{1 + \epsilon_0 \tan \beta} \right]^2$$

The observable:

$$R_{l23} = \left| \frac{V_{us}(K_{\mu 2})}{V_{us}(K_{l3})} \times \frac{V_{ud}(0^+ \rightarrow 0^+)}{V_{ud}(\pi_{\mu 2})} \right|$$

$$= \left| 1 - \frac{m_{K^+}^2}{m_{H^+}^2} \left(1 - \frac{m_{\pi^+}^2}{m_{K^+}^2} \right) \frac{\tan^2 \beta}{1 + \epsilon_0 \tan \beta} \right|$$

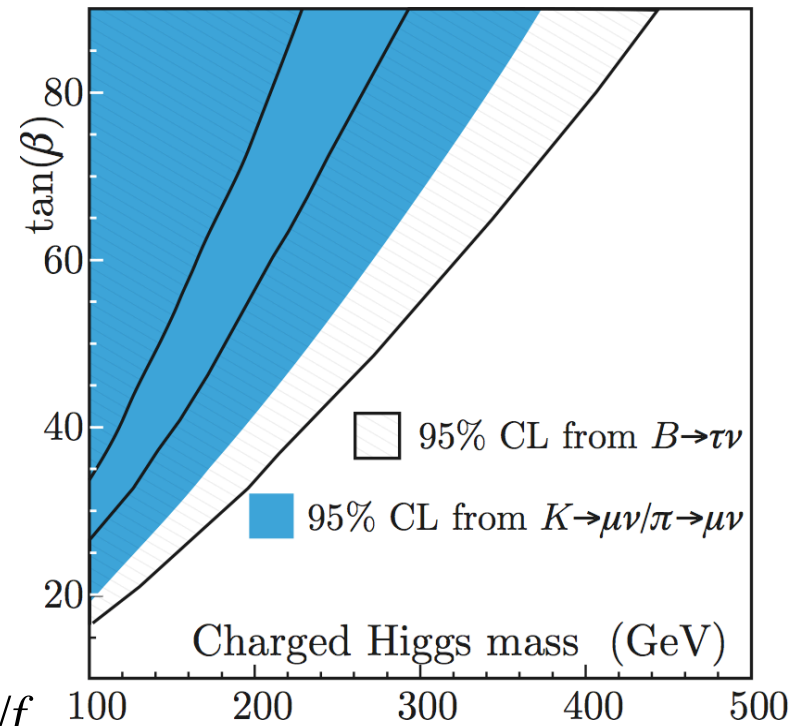
(Hou, Isidori-Paradisi)

we find

$$R_{l23} = 1.008 \pm 0.008$$

(KLOE only)

limited by lattice uncertainty on $f_+(0)$ and f_K/f_π



Fit to determine $f_+(0)$ and f_K/f_π

Assuming the SM, we can extract the decay constants

Start from $K_{\ell 2}$ rate expression:

$$\frac{\Gamma(K_{\mu 2}(\gamma))}{\Gamma(\pi_{\mu 2}(\gamma))} = \frac{|V_{us}|^2}{|V_{ud}|^2} \frac{f_K^2}{f_\pi^2} \frac{m_K(1 - m_\mu^2/m_K^2)^2}{m_\pi(1 - m_\mu^2/m_\pi^2)^2} [1 + \alpha(C_K - C_\pi)]$$

Rewrite as:

$$Q_{K\ell 2} = \frac{|V_{us} f_+(0)|_{K\ell 3}^2}{|V_{ud}|_{0^+ \rightarrow 0^+}^2} \times \frac{1}{f_+(0)^2} \times \frac{f_K^2}{f_\pi^2}$$

Experimental inputs

up to IB and radiative corrections

V_{ud} from $0^+ \rightarrow 0^+$

$V_{us} f_+(0)$ from $K_{\ell 3}$

$Q_{K\ell 2}$ rate ratio



Parameters from theory

$f_+(0)$

f_K/f_π

Constraints from: $Q_{K\ell 2}$ rate expression and 1st row CKM unitarity

Fit to determine $f_+(0)$ and f_K/f_π

5 parameters:

$$V_{ud}, V_{us}, Q_{K\ell 2}, f_+(0), f_K/f_\pi$$

3 inputs:

$$V_{ud}, V_{us}f_+(0), Q_{K\ell 2}$$

2 constraints:

