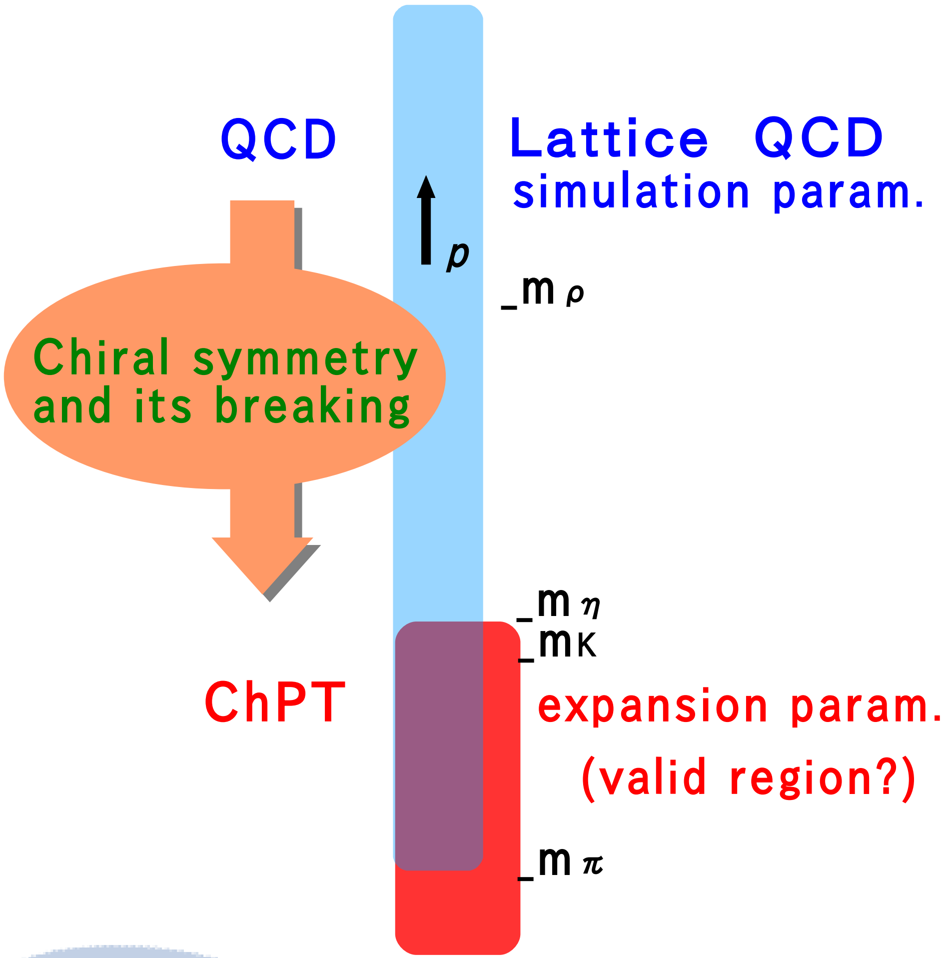
Convergence of ChPT in dynamical lattice QCD with exact chiral symmetry

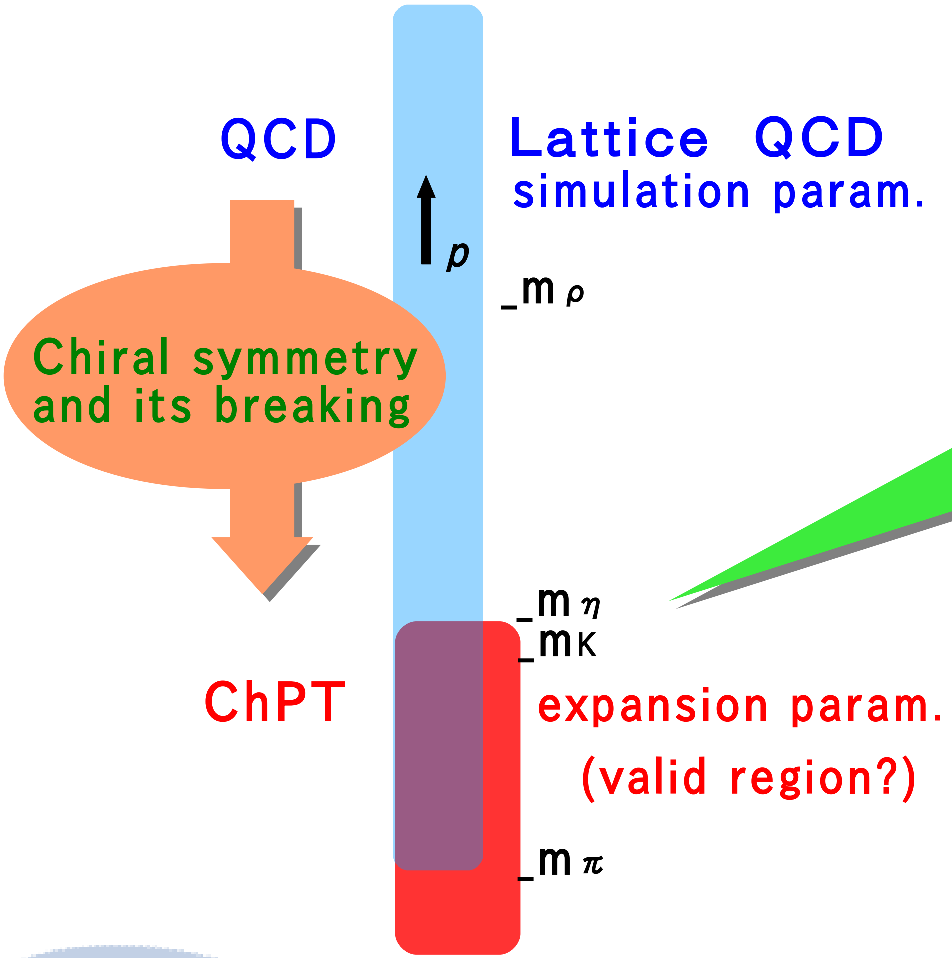
Jun Noaki for JLQCD Collaboration

High Energy Accerelator Research Organization (KEK)

