Outline

- Introduction
- Mass extrapolation & chiral lagrangian
- V_{us}
    - f_K/f_π
    - K_{l3}
- Kaon bag parameters
    - B_K
    - BSM
- Other topics

Current simulations

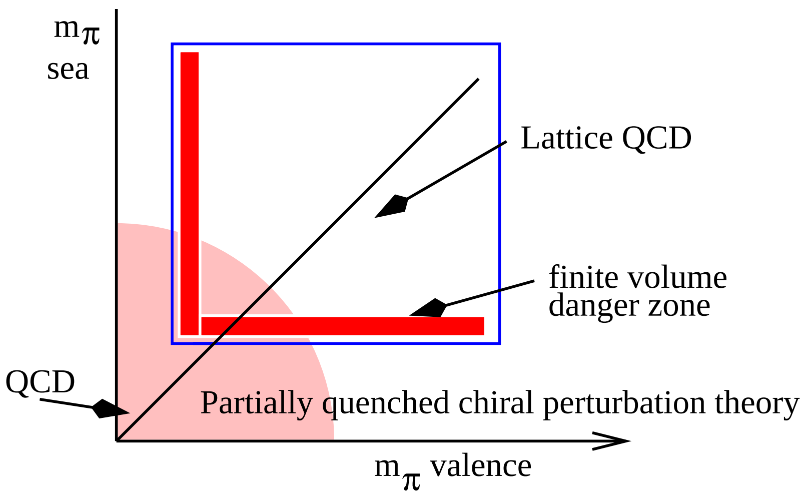
Introduction

- Wholeheartedly support FLAG's aims
  perhaps some disagreement in detail, but it is a very important effort!
- Unfortunately can not match this level of detail
- Will rather try to emphasize the most interesting activity in the field
- Also recomend excellent Lattice 2008 review by Lellouch arXiv:0902.4545

Introduction

Lattice QCD simulations not (currently) at physical masses in a large volume.

Interpolations are performed in m_s
Extrapolations are performed in m_{ud} from larger masses $m_\pi \in [200 - 500] MeV$ down.



Some calculations follow unitary line $m_\pi^{\rm sea} = m_\pi^{\rm valence}$

Some calculations explore orthogonal direction and include unitary data as a subset: “partially quenched”

$$\text{quenched} \neq \text{unitary} \subset \text{partially quenched}$$

Finite volume corrections sometimes applied affects smallest masses. Most popular model: Colangelo, Durr, Haefeli (hep-lat/0503014).

Introduction

List simulations covering broad physics programme relevant to this conference

Collaboration	Action	n_f	Chirality	Cost
JLQCD	Overlap	2 (2+1)	Exact	$ 50
RBC-UKQCD	DWF	2+1	Good	$ 16
ETMC	Twisted mass	2	Poor	$ 1
PACS-CS	Clover	2+1	Poor	$ 1
BMW	smeared clover	2+1	Poor	$ 1
MILC	Staggered	2+1	Poor	$ 0.25

Also: QCDSF, CLS collaborations.

Overlap and DWF have good lattice chiral symmetry but are expensive
$\longrightarrow$ weak matrix elements have continuum renormalisation structure
$\longrightarrow$ widest range of chiral weak operators
$\longrightarrow$ "V - A " does exactly what it says

PACS-CS use perturbative renormalisation for Z_A
BMW have not yet calculated Z_A but plan to do so non-perturbatively.

Introduction

List simulations covering broad physics programme relevant to this conference

Collaboration	Action	L(fm)	$m_\pi^{\rm min}$ (MeV)	a^{-1} GeV
JLQCD	Overlap	1.7	310	1.8
RBC-UKQCD	DWF	2.0,2.7	330	1.7,2.3
ETMC	Twisted mass	2.0	300	1.9,2.2,2.8
PACS-CS	Clover	3.1	156	2.2
BMW	smeared clover	4.0	190	1.6,2.3,3.03
MILC	Staggered	2.4,2.9,3.4	320	1.1,1.3,1.6,2.2,3.3,4.4

Also: QCDSF, CLS collaborations.

PACS-CS and BMW are getting very close the physical point

Exciting times!

Introduction

List simulations covering broad physics programme relevant to this conference

Collaboration	Action	ChPT	Unitary	Finite vol. corr.
JLQCD	Overlap	$\mathrm{SU}(2)_{\mathrm{n_f}}$	U	CDH
RBC-UKQCD	DWF	$\mathrm{SU}(2)_{\mathrm{n_f}}, \mathrm{SU}(3)_{\mathrm{n_f}}$	PQ	-
ETMC	Twisted mass	$\mathrm{SU}(2)_{\mathrm{n_f}}$	U	CDH
PACS-CS	Clover	$\mathrm{SU}(2)_{\mathrm{n_f}}, \mathrm{SU}(3)_{\mathrm{n_f}}$	U	CDH
BMW	smeared clover	polynomial	-	-
MILC	Staggered	rs-$\mathrm{SU}(3)_{\mathrm{n_f}}$	PQ	-

Also: QCDSF, CLS collaborations.

Colangelo, Durr, Haefeli used to “correct” volume effects

Outline

- Introduction
- Mass extrapolation & chiral lagrangian
- V_{us}
  - f_K/f_π
  - K_{l3}
- Kaon bag parameters
  - B_K
  - BSM
- Other topics

$$
\begin{aligned}
\chi_q &= 2Bm_q \\
M_P^2 &= \chi_q + \frac{\chi_q^2}{32\pi^2F^2}\left[\log\frac{\chi_q}{m_\pi^2} - \bar{l}_3\right] \\
F_P^2 &= F - \frac{\chi_q}{16\pi^2F^2}\left[\log\frac{\chi_q}{m_\pi^2} - \bar{l}_4\right] \\
F^{\pi\pi}(q^2) &= 1 + \frac{1}{F^2}\left[-2l_6^r q^2 + 4\tilde{\mathcal{H}}(\chi_q, q^2, \mu^2)\right] \\
\Pi_{V-A}^{(1)} &= -\frac{f_\pi^2}{q^2} - 8L_{10}^r(\mu) - \frac{\log(\frac{m_\pi^2}{\mu^2} + \frac{1}{3} - H(x))}{24\pi^2}
\end{aligned}
$$

2+1 flavors : $SU(3)$ or $SU(2)$?

Debate: best mass extrapolation method for 2+1f?