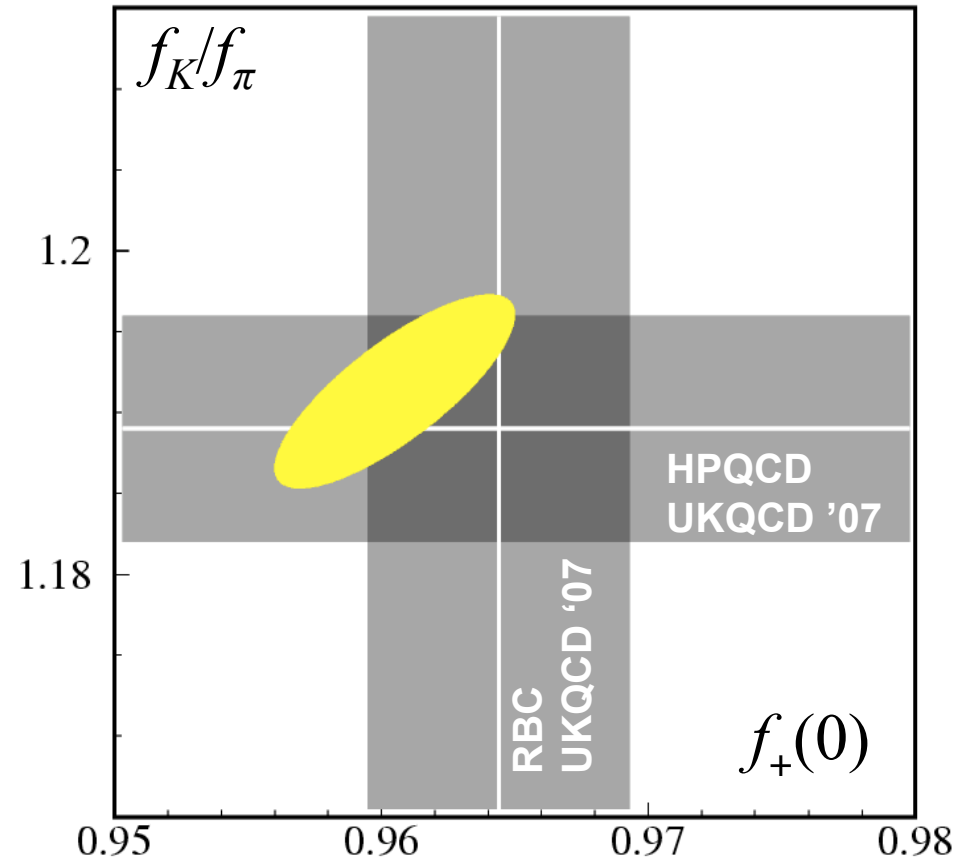
$Q_{K\ell 2}$ rate relation
1st row unitarity

$$Q_{K\ell 2} = \frac{|V_{us} f_+(0)|_{K\ell 3}^2}{|V_{ud}|_{0^+ \rightarrow 0^+}^2} \times \frac{1}{f_+(0)^2} \times \frac{f_K^2}{f_\pi^2}$$

$$f_+(0) = 0.9605(45)$$

$$f_K/f_\pi = 1.1913(60)$$

$$\rho = +0.79$$



Summary

$$K_{\ell 3} \text{ average: } |V_{us}| f_+(0) = 0.21660(47)$$

With V_{ud} from $0^+ \rightarrow 0^+$ β decays and hadronic parameters from lattice QCD, first-row CKM unitarity is verified:

$$V_{ud}^2 + V_{us}^2 - 1 = -0.0004(7)$$

Compatibility with unitarity -0.6σ

This allows precision tests of the SM and places important limits on universality-breaking new physics

Very rich new kaon data set; expect to improve on K^\pm BRs and semileptonic slopes

Impressive results from lattice QCD: very interesting prospects for precision test of the SM

Additional information

WHAT'S NEW SINCE CIPANP 2006?

Important new experimental data:

Total of 27 new published experiments (most significant are precise Penning-trap Q-value measurements). New survey in:
Hardy & Towner, PRC in press (2009)

Inner radiative correction, Δ_R :

Theoretical uncertainty reduced by a factor of 2.
Marciano & Sirlin, PRL 96, 032002 (2006)

Isospin symmetry-breaking correction, δ_C :

Improved by addition of core states.
Towner & Hardy, PRC 77, 025501 (2008)

New calculations with Hartree-Fock radial wave functions as test of uncertainty.
Hardy & Towner, PRC in press (2009)

Statistical rate function, f :

Small correction added for atomic excitation of daughter atom
Hardy & Towner, PRC in press (2009)

EXPERIMENTS PUBLISHED SINCE 2005

Q_{EC} values:

Argonne (Canadian Penning trap)

^{46}V Savard *et al.*, PRL 95, 102501 (2005)