ChPT vs Lattice QCD



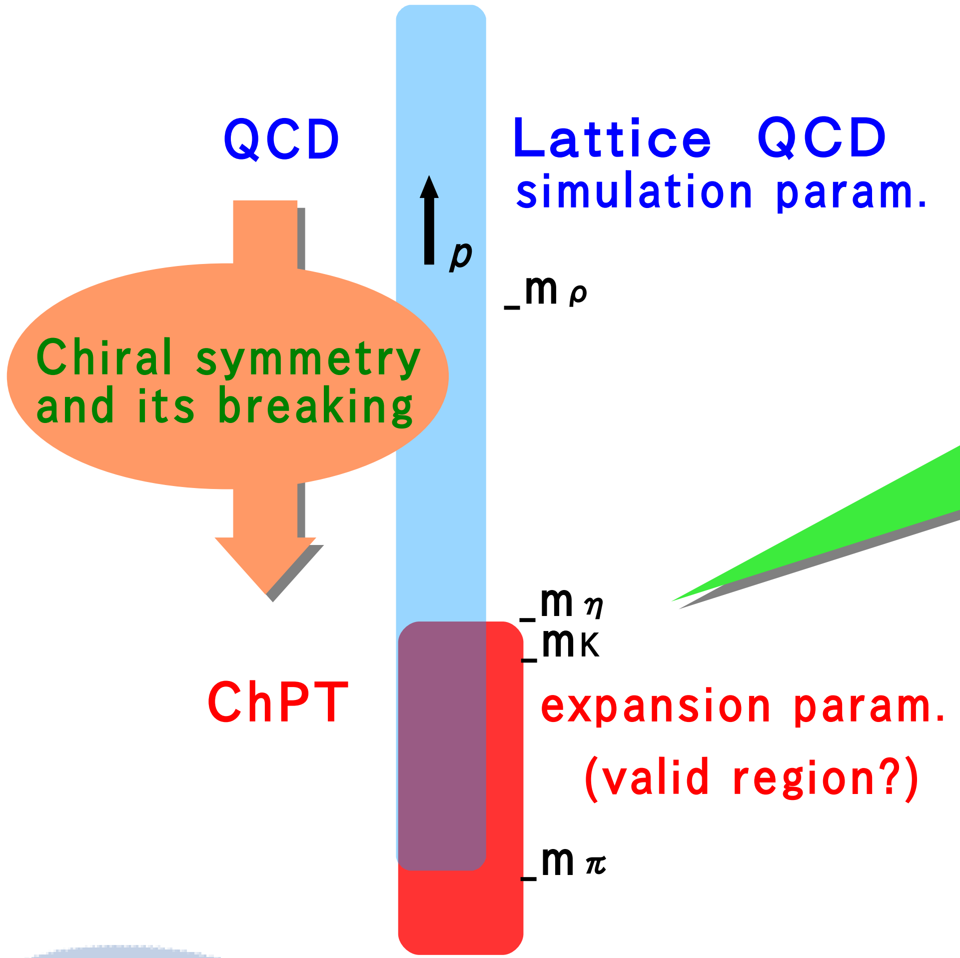
ChPT vs Lattice QCD



Comvergence of ChPT?

We compare ChPT predictions and results of lattice QCD simulation.

ChPT vs Lattice QCD



Comvergence of ChPT?
We compare ChPT predictions and results of lattice QCD simulation.

- Chiral behavior of m_π, f_π
- Exact chiral symmetry is crutial.
- Overlap fermion is promissing.
- Dynamical simulation by JLQCD collaboration

- ChPT does not converge at NLO
- Kaon physics requires NNLO ChPT.

Members

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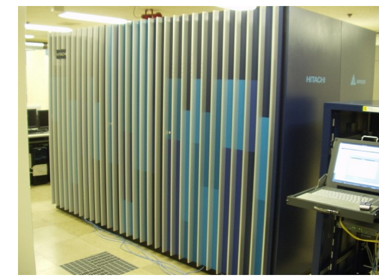
IBM BG/L, 57.3 Tflops



HITACHI, SR11000, 2.1 Tflops

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Plan

- Numerical simulation 5mins

  Overlap fermion /simulation setup

- Convergence of ChPT (N_f=2) 10mins

  ChPT vs LQCD

- Results in the N_f=2+1simulation 5mins

  Extrapolation of (m_π, m_K, f_π, f_K)
  to the physical point

- Summary

Chiral fermion on the lattice

Overlap operator $D_{ov}=m_0\left(1+\gamma_5 sign(H_W)\right),\ H_W=\gamma_5 D_W(-m_0)$

- Ginsparg-Wilson relation satisfied — Neuberger, 1998
  - Exact chiral symm for any N_f
  - Index theorem holds

Theoretical studies

- Chiral symmetry Breaking
  - Banks-Casher relation
  - Chiral RMT
  - Chiral properties
- Topology
  - θ-vacuum, χ_{top}

Phenomenological studies

- Coordinated chiral extrapolation
  - LECs in continuum ChPT
- Flavor physics
  - m_s, f_K/f_π, B_K, form factors
- OPE, vacuum polarization
  - m_d-m_u, α_s, S-parameter

JLQCD's overlap simulation

- Iwasaki glue + Overlap quarks

- $N_f = 2$

a / a^{-1}	size [fm^4]	Q	am_{sea}
0.09 fm/1.85 GeV	1.5^3x2.9	0	0.0020 (ε -regime)
0.12 fm/1.67 GeV	1.9^3x3.8	0	0.015-0.100 (6pts)
		-2,-4	0.050

- $N_f = 2+1$

a / a^{-1}	size [fm^4]	Q	am_s	am_{ud}	
0.11 fm/1.83 GeV	1.8^3x5.3	0	0.080	0.015-0.080 (5pts)	
		0	0.100	0.015-0.100 (5pts)	
		0	0.080	0.002 (ε -regime)	
		1	0.080	0.015	
	2.6^3x5.3	0	0.080	0.015,0.025	running

JLQCD's overlap simulation

- Iwasaki glue + Overlap quarks

- $N_f = 2$

a / a^{-1}	size [fm^4]	Q	am_{sea}
0.09 fm/1.85 GeV	1.5^3x2.9	0	0.0020 (ε -regime)
0.12 fm/1.67 GeV	1.9^3x3.8	0	0.015-0.100 (6pts)
290 MeV < m_π < 750 MeV		-2,-4	0.050

- $N_f = 2+1$

a / a^{-1}	size [fm^4]	Q	am_s	am_{ud}	
0.11 fm/1.83 GeV	1.8^3x5.3	0	0.080	0.015-0.080 (5pts)	
		0	0.100	0.015-0.100 (5pts)	
310 MeV < m_π < 800 MeV		0	0.080	0.002 (ε -regime)	
		1	0.080	0.015	
	2.6^3x5.3	0	0.080	0.015,0.025	running

Convergence of ChPT from light meson spectrum

JLQCD+TWQCD PRL101, 202004 (2008)

6/10/2009, EPOCHAL TSUKUBA

J.Noaki for JLQCD

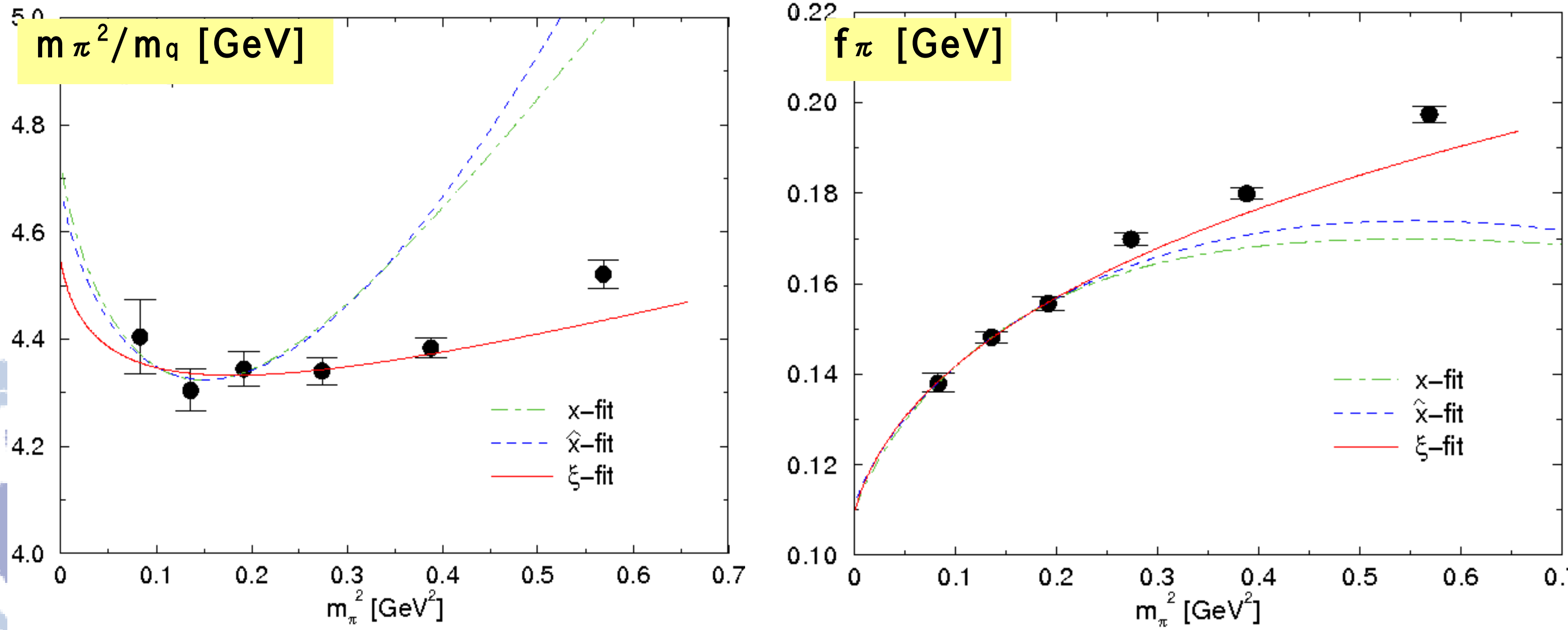
Test of NLO ChPT

- Useful guide of lattice calc.
  - important question: “mK＝450 MeV is accommodated?”
- Stability in variation of expansion parameters.