- $\mathrm{SU}(3)_{\mathrm{n_f}}$ – Treats the Kaon as a dynamical chiral pseudoscalar
  Least convergent terms expand in $(M_\eta/4\pi f)^2$
- $\mathrm{SU}(2)_{\mathrm{n_f}}$ **RBC & UKQCD hep-ph/0702042,arXiv:0804.0473**
  Treats the Kaon as a heavy spectator
  Least convergent terms expand in $(M_\pi/M_K)^2$
- Analytic expansion around all physical quark masses **BMW**
  Discussed Lellouch, Kaon review Lattice 2008 arXiv:0902.4545
  Argument compelling near physical point *if volume big enough*

Chiral effective theory guides extrapolations to physical masses (and in some cases volumes).

Convergence radius of chiral effective theory critical component of physical predictions.

Knowledge of LEC's yields good predictive tool with different scope than direct lattice simulation.

Debate:

- Inclusion of partial NNLO analytic terms?
- NNLO with non-lattice input for LEC's?
- Finite volume correction of data?

2+1 flavors : $SU(3)$ or $SU(2)$?

Both RBC-UKQCD and PACS-CS have recently found (arXiv:0804.0473 , arXiv:0811.2630):

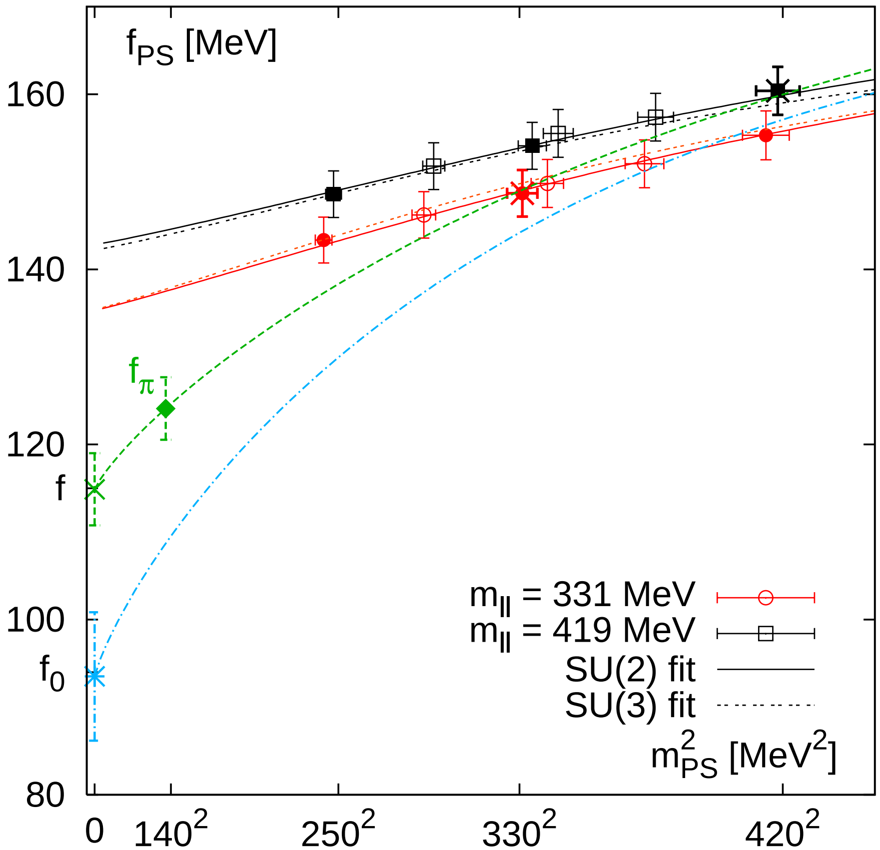
- Kaons cannot be well described by NLO $\chi_3^{\rm PT}$
- Both $\chi_3^{\rm PT}$ and $\chi_2^{\rm PT}$ can fit pionic lattice data $160 \le m_\pi \le 400$ MeV

Collaboration	F_0 (MeV)	F/F_0
MILC	106(8)	1.15(5)$\left(^{+13}_{-3}\right)$
RBC/UKQCD	93.5(7.3)	1.229(59)
PACS-CS	118.5(9)	1.078(58)

- Pseudoscalar decay constant F_0 in 3 flavor chiral limit may be quite low
- Simulated pions ($\simeq 310$ MeV) receive worryingly large NLO correction to this low F_0
- Some spread. PACS-CS use a LPT one-loop Z_A - however cancels in F/F_0

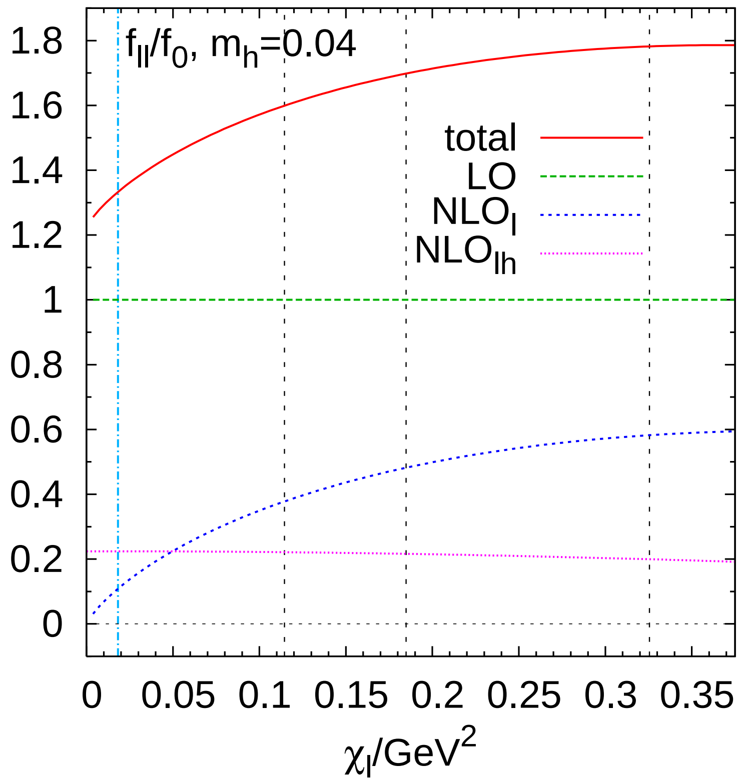
symmetrised MILC sys err in quad w. stat