Jyvaskyla (JYFLTRAP)

^{62}Ga Eronen *et al.* PLB 636, 191 (2006)

$^{26}\text{Al}^m$, ^{42}Sc , ^{46}V Eronen *et al.*,
PRL 97, 232501 (2006)

^{50}Mn , ^{54}Co Eronen *et al.*,
PRL 100, 132502 (2008)

NSCL (LEBIT)

^{38}Ca Bollen *et al.*, PRL 96, 152501 (2006)

^{66}As Schury *et al.*, PRC 75, 055801 (2007)

Munich Tandem

^{46}V Faestermann *et al.*, Progress Report

ISOLTRAP

^{38}Ca George *et al.*, PRL 98, 162501 (2007)

$^{26}\text{Al}^m$ George *et al.*, EPL 82, 50005 (2008)

Half-lives:

Auckland/Canberra

^{50}Mn Barker & Byrne, PRC 73, 064306 (2006)

LBNL

^{14}O Burke *et al.*, PRC 74, 025501 (2006)

Texas A&M

^{34}Cl , ^{34}Ar Iacob *et al.*, PRC 74, 055502 (2006)

^{10}C Iacob *et al.*, PRC 77, 045501 (2008)

TRIUMF

^{18}Ne Grinyer *et al.*, PRC 76, 025503 (2007)

^{62}Ga Grinyer *et al.*, PRC 77, 015501 (2008)

Jyvaskyla

^{26}Si Matea *et al.*, EPJA to be pub. (2008)

Melbourne

$^{26}\text{Al}^m$ Scott *et al.*, NIMPRA 539, 191 (2005)

Branching ratios:

TRIUMF

^{62}Ga Finlay *et al.*, PRC 78, 025502 (2008)

$^{38}\text{K}^m$ Leach *et al.*, PRL 100, 192504 (2008)

Jyvaskyla

^{26}Si Matea *et al.*, EPJA to be pub. (2008)

^{62}Ga Bey *et al.*, EPJA 36, 121 (2008)

Texas A&M

^{14}O Towner & Hardy, PRC 72, 055501 (2005)

T=1/2 SUPERALLOWED BETA DECAY

BASIC WEAK-DECAY EQUATION

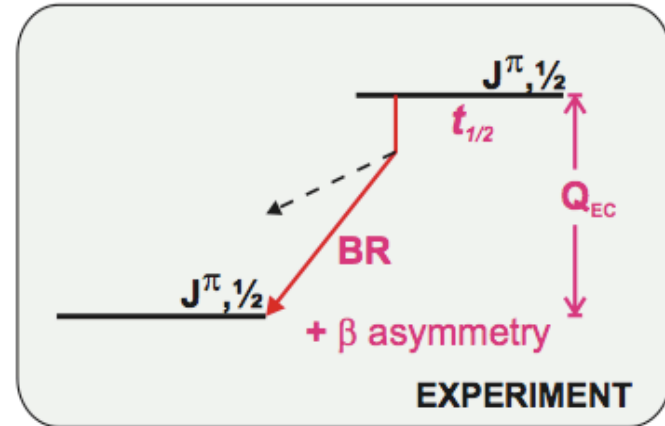
$$ft = \frac{K}{G_V^2 \langle \tau \rangle^2 + G_A^2 \langle \sigma \tau \rangle^2}$$

f = statistical rate function: $f(Z, Q_{EC})$

t = partial half-life: $t(t_{1/2}, BR)$

$G_{V,A}$ = coupling constants

$\langle \rangle$ = Fermi, Gamow-Teller matrix elements



INCLUDING RADIATIVE CORRECTIONS

$$\mathcal{F}t = ft (1 + \delta'_R) [1 - (\delta_\sigma - \delta_{NS})] = \frac{K}{G_V^2 (1 + \Delta_R) (1 + \lambda^2 \langle \sigma \tau \rangle^2)}$$

$\lambda = G_A / G_V$

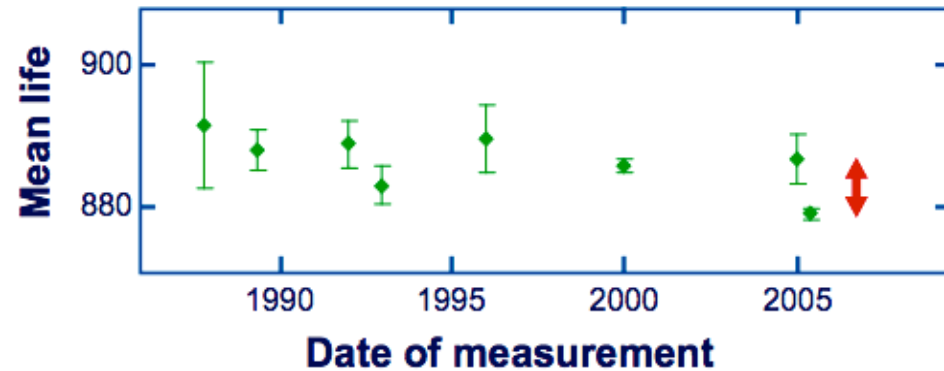
Requires additional experiment:
for example, β asymmetry