$$m_\pi^2/m_q = 2B\left(1+\frac{1}{2}x\ln x\right)+c_3 x \qquad x = 2\frac{2Bm_q}{(4\pi f)^2} \qquad \hat{x} = 2\left(\frac{m_\pi}{4\pi f}\right)^2 \qquad \xi = 2\left(\frac{m_\pi}{4\pi f_\pi}\right)^2$$

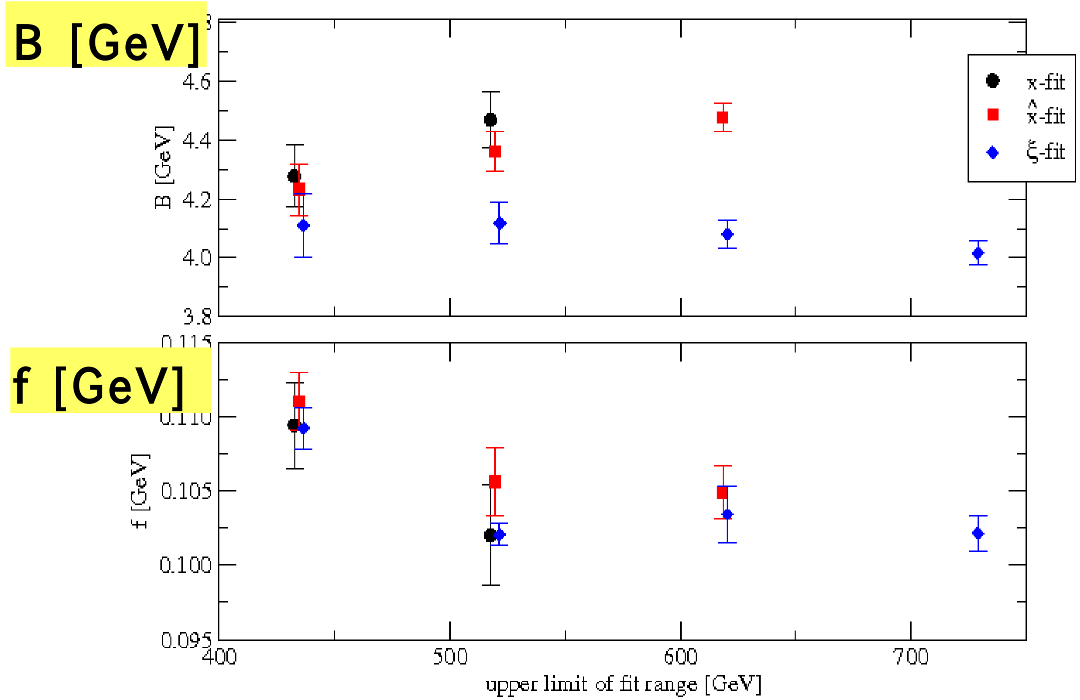
$$f_\pi = f\left(1-x\ln x\right)+c_4 x$$

- Simul. fit of the lightest 3 data.



Test of NLO ChPT

- Mass region where NLO ChPT is valid



fit with χ^2/dof < 2.0

- NLO fails around m_π = 450 MeV
- ξ is most promising. (resummation through f_π)
- For better precision,
  - Lighter m_π points (expensive)
  - Take NNLO effect into account → next step

N_f = 2, NNLO ChPT

- NNLO with ξ Colangelo et al., 1997

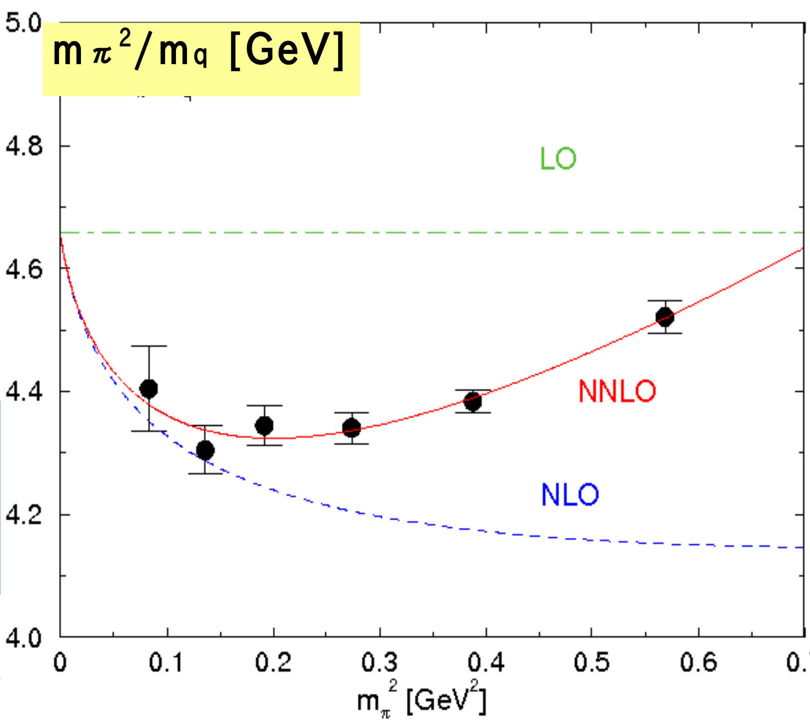
$$m_\pi^2/m_q = 2B\left[1+\frac{1}{2}\xi\ln\xi+\frac{7}{8}(\xi\ln\xi)^2+\left(\frac{c_4}{f}-\frac{1}{3}(\tilde{l}+16)\right)\xi^2\ln\xi\right]+c_3\xi\left(1-\frac{9}{2}\xi\ln\xi\right)+\alpha\xi^2$$

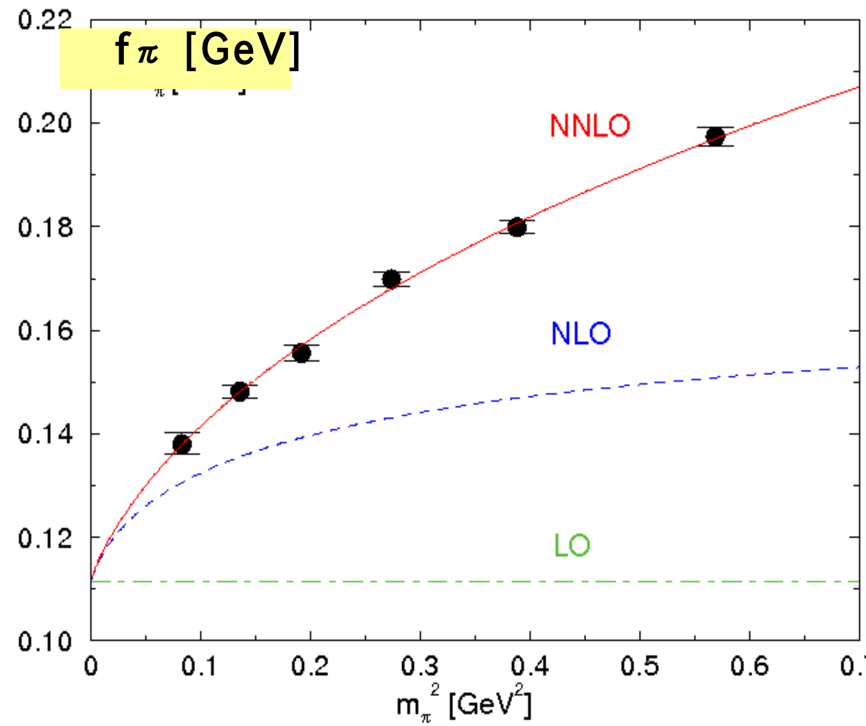
$$f_\pi = f\left[1-\xi\ln\xi+\frac{5}{4}(\xi\ln\xi)^2+\frac{1}{6}\left(\tilde{l}+\frac{53}{2}\right)\xi^2\ln\xi\right]+c_4\xi(1-5\xi\ln\xi)+\beta\xi^2$$

input: $\tilde{l}=7\bar{l}_1+8\bar{l}_2+$cnst.