2+1f DWF RBC-UKQCD f_π



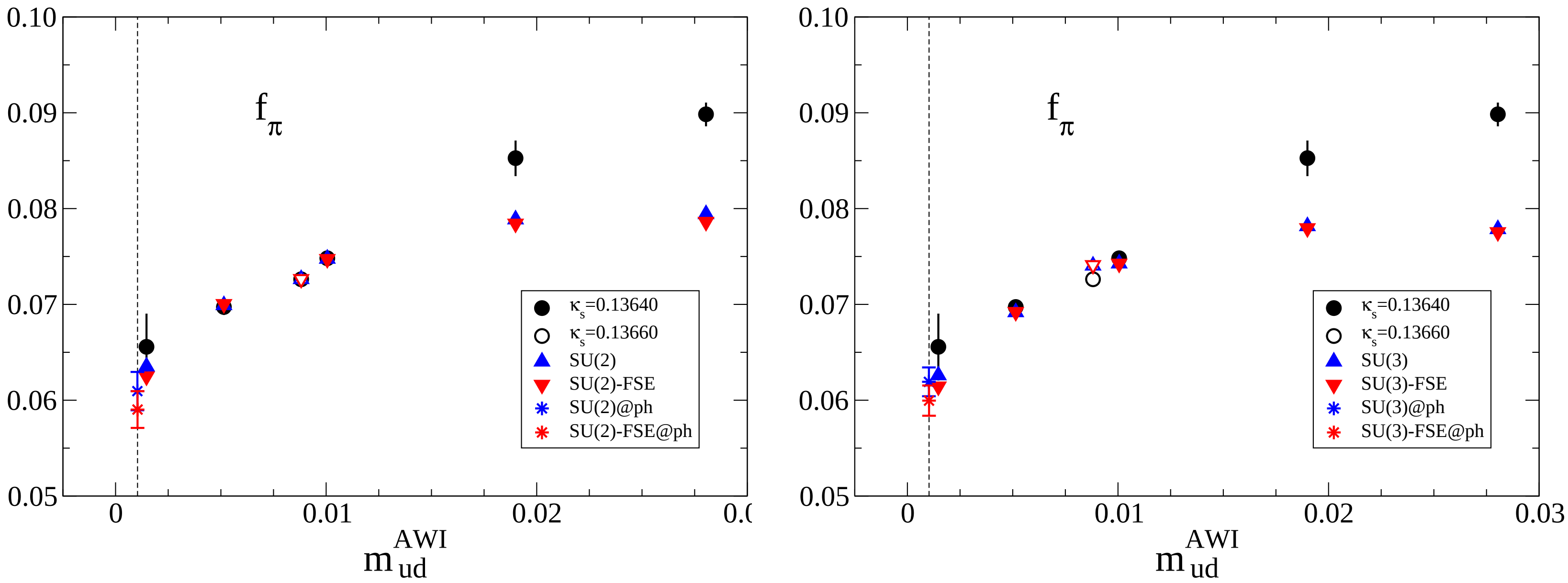
- Partially quenched & simultaneous fit with pion mass
- DWF axial current is conserved ⇒ renormalisation controlled
- Single lattice spacing

2+1f DWF RBC-UKQCD f_π



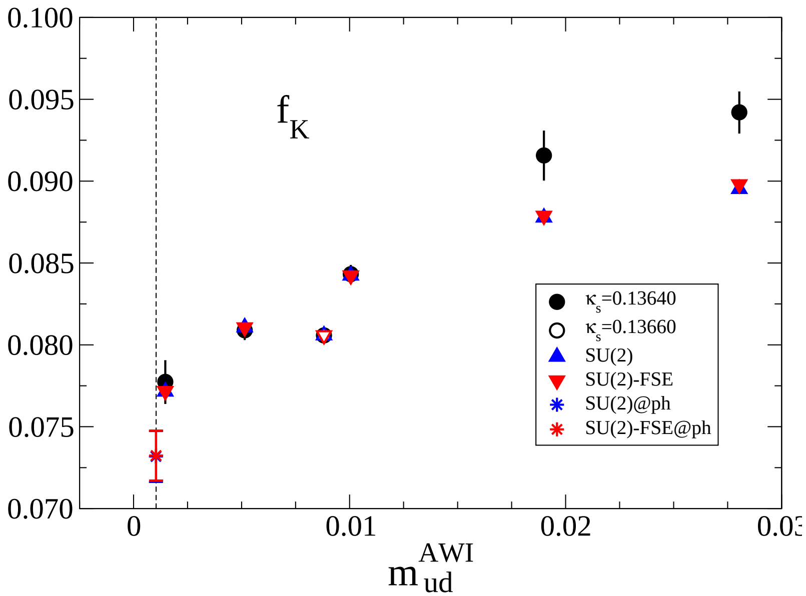
- Partially quenched & simultaneous fit with pion mass
- DWF axial current is conserved $\Rightarrow$ renormalisation controlled
- Single lattice spacing

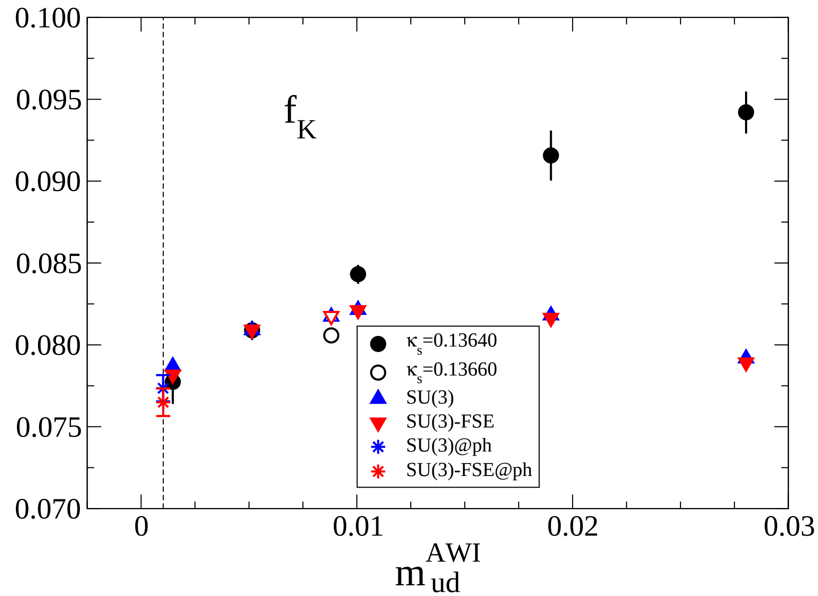
2+1f Clover PACS-CS f_π



- Unitary & simultaneous fit with pion mass
- Not far from direct simulation at the physical mass (but small vol)
  Finite volume CDH adjust at lightest masses