NEUTRON DECAY

NEUTRON DECAY DATA 2009

Mean life:

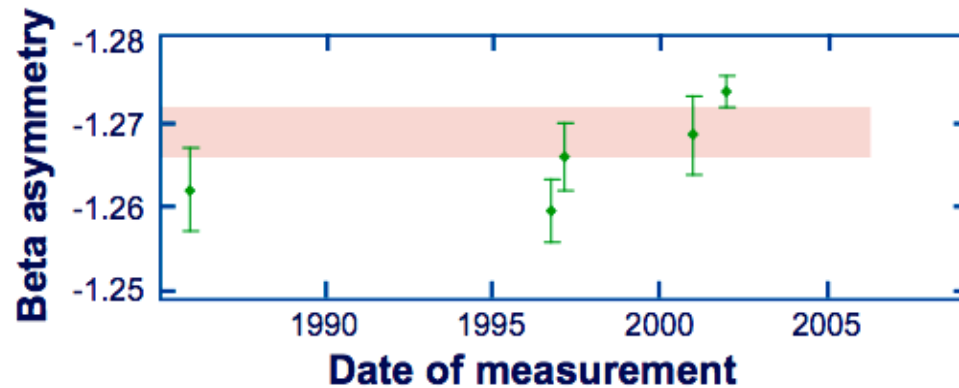
$$878 < \tau < 886 \text{ s}$$



β asymmetry:

$$\lambda = -1.269 \pm 0.003$$

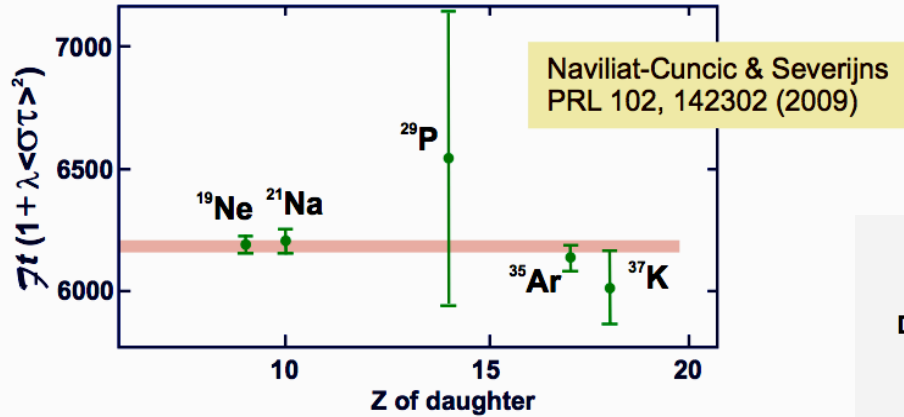
$$\chi^2/N = 3.9$$



$$0.9727 < V_{ud} < 0.9807$$

most recent values only: $V_{ud} = 0.9758 \pm 0.0013$

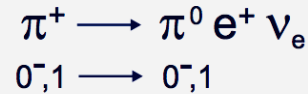
$$\mathcal{F}t = ft(1 + \delta_R') [1 - (\delta_C - \delta_{NS})] = \frac{K}{G_V^2 (1 + \Delta_R)(1 + \lambda^2 \langle \sigma\tau \rangle^2)}$$



$$V_{ud} = 0.9719 \pm 0.0017$$

PION BETA DECAY

Decay process:



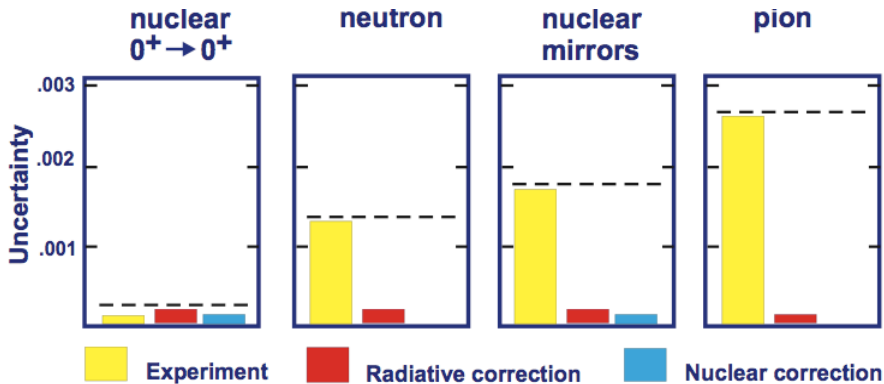
Experimental data:

$$\tau = 2.6033 \pm 0.0005 \times 10^{-8} \text{ s} \quad (\text{PDG 2009})$$

$$\text{BR} = 1.036 \pm 0.007 \times 10^{-8} \quad \text{Pocanic et al, PRL 93, 181803 (2004)}$$

Result:

$$V_{ud} = 0.9749 \pm 0.0026$$



K_L branching ratios from KTeV

5 ratios of main BRs from independent samples of 10^5 - 10^6 events collected with a single trigger

KTeV, PRD 70 (2004)

2-track ratios

$$\text{BR}(K_{\mu 3}) / \text{BR}(K_{e3}) = 0.6640(26)$$

$$\text{BR}(\pi^+\pi^-\pi^0) / \text{BR}(K_{e3}) = 0.3078(18)$$

$$\text{BR}(\pi^+\pi^-) / \text{BR}(K_{e3}) = 0.004856(28)$$

Neutral ratio

$$\text{BR}(2\pi^0) / \text{BR}(3\pi^0) = 0.004446(25)$$

Mixed ratio

$$\text{BR}(3\pi^0) / \text{BR}(K_{e3}) = 0.4782(55)$$

- 6 decays = 99.93% of K_L width, KTeV combines ratios to extract BRs
- Fit to BRs uses the 5 ratios (correlations available)

K_L branching ratios from NA48

1) K_L beam, 2-track sample, 80×10^6 events (6×10^6 signal)

NA48, PLB 602 (2004)

$$\text{BR}(K_{e3}) / \text{BR}(2 \text{ track}) = 0.4978(35)$$

- NA48 evaluates $\text{BR}(K_{e3})$ from $\text{BR}(2 \text{ track}) = 1.0048 - \text{BR}(3\pi^0)_{K_{TeV}}$
- The measured ratio used in fit to BRs

2) Measurement of $\Gamma(K_L \rightarrow 3\pi^0) / \Gamma(K_S \rightarrow 2\pi^0)$ with same number of K_L and K_S produced on target, $2-6 \times 10^5$ signal events; use $\Gamma(K_S \rightarrow 2\pi^0)$ to extract $\Gamma(K_L \rightarrow 3\pi^0)$

NA48, 2004 preliminary

$$\text{BR}(K_L \rightarrow 3\pi^0) / \tau_L = 3.795(58) \text{ MHz}$$

K_L branching ratios and lifetime from KLOE

- 1) Absolute BRs: K_L decays tagged by $K_S \rightarrow \pi^+ \pi^-$
 13×10^6 tagged, $10^5 - 10^6$ signal

KLOE, PLB 632 (2006)

$$\text{BR}^{(0)}(K_{e3}) = 0.4049(21)$$

$$\text{BR}^{(0)}(K_{\mu3}) = 0.2726(16)$$

$$\text{BR}^{(0)}(3\pi^0) = 0.2018(24)$$

$$\text{BR}^{(0)}(\pi^+ \pi^- \pi^0) = 0.1276(15)$$

at $\tau_L^{(0)} = 51.54$ ns, with
 $d\text{BR}/\text{BR} = 0.67 d\tau_L/\tau_L$
(geometrical acceptance)

Correlations available

KLOE results: set $\Sigma \text{BR}(i) = 1$ and solve for τ_L

Fit to BRs: use unconstrained BRs with dependence on τ_L

- 2) Lifetime: measurement from an independent sample of

$15 \times 10^6 K_L \rightarrow \pi^0 \pi^0 \pi^0$ events

uniform reconstruction eff. over $0.4\tau_L$

KLOE, PLB 626 (2005)

$$\tau_L = 50.92(30) \text{ ns}$$

$$K_L \rightarrow \pi^+\pi^-$$

New measurements of $K_L \rightarrow \pi^+\pi^-(\gamma)$ also useful in global fit