- Reasonable quality of fit: χ^2/dof = 1.40
- Correction at 500 MeV:

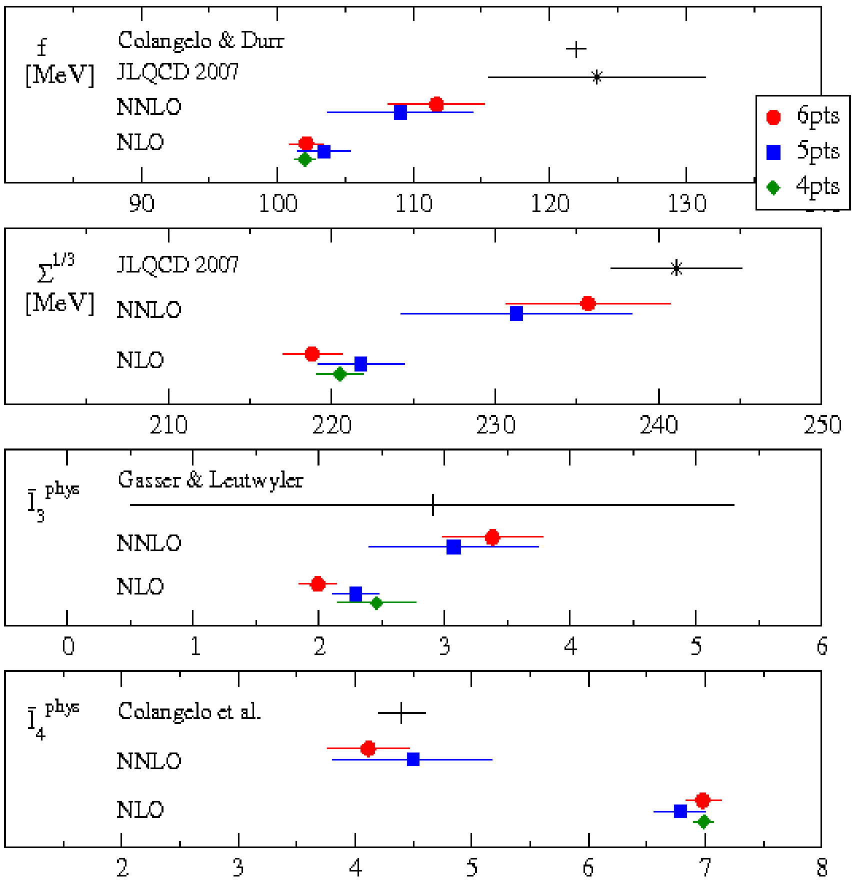
LO → NLO	-10%	+28%
NLO → NNLO	+3%	+18%





NLO vs NNLO

- Test of the ξ -expansion
  - Heavior data fit to NLO (NLO is OK?)
  - Check of LECs needed
  - Significant deviations from NNLO fit



- NNLO is required for consistency with phenomenology/independent calcs.

f = 111.7(3.5)(1.0)(+6.0/-0.0) MeV

$l^r_3(m_\pi)$ = 3.38(40)(24)(+31/-0)

$l^r_4(m_\pi)$ = 4.12(35)(30)(+31/-0)

Σ = [235.7(5.0)(2.0)(+12.7/-0.0) MeV]^3

m_{ud} (2GeV) = 4.452(81)(38)(+0/-227) MeV

f_π = 119.6(3.0)(1.0)(+6.4/-0.0) MeV

Results in N_f=2+1

～Application to Kaon physics ～

6/10/2009, EPOCHAL TSUKUBA

J.Noaki for JLQCD

Kaon Physics on LQCD

- Kaon is out of NLO ChPT.
- Possible lattice stragegies:
  - Simulation on the physical point (m_{ud}, m_s)
    - Very expensive
    - No extrapolation, give up to determine LECs.
  - Integrate out the strange quark: SU(2) +O(m_{ud}/m_s)

    Gasser et al, 2007; RBC+UKQCD, 2008; PACS-CS, 2008
    - NLO extrapolation possible, SU(2) LECs are determined.
  - Inclusion of the higher order effect (NNLO)
    - Requires two-loop calculation
    - Only way to determine SU(3) LECs
    - Some LECs are imported as inputs.

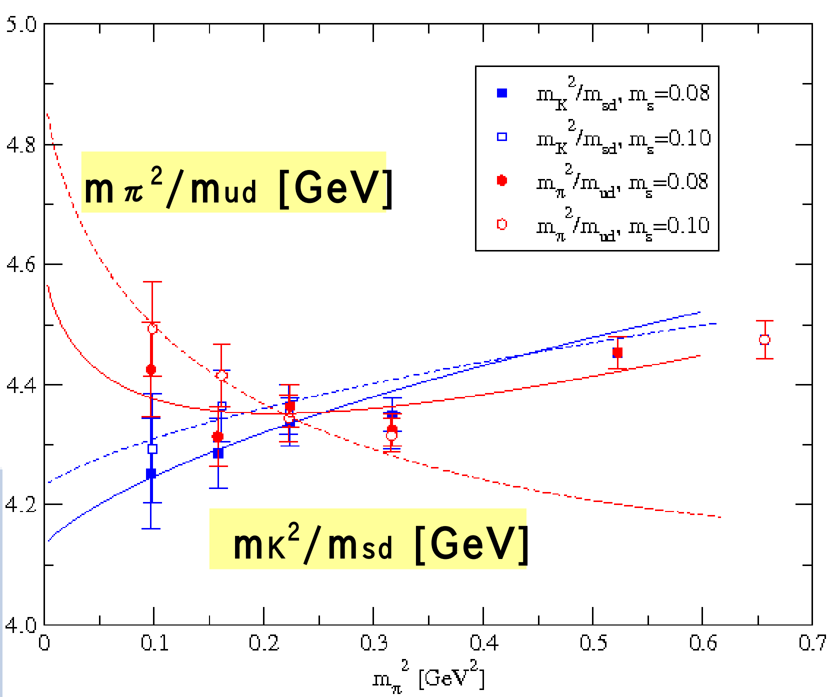
Reduced SU(2) ChPT

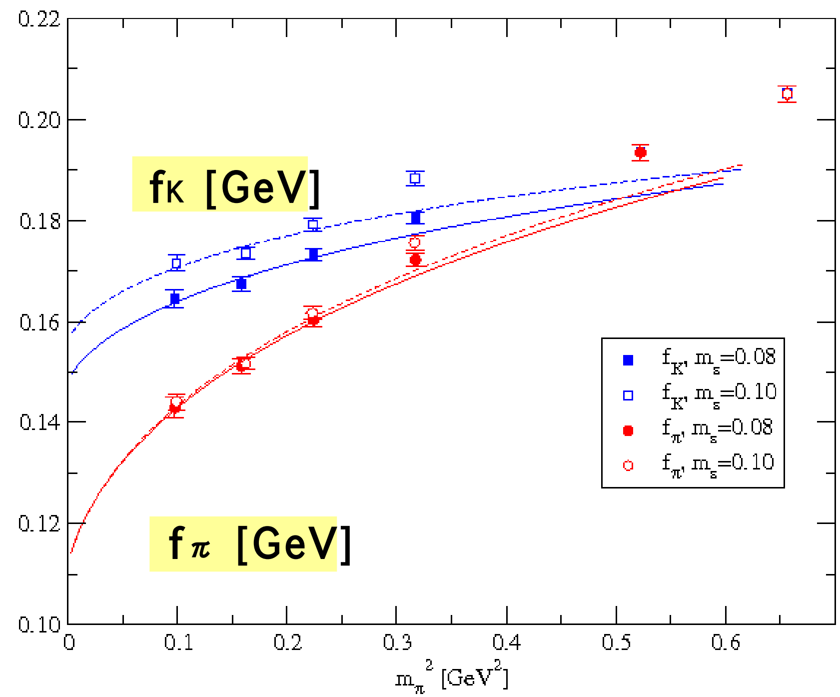
- **NLO formulae** Gasser et al, 2007; RBC+UKQCD, 2008