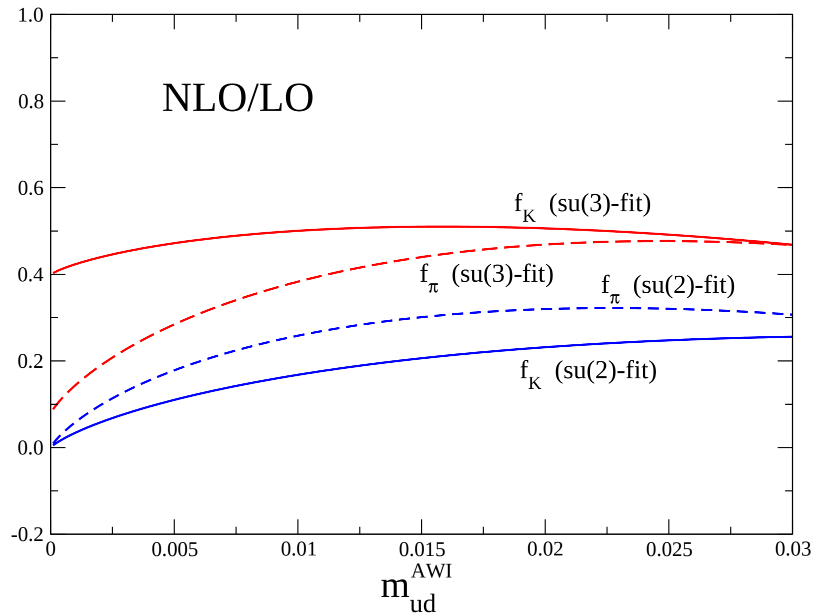
2+1f Clover PACS-CS f_π





- Kaon is poorly described by SU(3)

2+1f Clover PACS-CS f_π



- PACS-CS see a smaller F/F_0 ratio than MILC and RBC-UKQCD
- SU(3) corrections to pionic data correspondingly smaller
- Z_A is only 1-loop LPT
  Physical values likely too high due to higher orders
- Does not affect decay constant ratios & convergence discussion

$SU(3)$ vs $SU(2)$

Conclusions:

- Broad agreement that SU(2) is better for Kaons
- Some disagreement about just how low F_0 is
- Convergence of $SU(3)$ χ^{PT}questioned even for pionic lattice data

As SU(3) reduces to SU(2) in large strange mass limit

- SU(3) form fits pionic data well but with large NLO terms
- F_0 & other LEC's from lattice data describe "real world" pions well
- These may not be *true* LEC's massless 3-flavour theory

Lookout for:

- JLQCD's 2+1f overlap results will be very interesting
- RBC-UKQCD's second lattice spacing later this year

Chiral effective lagrangian

Quote SU(2) LEC's as more meaningful – perturbative conversion of SU(3) results

From S. Necco arXiv:0901.4257 (excellent recent review)
F in convention where $f_\pi \simeq 92$MeV

Collaboration	N_f	F (MeV)	F_0 (MeV)	$\Sigma^{1/3}$ (MeV)	$\Sigma_0^{1/3}$ (MeV)
ETM	2	86.03(5)		267(2)(9)(4)	
JLQCD/TWQCD	2	79.0(2.5)(0.7)$\left(^{+4.2}_{-0.0}\right)$		235.7(5.0)(2.0)$\left(^{+12.7}_{-0.0}\right)$	
MILC	2+1			$278(1)\left(^{+2}_{-3}\right)(5)$	$242(9)\left(^{+5}_{-17}\right)(4)$
RBC/UKQCD	2+1	81.2(2.9)(5.7)		255(8)(8)(13)	
PACS-CS	2+1	90.3(3.6)	83.8(6.4)	309(7)	290(15)
Giusti, Lüscher	2			276(3)(4)(5)	
ETM	2	86.6(4)(7)		264(2)(5)	
Colangelo, Dürr	phen.	86.2(5)			

+ omitted large body of ϵ-regime work. See Necco.

Chiral effective lagrangian

Quote SU(2) LEC's as more meaningful – perturbative conversion of SU(3) results

From S. Necco arXiv:0901.4257 (excellent recent review)

F in convention where $f_\pi \simeq 92$MeV

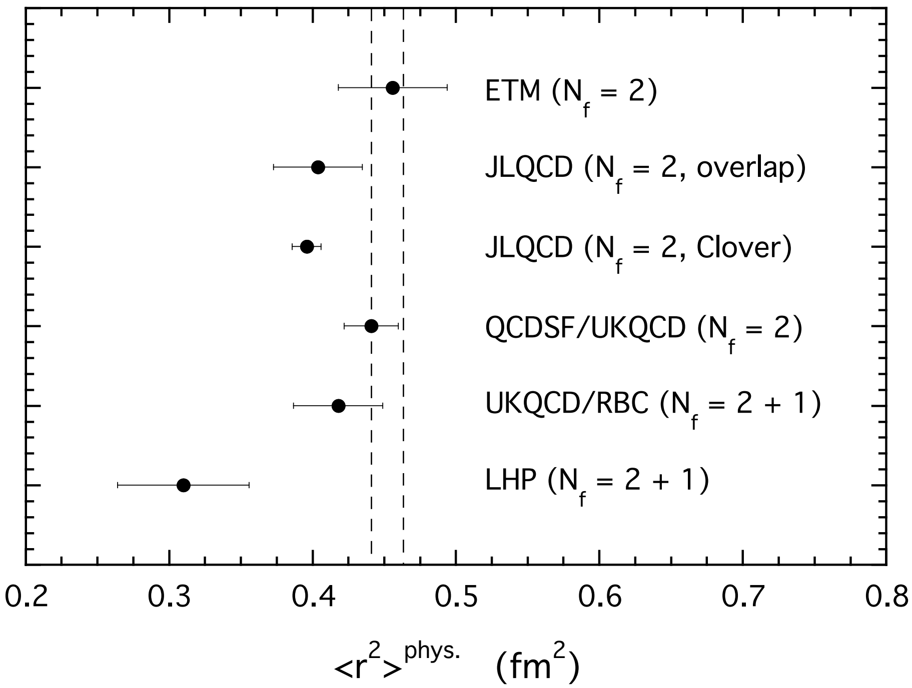
Collaboration	N_f	$\bar{l}_3$ (SU(2))	$\bar{l}_4$ (SU(2))	$\bar{l}_3$ (SU(3))	$\bar{l}_4$ (SU(3))
CERN-TOV	2	3.0(5)(1)			
ETM	2	3.42(8)(10)(27)	4.59(4)(2)(13)		
ETM	2	3.2(4)(2)	4.4(1)(1)		
JLQCD/TWQCD	2	3.44(57)$\binom{+0}{-68}\binom{+32}{-0}$	4.14(26)$\binom{+49}{-0}\binom{+32}{-0}$		
MILC	2+1			$1.1(6)\binom{+1.0}{-1.5}$	$4.4(4)\binom{+4}{-1}$
RBC/UKQCD	2+1	3.13(33)(24)	4.43(14)(77)	2.87(28)	4.10(5)
PACS-CS	2+1	3.14(23)	4.04(19)	3.47(11)	4.21(11)
Gasser, Leutwyler	phen.	2.9(2.4)	4.3(9)		
Colangelo *et al*	phen.		4.4(2)		

+ omitted large body of ϵ-regime work. See Necco.