KTeV
PRD 70 (2004)

$$\text{BR}(\pi^+\pi^-/Ke3) = 4.856(29) \times 10^{-3}$$

1 of 5 ratios in K_L BR analysis

Contribution from direct emission (DE) negligible

KLOE
PLB 638 (2006)

$$\text{BR}(\pi^+\pi^-/K\mu3) = 7.275(68) \times 10^{-3}$$

Fully inclusive of DE component

NA48
PLB 645 (2007)

$$\text{BR}(\pi^+\pi^-/Ke3) = 4.826(27) \times 10^{-3}$$

Residual DE contribution of 0.19% subtracted

For consistency and to better satisfy S BR = 1 in global fit,
DE contribution of **1.52(7)%*** added to **KTeV** and **NA48** results

* From E731 '93, KTeV '01 and KTeV '06 $K_L \rightarrow \pi^+\pi^-\gamma$ results

Can we observe λ_0'' ?

Determination of $f_+(0)$ with ChPT calculations at $O(p^6)$, would benefit from the measurement of scalar form factor curvature (Bijnens and Talavera, 2003):

$$2C_{12}^r + C_{34}^r \sim \lambda_0' \quad C_{12}^r \sim \lambda_0''$$

[C_{ij}^r = LEC's at order $O(p^6)$]

Unfortunately, for $K\mu 3$ there are two FF and 4 parameters \rightarrow high correlations \rightarrow big errors. If we compute errors on λ_0'' from the (no experimental resolution) we get:

$$\rho(\lambda_0', \lambda_0'') = -0.999$$

$$d\lambda_0'' \sim 20/\sqrt{N}$$

Fit to $\lambda_+' , \lambda_+'' , \lambda_0' , \lambda_0''$

impossible

$$d\lambda_0'' \sim 2.5/\sqrt{N}$$

Fit to λ_0' , λ_0'' (λ_+' , λ_+'' fixed)

25M events for $d\lambda_0'' = 0.5 \times 10^{-3}$

Dispersive treatment of form factors

Ignoring λ_0'' changes $K_{\mu 3}$ phase space by -0.1% (0.2σ KLOE)

Better parameterization allows tests of ChPT & low-energy dynamics

Use parameterizations of f_+, f_0 from twice-subtracted dispersion relations constrained by fits to $K\pi$ scattering data

Bernard et al. '06

Expansion gives:

$$\tilde{f}_{+,0}(t) = 1 + \lambda_{+,0}^C \left(\frac{t}{m_{\pi^+}^2} \right) + \frac{(\lambda_{+,0}^C)^2 + p_2}{2} \left(\frac{t}{m_{\pi^+}^2} \right)^2 + \frac{(\lambda_{+,0}^C)^3 + 3\lambda_{+,0}^C p_2 + p_3}{6} \left(\frac{t}{m_{\pi^+}^2} \right)^3$$

Fit results:

$I(K_{L\mu 3})$ phase-space integrals

**KLOE $e3-\mu 3$
JHEP 12(2007)**

$$\begin{aligned} \lambda_+^C &= (25.7 \pm 0.6) \times 10^{-3} \\ \lambda_0^C &= (14.0 \pm 2.1) \times 10^{-3} \\ \rho &= -0.26 \end{aligned}$$

$$\begin{aligned} \text{Quad-lin: } & 0.10271(52) \\ \text{Disp: } & 0.10262(47) \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Quad-lin: } \\ \text{Disp: } \end{aligned}} \right\} 0.1\%$$

**NA48 $\mu 3$ only
PLB 647 (2007)**

$$\begin{aligned} \lambda_+^C &= (23.3 \pm 0.9) \times 10^{-3} \\ \lambda_0^C &= (8.8 \pm 1.2) \times 10^{-3} \\ \rho &= -0.44 \end{aligned}$$

$$\begin{aligned} \text{Quad-lin: } & 0.10039(58) \\ \text{Disp: } & 0.10063(31) \end{aligned} \quad \left. \vphantom{\begin{aligned} \text{Quad-lin: } \\ \text{Disp: } \end{aligned}} \right\} 0.1\%$$