$$m_\pi^2/m_{ud} = 2B\left(1+\frac{1}{2}\xi\ln\xi\right)+c_3\xi \qquad m_K^2/m_{sd} = 2B^{(K)}\left(1+l_1^{(K)}\xi\right)$$

$$f_\pi = f\left(1-\xi\ln\xi\right)+c_4\xi \qquad f_K = f^{(K)}\left(1-\frac{3}{8}\xi\ln\xi+l_2^{(K)}\xi\right)$$

- **Fit with the 3 lightest points: χ^2/dof < 2.0**





N_f=2+1, NNLO ChPT

- m_π^2/m_{ud}, m_K^2/m_{sd} Amoros, Bijnens and Talavera, 2000

$$
\begin{aligned}
m_\pi^2/m_{ud} = 2B_0\Big[&1 + \tfrac{1}{2}\xi_\pi \ln\xi_\pi - \tfrac{1}{6}\xi_\eta \ln\xi_\eta - (\xi_\pi + \xi_K)\xi_\pi \ln\xi_\pi - \xi_K^2 \ln\xi_K - (\tfrac{553}{144} + \tfrac{1}{2}\xi_K/\xi_\pi)(\xi_\pi \ln\xi_\pi)^2 \\
&-\xi_\pi\xi_K \ln\xi_\pi \ln\xi_K + \tfrac{1}{4}\xi_\pi\xi_\eta \ln\xi_\pi \ln\xi_\eta - (\tfrac{1}{6}\xi_\pi/\xi_K + \tfrac{7}{4})(\xi_K \ln\xi_K)^2 - \tfrac{1}{3}\xi_K\xi_\eta \ln\xi_K \ln\xi_\eta \\
&-\tfrac{5}{36}(\xi_\pi/\xi_\eta + \tfrac{5}{4})(\xi_\eta \ln\xi_\eta)^2 \\
&+\tfrac{5}{6}H(\xi_\pi,\xi_\pi,\xi_\pi,\xi_\pi)\xi_\pi - \tfrac{5}{8}H(\xi_\pi,\xi_K,\xi_K,\xi_\pi)\xi_\pi + \tfrac{1}{18}H(\xi_\pi,\xi_\eta,\xi_\eta,\xi_\pi)\xi_\pi \\
&+H(\xi_K,\xi_\pi,\xi_K,\xi_\pi)\xi_K - \tfrac{5}{6}H(\xi_K,\xi_K,\xi_\eta,\xi_\pi)\xi_\pi + \tfrac{3}{8}H(\xi_\eta,\xi_K,\xi_K,\xi_\pi)\xi_\eta \\
&+H_1(\xi_\pi,\xi_K,\xi_K,\xi_\pi)\xi_\pi + 2H_1(\xi_K,\xi_K,\xi_\eta,\xi_\pi)\xi_\pi + 3H_{21}(\xi_\pi,\xi_\pi,\xi_\pi,\xi_\pi)\xi_\pi \\
&-\tfrac{3}{8}H_{21}(\xi_\pi,\xi_K,\xi_K,\xi_\pi)\xi_\pi + 3H_{21}(\xi_K,\xi_\pi,\xi_K,\xi_\pi)\xi_\pi + \tfrac{9}{8}H_{21}(\xi_\eta,\xi_K,\xi_K,\xi_\pi)\xi_\pi \\
&-L_1^r\,(56\xi_\pi^2 \ln\xi_\pi + 64\xi_K^2 \ln\xi_K + 16\xi_\eta^2 \ln\xi_\eta) - L_2^r\,(32\xi_\pi^2 \ln\xi_\pi + 16\xi_K^2 \ln\xi_K + 4\xi_\eta^2 \ln\xi_\eta) \\
&-L_3^r\,(28\xi_\pi^2 \ln\xi_\pi + 20\xi_K^2 \ln\xi_K + 4\xi_\eta^2 \ln\xi_\eta) - L_7^r \tfrac{64}{3}(\xi_\pi - \xi_K)\xi_\eta \ln\xi_\eta \\
&+L_4^r\,(8\xi_\pi^2 \ln\xi_\pi + 32\xi_K^2 \ln\xi_K + 8\xi_\eta^2 \ln\xi_\eta) + L_5^r\,(4\xi_\pi^2 \ln\xi_\pi - \tfrac{4}{9}(11\xi_\pi - 8\xi_K)\xi_\eta \ln\xi_\eta) \\
&+(L_4^r - 2L_6^r)(-8\xi_\pi - 16\xi_K + 8(7\xi_\pi + 3\xi_K)\xi_\pi \ln\xi_\pi + 8(\xi_\pi + 6\xi_K)\xi_K \ln\xi_K - \tfrac{8}{3}(\xi_\pi - 7\xi_K)\xi_\eta \ln\xi_\eta) \\
&+(L_5^r - 2L_8^r)\,(-8\xi_\pi + 36\xi_\pi^2 \ln\xi_\pi + 8(\xi_\pi + 2\xi_K)\xi_K \ln\xi_K + 4\xi_\pi\xi_\eta \ln\xi_\eta)\Big] \\
&+\alpha_1^\pi \xi_\pi^2 + \alpha_2^\pi \xi_\pi \xi_K + \alpha_3^\pi \xi_K^2
\end{aligned}
$$