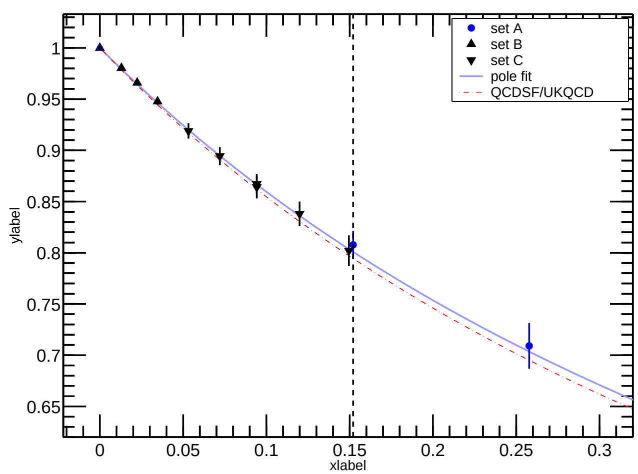
Pion form factors & l_6

$$\langle \pi^+(p')|V_\mu|\pi^+(p)\rangle = F^{\pi\pi}(q^2)(p_\mu + p'_\mu)$$

ETMC arXiv:0812.4042



RBC-UKQCD arXiv:0804.3971



- Simulation required at low mass *and* q^2
- Twisted BC's (ETMC'07,RBC-UKQCD'08) get low q^2 region
- ETMC concluded NNLO fits needed in mass extrap

Pion form factors & l_6

$$\langle \pi^+(p')|V_\mu|\pi^+(p)\rangle = F^{\pi\pi}(q^2)(p_\mu + p'_\mu)$$

$$\begin{aligned}\langle r_\pi^2\rangle &= 6\frac{d}{dq^2}f^{\pi\pi}(q^2)|_{q^2=0}\\ &= -\frac{12l_6^r}{f^2} - \frac{1}{8\pi^2 f^2}\left(\log\frac{m_\pi^2}{\mu^2}+1\right)\end{aligned}$$

Collab	Action	$\langle r_\pi^2\rangle_V$	$\langle r_\pi^2\rangle_S$
ETMC	2f TM	0.456(38)	
JLQCD	2f Overlap	0.409(23)(37)	0.617(79)(66)
JLQCD	2f Clover	0.396(10)	
QCDSF	2f Clover	0.441(19)	
RBC-UKQCD	2+1f DWF	0.418(31)	
LHPC	2+1f MILC/DWF	0.310(46)	

Pion form factors & l_6

$$\langle \pi^+(p')|V_\mu|\pi^+(p)\rangle = F^{\pi\pi}(q^2)(p_\mu + p'_\mu)$$

Scalar form factor:

- JLQCD **arXiv:0905.2465** have determined the scalar form factor.
- Useful complement to f_π to determine $\bar{l}4$
- ETMC used experiment & χ^{PT}as input to better constrain NNLO fits.
- *Both* use NNLO χ^{PT}analyses with non-lattice input

L_{10}^r and chiral fermions

Determine L_{10}^r from difference in vector and axial vacuum polarisation

JLQCD arXiv:0806.4222, $N_f = 2$

$$\Pi_{V_{\mu\nu}} - \Pi_{A_{\mu\nu}} = \left(q^2\delta_{\mu\nu} - q_\mu q_\nu\right)\Pi^{(1)}_{V-A} - q_\mu q_\nu \Pi^{(0)}_{V-A}$$

$$\Pi^{(1)}_{V-A} = -\frac{f_\pi^2}{q^2} - 8L_{10}^r(\mu) - \frac{\log(\frac{m_\pi^2}{\mu^2} + \frac{1}{3} - H(x))}{24\pi^2}$$

Only accessible with lattice chiral fermions (Overlap, DWF)

JLQCD, $N_f = 2$

$$L_{10}^r m_\rho = -5.2(2)^{+5}_{-3} \times 10^{-3}$$

Pion mass squared e-m splitting (in chiral limit):

$$m^2_{\pi^\pm} = 993(12)(^{+0}_{-135})(149)\mathrm{MeV}^2$$

PDG (physical masses): $1261\mathrm{MeV}^2$

L_{10}^r and chiral fermions

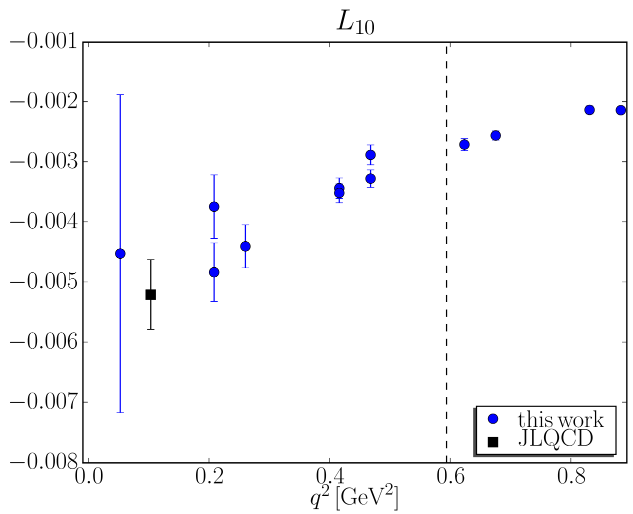
Determine L_{10}^r from difference in vector and axial vacuum polarisation