NB: NA48 value for λ_+^C converted from $\ln C = 0.1438(138)$

Form factors and the Callan-Treiman relation

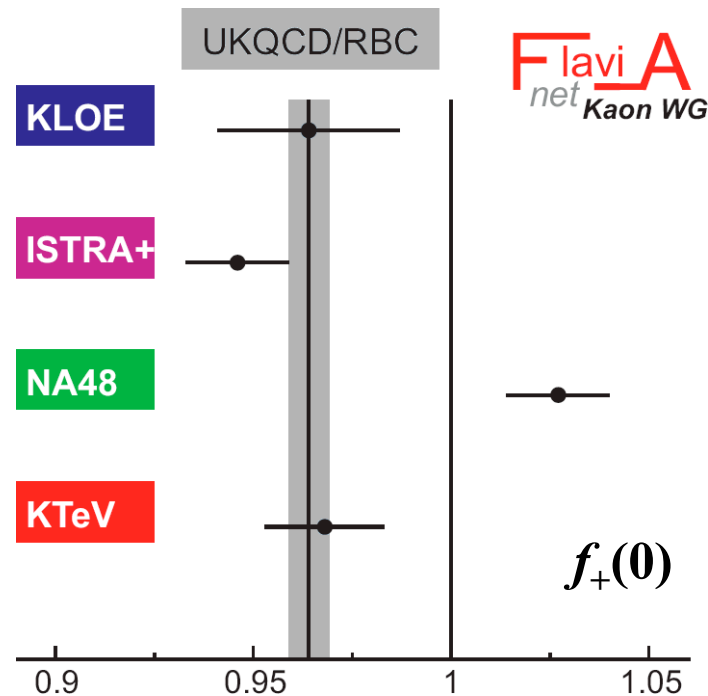
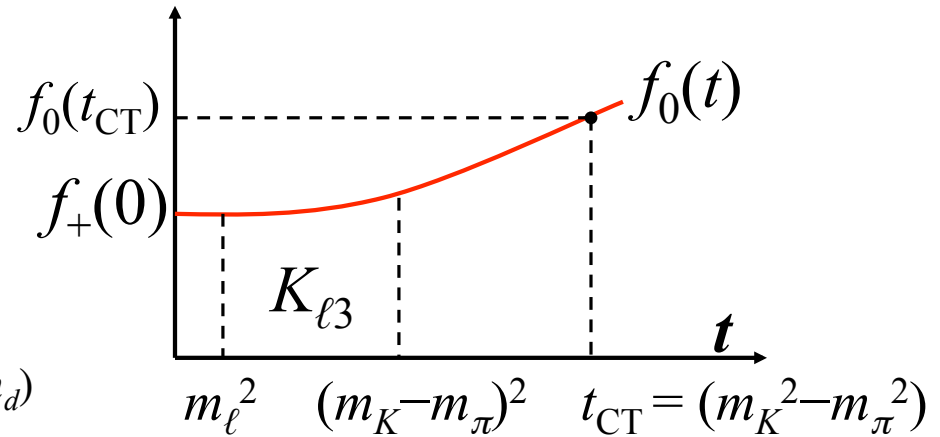
Callan-Treiman relation:

$$\tilde{f}_0(t_{CT}) = \frac{f_K}{f_\pi} \frac{1}{f_+(0)} + \Delta_{CT}$$

$$t_{CT} = m_K^2 - m_\pi^2$$

$\Delta_{CT} = SU(2)$ -breaking correction

$$= -(3.5 \pm 8.0) \times 10^{-3} \text{ in NLO ChPT } (m_u = m_d)$$



Use dispersive parameterization of $f_0(t)$ and statistical considerations to extrapolate to t_{CT}

Lattice QCD

$$f_K/f_\pi = 1.190(15)$$

KLOE $K_{e3-\mu3}$ data:

$$\lambda_0^C = (14.0 \pm 2.1) \times 10^{-3}$$

$$\rightarrow f_+(0) = 0.968(28)$$

NA48 $K_{\mu3}$ data:

$$\ln C = 0.1438(138)$$

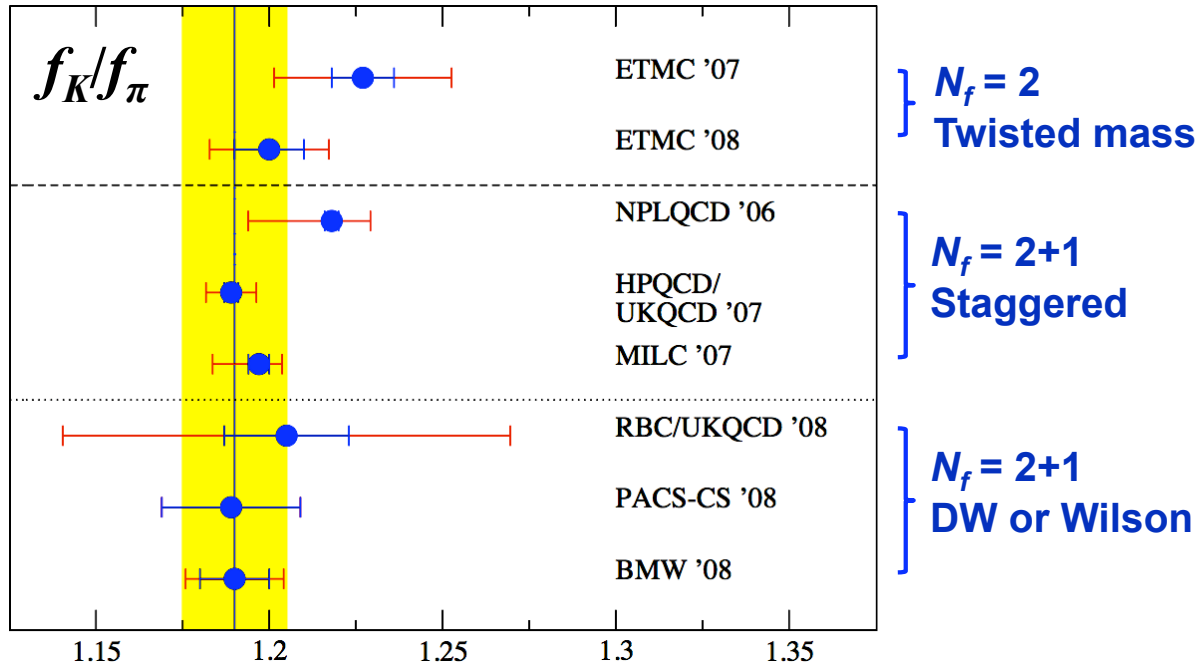
$$\rightarrow f_+(0) = 1.027(20)$$

Lattice QCD

$$f_+(0) = 0.964(5)$$

Lattice evaluations of f_K/f_π

Compilations: L. Lellouch, Lattice '08



Only *published* result with full evaluation of systematic errors is

HPQCD/UKQCD '07
 $N_f = 2+1_{\text{HISQ}}$
 $f_K/f_\pi = 1.189(7)$

Results without staggered quarks have large errors (PACS '08 has stat errs only) or are preliminary (BMW '08)

Provisional coding

ref.	publication	N_f , action, etc	mass extrap	$a \rightarrow 0$	finite volume
ETM '08	●	●	●	●	●
NPLQCD '06	●	●	●	●	●
MILC '04-07	●	●	●	●	●
HPQCD/UKQCD '07	●	●	●	●	●
RBC/UKQCD '08	●	●	●	●	●
PACS-CS '08	●	●	●	●	●
BMW '08	●	●	●	●	●

For now use HPQCD/UKQCD'07

V_{us} from τ

V_{us} from inclusive $\tau \rightarrow \nu X_{us}$ involves PQCD

S. Banerjee arXiv:0811.1429

$$|V_{us}|^2 = \frac{R_{\tau,S}^{00}}{\frac{R_{\tau,V+A}^{00}}{|V_{ud}|^2} - \delta R_{\tau,th}^{00}}$$

Gámiz-Jamin-Pich-Prades-Schwab

$$V_{us} = 0.2159 (30_{\text{exp}})(5_{\text{th}})$$

$\sim 3 \sigma$ lower wrt kaons (same fitting m_s, V_{us})

Theory? Exp.?

check with kaons on exclusive modes ($\sim 70\%$ of R_s):

(24% of R_s) $\text{BR}(K\nu) = 0.69(1)$ vs $0.715(4)$ from $K\mu 2$

but $\text{BR}(K\nu)/\text{BR}(\pi\nu)$ ok

X_{us}^-	$\mathcal{B}_{\text{World Averages}} (\%)$
$K^- [\tau \text{ decay}]$	0.690 ± 0.010
$([K_{\mu 2}])$	(0.715 ± 0.004)
$K^- \pi^0$	0.426 ± 0.016
$\bar{K}^0 \pi^-$	$0.835 \pm 0.022 (S = 1.4)$
$K^- \pi^0 \pi^0$	0.058 ± 0.024
$\bar{K}^0 \pi^0 \pi^-$	0.360 ± 0.040
$K^- \pi^- \pi^+$	$0.290 \pm 0.018 (S = 2.3)$
$K^- \eta$	0.016 ± 0.001
$(\bar{K}^0 3\pi)^- \text{ (est'd)}$	0.074 ± 0.030
$K_1(1270) \rightarrow K^- \omega$	0.067 ± 0.021
$(\bar{K}^0 4\pi)^- \text{ (est'd)}$	0.011 ± 0.007
$K^{*-} \eta$	0.014 ± 0.001
$K^- \phi$	$0.0037 \pm 0.0003 (S = 1.3)$
TOTAL	2.8447 ± 0.0688
	(2.8697 ± 0.0680)