$$
\begin{aligned}
m_K^2/m_{sd} = 2B_0\Big[&1 + \tfrac{1}{12}(\xi_\pi + 3\xi_\eta)/\xi_K \cdot \xi_\eta \ln\xi_\eta - \tfrac{3}{4}\xi_\pi\xi_K \ln\xi_\pi - \tfrac{3}{4}\xi_K(\xi_\pi + \xi_K)\ln\xi_K \\
&-\tfrac{1}{4}\xi_\eta(\xi_\pi + 2\xi_K)\ln\xi_\eta - (\tfrac{1}{2}\xi_K/\xi_\pi + \tfrac{27}{32})(\xi_\pi \ln\xi_\pi)^2 - \tfrac{3}{4}\xi_\pi\xi_K \ln\xi_\pi \ln\xi_K \\
&+\tfrac{1}{12}(\xi_\pi/\xi_K - \tfrac{41}{4})\xi_\pi\xi_\eta \ln\xi_\pi \ln\xi_\eta - \tfrac{1}{8}(3\xi_\pi/\xi_K + \tfrac{251}{9})(\xi_K \ln\xi_K)^2 - \tfrac{2}{3}\xi_K\xi_\eta \ln\xi_K \ln\xi_\eta \\
&-\tfrac{1}{1152}\,(43\xi_\pi^2/(\xi_\eta\xi_K) + 225\xi_\pi/\xi_K + 32)\,(\xi_\eta \ln\xi_\eta)^2 \\
&+\tfrac{3}{8}H(\xi_\pi,\xi_\pi,\xi_K,\xi_K)(2\xi_\pi + \xi_K) + \tfrac{1}{4}H(\xi_\pi,\xi_K,\xi_\eta,\xi_K)\xi_K \\
&-\tfrac{3}{32}H(\xi_K,\xi_\pi,\xi_\pi,\xi_K)\xi_K + \tfrac{9}{16}H(\xi_K,\xi_\pi,\xi_\eta,\xi_K)\xi_K + \tfrac{3}{4}H(\xi_K,\xi_K,\xi_K,\xi_K)\xi_K \\
&+\tfrac{181}{288}H(\xi_K,\xi_\eta,\xi_\eta,\xi_K)\xi_K - \tfrac{3}{2}H_1(\xi_\pi,\xi_\pi,\xi_K,\xi_K)\xi_K - \tfrac{3}{2}H_1(\xi_K,\xi_\pi,\xi_\eta,\xi_K)\xi_K \\
&-\tfrac{5}{4}H_1(\xi_K,\xi_\eta,\xi_\eta,\xi_K)\xi_K + \tfrac{9}{4}H_{21}(\xi_\pi,\xi_\pi,\xi_K,\xi_K)\xi_K - \tfrac{9}{32}H_{21}(\xi_K,\xi_\pi,\xi_\pi,\xi_K)\xi_K \\
&+\tfrac{27}{16}H_{21}(\xi_K,\xi_\pi,\xi_\eta,\xi_K)\xi_K + \tfrac{9}{4}H_{21}(\xi_K,\xi_K,\xi_K,\xi_K)\xi_K + \tfrac{27}{32}H_{21}(\xi_K,\xi_\eta,\xi_\eta,\xi_K)\xi_K \\
&-L_1^r(48\xi_\pi^2 \ln\xi_\pi + 72\xi_K^2 \ln\xi_K + 16\xi_\eta^2 \ln\xi_\eta) - L_2^r(12\xi_\pi^2 \ln\xi_\pi + 36\xi_K^2 \ln\xi_K + 4\xi_\eta^2 \ln\xi_\eta) \\
&-L_3^r(15\xi_\pi^2 \ln\xi_\pi + 30\xi_K^2 \ln\xi_K + 7\xi_\eta^2 \ln\xi_\eta) - L_7^r \tfrac{32}{3}(\xi_\pi^2/\xi_K - 3\xi_\pi + 2\xi_K)\xi_\eta \ln\xi_\eta \\
&+L_4^r(24\xi_\pi^2 \ln\xi_\pi + 16\xi_K^2 \ln\xi_K + 8\xi_\eta^2 \ln\xi_\eta) - L_5^r \tfrac{8}{9}(2\xi_\pi^2/\xi_K - 9\xi_\pi + 4\xi_K)\xi_\eta \ln\xi_\eta \\
&-4(L_4^r - 2L_6^r)(2\xi_\pi + 4\xi_K - (11\xi_\pi + 8\xi_K)\xi_\pi \ln\xi_\pi - 2(\xi_\pi + 8\xi_K)\xi_K \ln\xi_K + \tfrac{1}{3}(5\xi_\pi - 8\xi_K)\xi_\eta \ln\xi_\eta) \\
&-4(L_5^r - 2L_8^r)(2\xi_K - (3\xi_\pi + 4\xi_K)\xi_\pi \ln\xi_\pi - 8\xi_K^2 \ln\xi_K + \tfrac{1}{3}(7\xi_\pi - 2\xi_\pi^2/\xi_K - 8\xi_K)\xi_\eta \ln\xi_\eta)\Big] \\
&+\alpha_1^K \xi_\pi(\xi_\pi - \xi_K) + \alpha_2^K \xi_K(\xi_K - \xi_\pi) + (\alpha_1^\pi + \alpha_2^\pi + \alpha_3^\pi)\xi_\pi\xi_K.
\end{aligned}
$$

N_f=2+1, NNLO ChPT

- f_π, f_K Amoros, Bijnens and Talavera, 2000

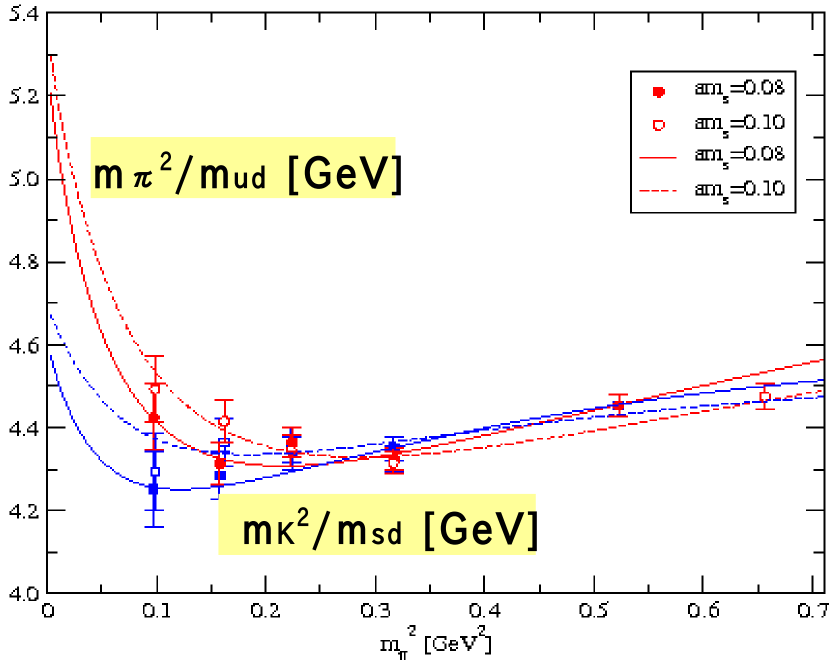
$$
\begin{aligned}
f_\pi = f_0\Big[&1 - \xi_\pi \ln\xi_\pi - \tfrac{1}{2}\xi_K \ln\xi_K + (\tfrac{3}{4}\xi_\pi + \tfrac{1}{2}\xi_K)\xi_\pi \ln\xi_\pi + (\tfrac{1}{8}\xi_\pi + \tfrac{1}{2}\xi_K)\xi_K \ln\xi_K \\
&+(\tfrac{87}{32} + \tfrac{1}{4}\xi_K/\xi_\pi)(\xi_\pi \ln\xi_\pi)^2 + 2\xi_\pi\xi_K \ln\xi_\pi \ln\xi_K + (\tfrac{5}{4} - \tfrac{1}{8}\xi_\pi/\xi_K)(\xi_K \ln\xi_K)^2 + \tfrac{3}{32}(\xi_\eta \ln\xi_\eta)^2 \\
&-\tfrac{1}{2}H(\xi_\pi,\xi_\pi,\xi_\pi,\xi_\pi)\xi_\pi + \tfrac{1}{16}H(\xi_\pi,\xi_K,\xi_K,\xi_\pi)(\xi_\pi - 8\xi_K) - \tfrac{3}{16}H(\xi_\eta,\xi_K,\xi_K,\xi_\pi)\xi_\eta \\
&+\tfrac{5}{12}H'(\xi_\pi,\xi_\pi,\xi_\pi,\xi_\pi)\xi_\pi^2 + \tfrac{1}{2}H'(\xi_\pi,\xi_K,\xi_K,\xi_\pi)\xi_\pi(\xi_K - \tfrac{5}{8}\xi_\pi) + \tfrac{1}{36}H'(\xi_\pi,\xi_\eta,\xi_\eta,\xi_\pi)\xi_\pi^2 \\
&+\tfrac{1}{48}H'(\xi_K,\xi_K,\xi_\eta,\xi_\pi)\xi_\pi(12\xi_K - 23\xi_\pi) + \tfrac{1}{2}H_1'(\xi_\pi,\xi_K,\xi_K,\xi_\pi)\xi_\pi^2 + H_1'(\xi_K,\xi_K,\xi_\eta,\xi_\pi)\xi_\pi^2 \\
&+\tfrac{3}{2}H_{21}'(\xi_\pi,\xi_\pi,\xi_\pi,\xi_\pi)\xi_\pi^2 - \tfrac{3}{16}H_{21}'(\xi_\pi,\xi_K,\xi_K,\xi_\pi)\xi_\pi^2 + \tfrac{3}{2}H_{21}'(\xi_K,\xi_\pi,\xi_K,\xi_\pi)\xi_\pi^2 \\
&+\tfrac{9}{16}H_{21}'(\xi_\eta,\xi_K,\xi_K,\xi_\pi)\xi_\pi^2 \\
&+L_1^r\,(28\xi_\pi^2 \ln\xi_\pi + 32\xi_K^2 \ln\xi_K + 8\xi_\eta^2 \ln\xi_\eta) + L_2^r\,(16\xi_\pi^2 \ln\xi_\pi + 8\xi_K^2 \ln\xi_K + 2\xi_\eta^2 \ln\xi_\eta) \\
&+L_3^r\,(14\xi_\pi^2 \ln\xi_\pi + 10\xi_K^2 \ln\xi_K + 2\xi_\eta^2 \ln\xi_\eta) \\
&+L_4^r\,(4\xi_\pi + 8\xi_K - (26\xi_\pi + 24\xi_K)\xi_\pi \ln\xi_\pi - (6\xi_\pi + 28\xi_K)\xi_K \ln\xi_K + (2\xi_\pi - 8\xi_K)\xi_\eta \ln\xi_\eta) \\
&+L_5^r\,(4\xi_\pi - 20\xi_\pi^2 \ln\xi_\pi - 10\xi_\pi\xi_K \ln\xi_K)\Big] + \beta_1^\pi\xi_\pi^2 + \beta_2^\pi\xi_\pi\xi_K + \beta_3^\pi\xi_K^2
\end{aligned}
$$