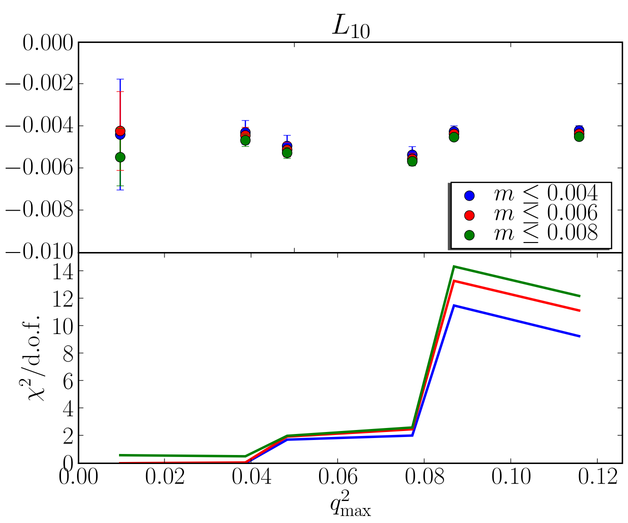
JLQCD arXiv:0806.4222, $N_f = 2$

$$\Pi_{V_{\mu\nu}} - \Pi_{A_{\mu\nu}} = \left(q^2\delta_{\mu\nu} - q_\mu q_\nu\right)\Pi^{(1)}_{V-A} - q_\mu q_\nu \Pi^{(0)}_{V-A}$$

$$\Pi^{(1)}_{V-A} = -\frac{f_\pi^2}{q^2} - 8L_{10}^r(\mu) - \frac{\log(\frac{m_\pi^2}{\mu^2} + \frac{1}{3} - H(x))}{24\pi^2}$$

Approach copied by RBC-UKQCD: preliminary $N_f = 2+1$ **(Wennekers)**



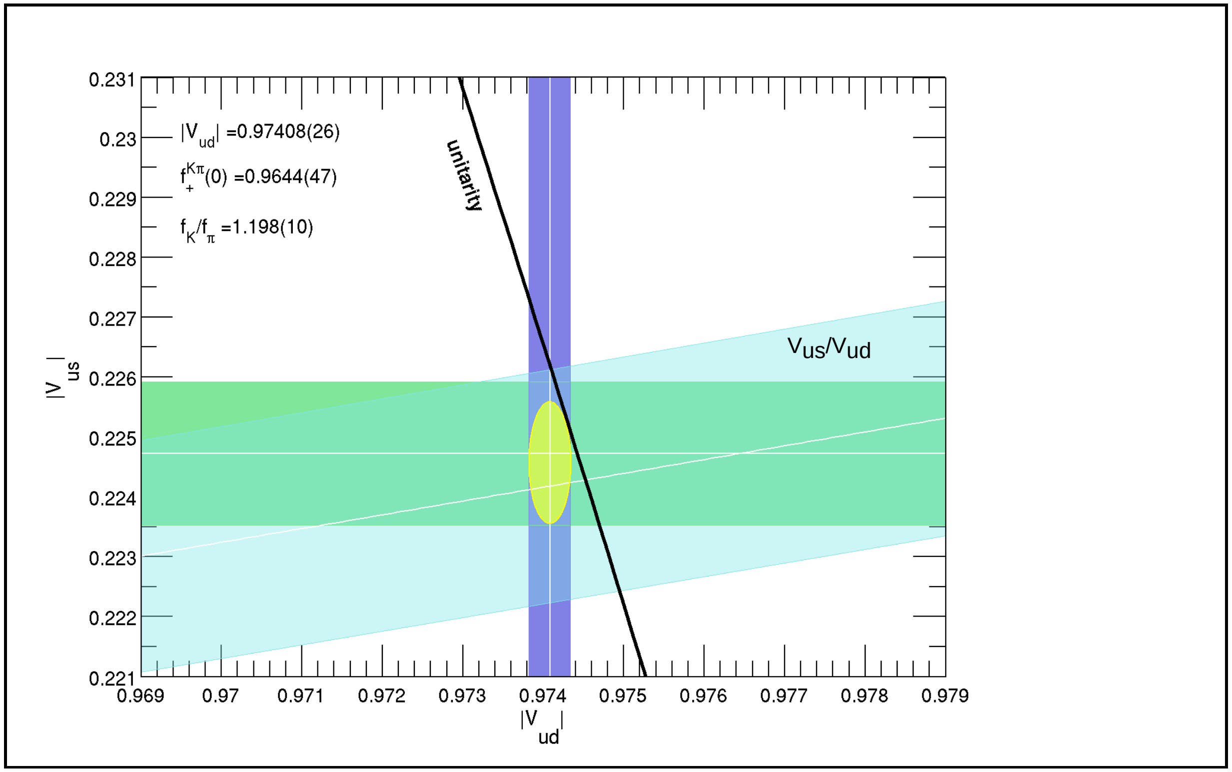


$$L_{10}^r = -4.7(3) \times 10^{-3}$$

Error is statistical only

Outline

- Introduction
- Mass extrapolation & chiral lagrangian
- V_{us}
  - f_K / f_π
  - K_{l3}
- Kaon bag parameters
  - B_K
  - BSM
- Other topics



V_{us}

Two best lattice constraints for V_{us} involve SU(3) breaking effects

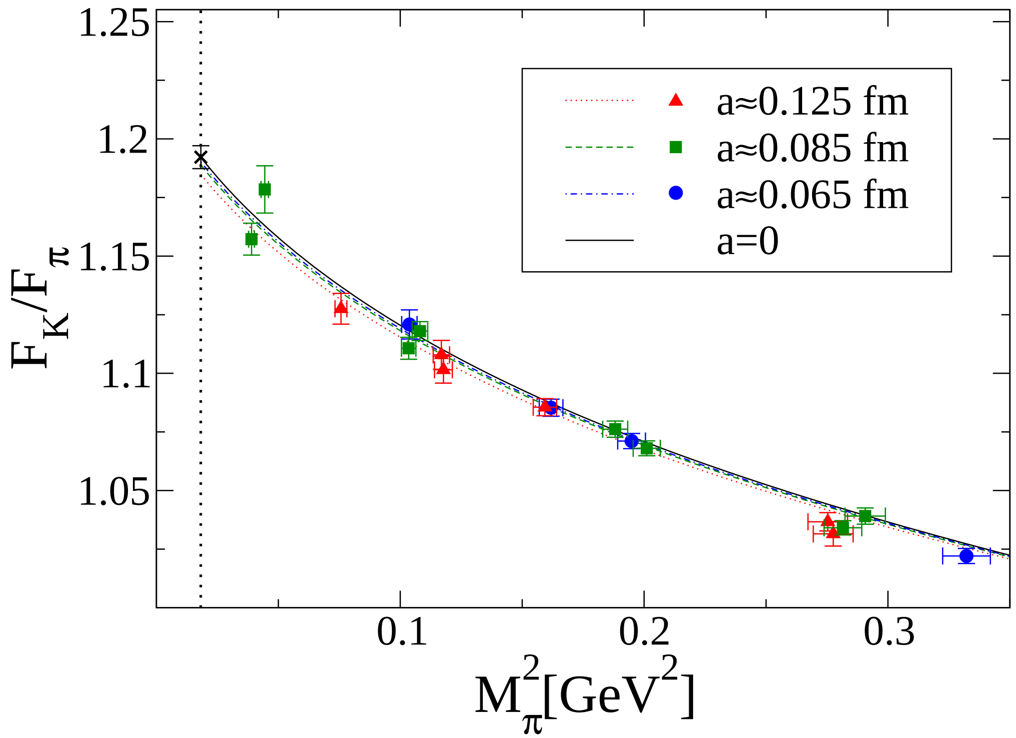
quantity	SU(3) breaking
$\frac{f_K}{f_\pi}$	20%
$f_0^{K\pi}(q^2 = 0)$	4%.