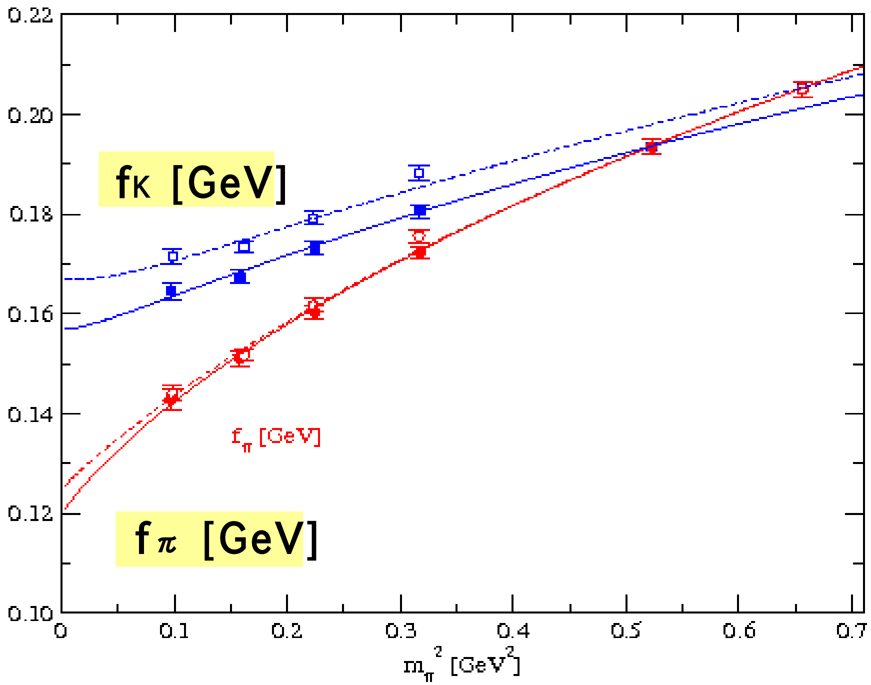
$$
\begin{aligned}
f_K = f_0\Big[&1 - \tfrac{3}{8}\xi_\pi \ln\xi_\pi - \tfrac{3}{4}\xi_K \ln\xi_K - \tfrac{3}{8}\xi_\eta \ln\xi_\eta + \tfrac{15}{32}\xi_K\xi_\pi \ln\xi_\pi + \tfrac{3}{8}(\xi_\pi + \tfrac{3}{2}\xi_K)\xi_\pi \ln\xi_\pi \\
&+\tfrac{1}{8}(\xi_\pi + \tfrac{11}{4}\xi_K)\xi_\eta \ln\xi_\eta + \tfrac{3}{32}(\tfrac{43}{4} + \xi_K/\xi_\pi)(\xi_\pi \ln\xi_\pi)^2 + \tfrac{63}{32}\xi_\pi\xi_K \ln\xi_\pi \ln\xi_K + \tfrac{57}{64}\xi_\pi\xi_\eta \ln\xi_\pi \ln\xi_\eta \\
&+\tfrac{3}{16}(\tfrac{17}{2} + \xi_\pi/\xi_K)(\xi_K \ln\xi_K)^2 + \tfrac{3}{32}\xi_K\xi_\eta \ln\xi_K \ln\xi_\eta + \tfrac{9}{32}(1 + \tfrac{1}{4}\xi_\pi/\xi_\eta)(\xi_\eta \ln\xi_\eta)^2 \\
&-\tfrac{3}{64}H(\xi_\pi,\xi_\pi,\xi_K,\xi_K)(8\xi_\pi - \xi_K) - \tfrac{9}{32}H(\xi_K,\xi_\pi,\xi_\eta,\xi_K)\xi_K - \tfrac{3}{8}H(\xi_K,\xi_K,\xi_K,\xi_K)\xi_K \\
&-\tfrac{9}{64}H(\xi_K,\xi_\eta,\xi_\eta,\xi_K)\xi_K + \tfrac{3}{16}H'(\xi_\pi,\xi_\pi,\xi_K,\xi_K)\xi_K(2\xi_\pi + \xi_K) + \tfrac{13}{32}H'(\xi_\pi,\xi_K,\xi_\eta,\xi_K)\xi_K^2 \\
&-\tfrac{3}{64}H'(\xi_K,\xi_\pi,\xi_\pi,\xi_K)\xi_K^2 + \tfrac{3}{8}H'(\xi_K,\xi_K,\xi_K,\xi_K)\xi_K^2 + \tfrac{181}{576}H'(\xi_K,\xi_\eta,\xi_\eta,\xi_K)\xi_K^2 \\
&-\tfrac{3}{4}H_1'(\xi_\pi,\xi_\pi,\xi_K,\xi_K)\xi_K^2 - \tfrac{3}{4}H_1'(\xi_K,\xi_\pi,\xi_\eta,\xi_K)\xi_K^2 - \tfrac{5}{8}H_1'(\xi_K,\xi_\eta,\xi_\eta,\xi_K)\xi_K^2 \\
&+\tfrac{9}{8}H_{21}'(\xi_\pi,\xi_\pi,\xi_K,\xi_K)\xi_K^2 - \tfrac{9}{64}H_{21}'(\xi_K,\xi_\pi,\xi_\pi,\xi_K)\xi_K^2 + \tfrac{27}{32}H_{21}'(\xi_K,\xi_\pi,\xi_\eta,\xi_K)\xi_K^2 \\
&+\tfrac{9}{8}H_{21}'(\xi_K,\xi_K,\xi_K,\xi_K)\xi_K^2 + \tfrac{27}{64}H_{21}'(\xi_K,\xi_\eta,\xi_\eta,\xi_K)\xi_K^2 \\
&+4L_1^r(6\xi_\pi^2 \ln\xi_\pi + 9\xi_K^2 \ln\xi_K + 2\xi_\eta^2 \ln\xi_\eta) + 2L_2^r(3\xi_\pi^2 \ln\xi_\pi + 9\xi_K^2 \ln\xi_K + \xi_\eta^2 \ln\xi_\eta) \\
&+L_3^r\,(\tfrac{15}{2}\xi_\pi^2 \ln\xi_\pi + 15\xi_K^2 \ln\xi_K + \tfrac{7}{2}\xi_\eta^2 \ln\xi_\eta) \\
&+L_4^r\,(4\xi_\pi + 8\xi_K - (\tfrac{47}{2}\xi_\pi + 19\xi_K)\xi_\pi \ln\xi_\pi - (7\xi_\pi + 30\xi_K)\xi_K \ln\xi_K + (\tfrac{1}{2}\xi_\pi - 11\xi_K)\xi_\eta \ln\xi_\eta) \\
&+L_5^r\,(4\xi_K - (3\xi_\pi + \tfrac{19}{2}\xi_K)\xi_\pi \ln\xi_\pi - (6\xi_\pi + 7\xi_K)\xi_K \ln\xi_K - (3\xi_\pi + \tfrac{3}{2}\xi_K)\xi_\eta \ln\xi_\eta)\Big] \\
&+\beta_1^K\xi_\pi(\xi_\pi - \xi_K) + \beta_2^K\xi_K(\xi_K - \xi_\pi) + (\beta_1^\pi + \beta_2^\pi + \beta_3^\pi)\xi_\pi\xi_K
\end{aligned}
$$