While less mature K_{l3} looks very promising and is rapidly becoming better studied.

Debate: Systematic error estimation:

- Exactly unity in the $m_s = m_u$ limit for $a \neq 0$ and $V \neq \infty$ and even configuration by configuration
- Lattice errors should be suppressed and are likely a fraction of the deviation from unity
- Associating a precise error is difficult

BMW



- No detailed exposition in preprints yet, but exciting
- 2+1f clover & $a^{-1} \in \{1.6, 2.3, 3.0\}$ GeV
- *No renormalisation yet*
- Is axial current improved non-peturbatively ?
- 190 MeV pion masses ... wow!

ETMC

ETMC have updated (arXiv:0904.0954) for $N_f = 2$ twisted mass:

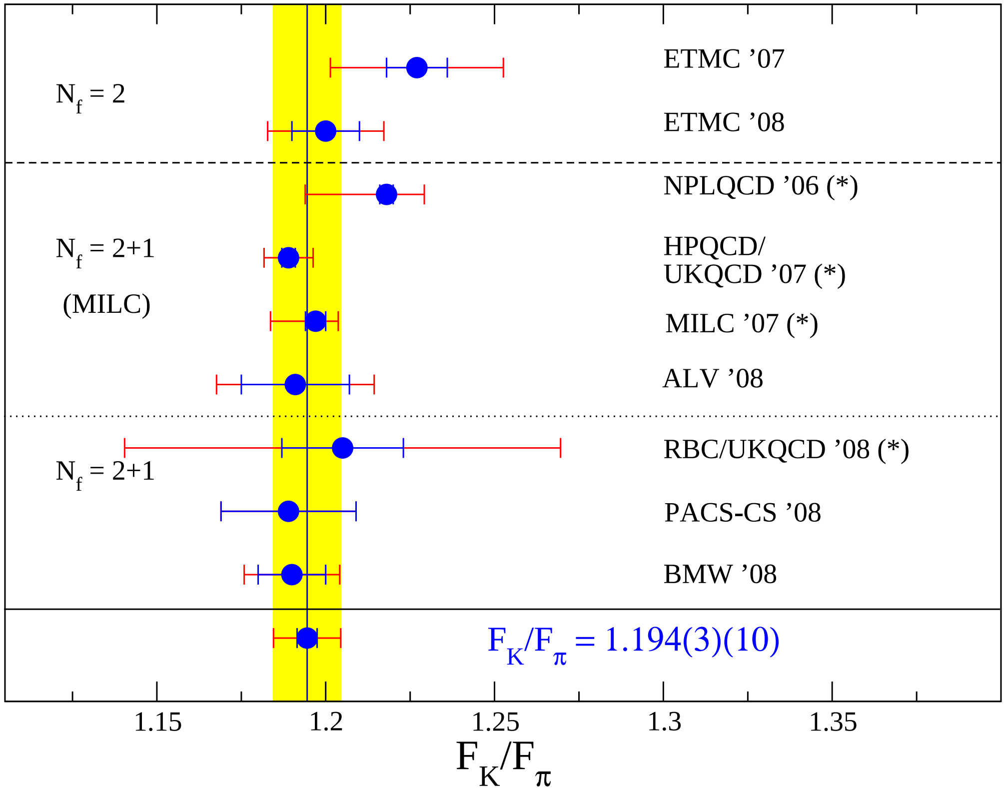
$$f_K/f_\pi = 1.210(18)$$

Analysis uses measured pseudoscalar meson masses in chiral formulae rather than quark masses.
Rearranges the $\chi^{\rm PT}$beyond LO
Easier to fit multiple lattice spacings simultaneously without computing change in Z_m

ETMC also follow the RBC-UKQCD approach and find SU(2)-ChPT better describes the kaon sector.

f_K/f_π

Lellouch Lattice 2008 world average:



- Here RBC-UKQCD estimated discretisation error as 4% of 1.2
  Likely pessimistic