- Inputs: L_1^r, L_2^r, L_3^r, L_7^r
- 16 fit parameters, 36 data points

N_f = 2+1, NNLO ChPT

- Fit curves (χ^2/dof = 1.36)





- Large statistical error in LECs
- More $m_{ud}=m_s$ data points (unitary points) wanted
- Finite size effects to be studied

Preliminaly results

f_0 = 110(40) MeV

Σ_0 = [214(24) MeV]^3

L^r_4 (m_ρ) = -1.17(82) x 10^{-3}

L^r_5 (m_ρ) = -1.1(1.3) x 10^{-3}

L^r_6 (m_ρ) = -0.40(33) x 10^{-3}

L^r_8 (m_ρ) = 0.59(16) x 10^{-3}

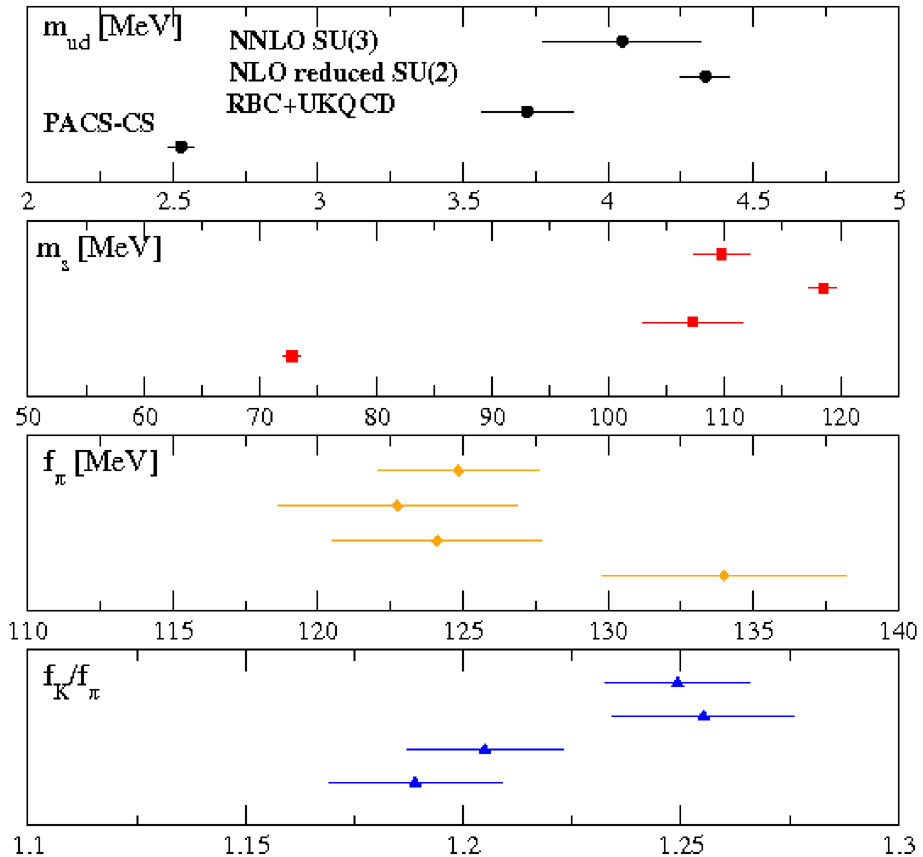
m_{ud}(2GeV) = 4.03(14) MeV

m_s(2GeV) = 108.9(1.3) MeV

f_π = 124.7(3.4) MeV

f_K = 154.8(2.7) MeV

f_K/f_π = 1.241(19)



- r_0 = 0.49 fm is used to determine a^{-1}.
- could be determined with f_π
  → r_0 = 0.47 fm: agrees with other lattice calcs.

Summary & Future plan

- Convergence of ChPT
  - Not clear at NLO.
  - Tested options for kaon physics on the lattice.
    - Reduced SU(2) ChPT
    - Full SU(3) to NNLO.

- Plans for near future
  - Finalization of planned projects (B_K, form factors, etc)
  - Unitary SU(3) points → Useful to determine LECs
  - Reweighing method to get physical strange mass point
  - Larger volume to check FSE: Coming soon

Thank you for your attention.

Backup slides

Data points

- Light meson spectrum: "A touchstone of LQCD"
- Data improved with low-lying modes

Giusti et al., 2003; DeGrand & Schaefer, 2004

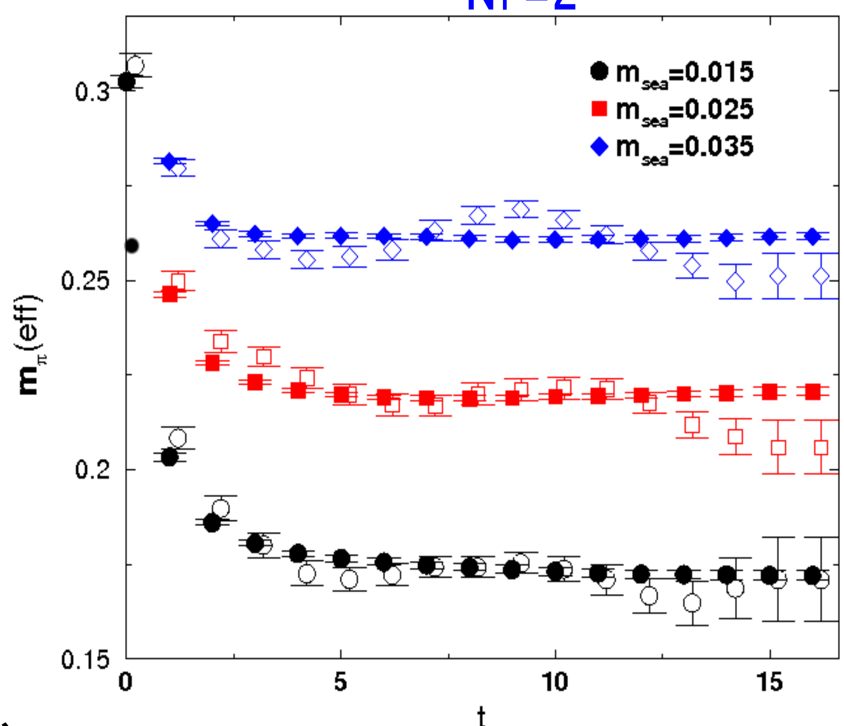
$$D_{ov} u_i = \lambda_i u_i$$

$$S_q(x,y) = \frac{\sum_{i=1}^{N} u_i(x) u_i(y)}{\lambda_i + m_q} + S_q^{High}(t)$$

$$C(t) = C^{HH}(t) + C^{HL}(t) + C^{LH}(t) + C^{LL}(t)$$

averaging

$N_f = 2$



- FSE corrections with ChPT ($m_\pi L < 3$)

Luscher, 1985; Colangelo et al.,2005

- Luscher's formulae at most -4% (m_π^2), +5% (f_π)
- Fixed topology collection +4% +0.1%
- Justified due to the exact chiral symmetry