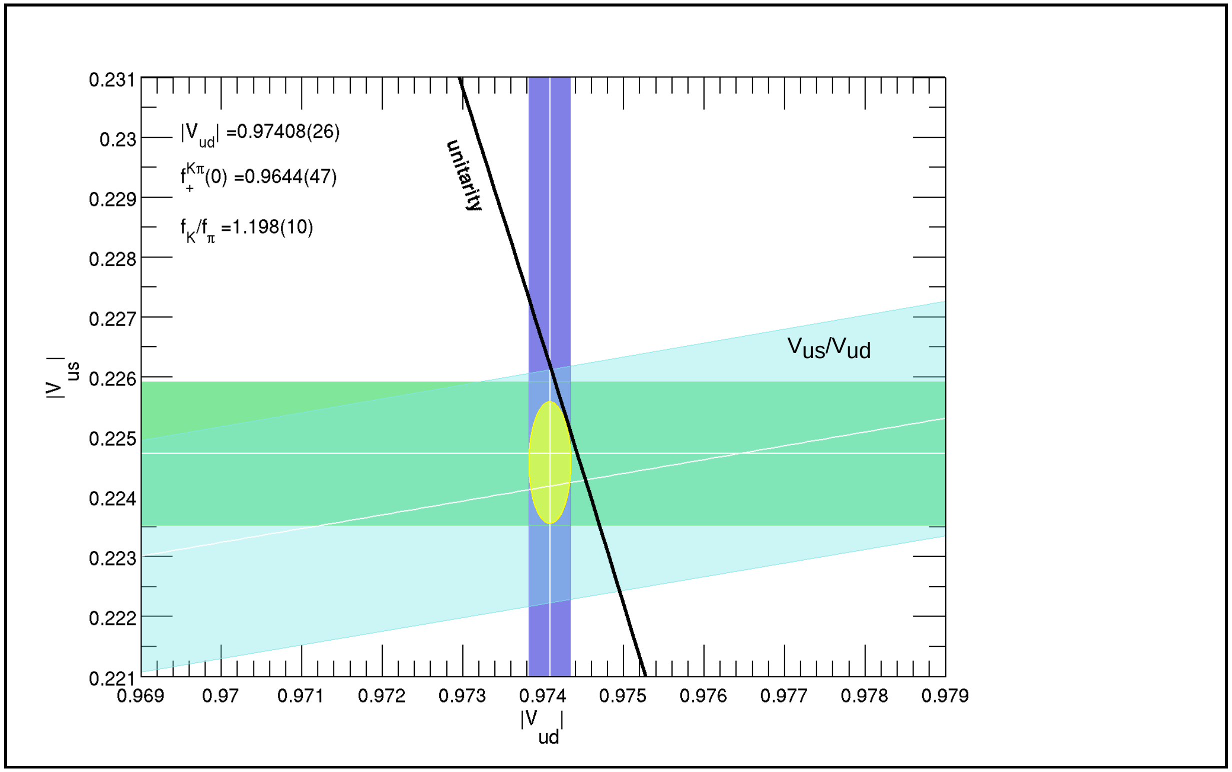
f_K/f_π

Update of Lellouch Lattice 2008 table:

Ref.	N_f	action	$a[\,\mathrm{fm}]$	LM_π^{min} typ/val	$M_\pi^{min}[\,\mathrm{MeV}]$ typ/val	F_K/F_π
PDG'08						1.193(6)
ETM'08	2	tmQCD	$0.07, 0.09, 0.10[F_\pi]$	3.6/3.6	$260/260$	*1.196(13)(7)(8)*
ETM'09	2	tmQCD	$0.07, 0.09, 0.10[F_\pi]$	3.6/3.6	$260/260$	*1.210(6)(15)(9)*
NPLQCD'06	2+1	DWF/ $\mathrm{KS_{MILC}}$	$0.13[r_0]$	5.1/3.5	$420/290$	$1.218(2)^{+11}_{-24}$
MILC'04-07	2+1	$\mathrm{KS^{AsqTad}_{MILC}}$	$0.06, 0.09, 0.012, 0.15[F_\pi]$	5.3/4.2	$300/240$	$1.197(3)^{+6}_{-13}$
HPQCD/'07 UKQCD	2+1	$\mathrm{KS^{HISQ}_{MILC}}$	$0.09, 0.12, 0.15[\Upsilon]$	4.8/4.1	$360/310$	1.189(2)(7)
RBC/'08 UKQCD	2+1	DWF	$0.11[\Omega]$	4.1/3.4	$290/240$	1.205(18)(62)
ALV'08	2+1	DWF/ $\mathrm{KS_{MILC}}$	$0.09, 0.12[\Upsilon/F_\pi]$	5.3/4.2	$300/240$	*1.191(16)(17)*
PACS-CS'08	2+1	NP-SW	$0.09[\Omega]$	2.3/2.3	$160/160$	1.189(20)
BMW'08	2+1	SW	$0.065, 0.085, 0.125[\Xi]$	$4/4$	$190/190$	*1.19(1)(1)*

Outline

- Introduction
- Mass extrapolation & chiral lagrangian
- V_{us}
  - f_K/f_π
  - K_{l3}
- Kaon bag parameters
  - B_K
  - BSM
- Other topics



K_{l3}

$$\langle \pi(p')|V_\mu|K(p)\rangle = f_+(q^2)(p_\mu + p'_\mu) + f_-(q^2)(p_\mu - p'_\mu)$$

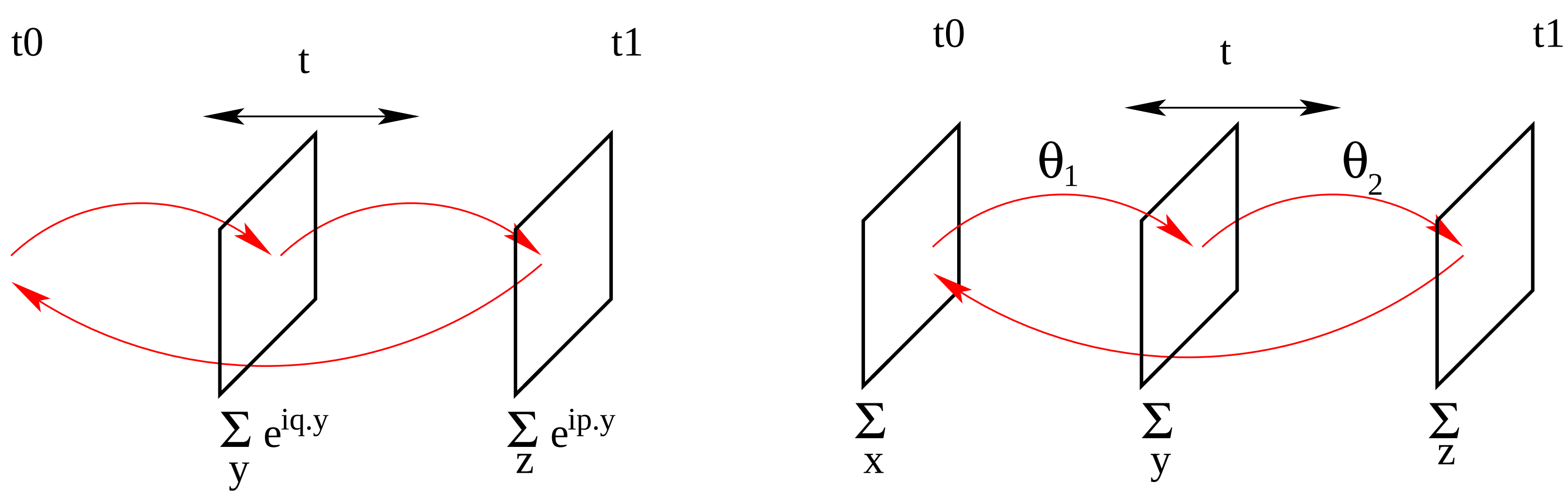
- Several “double ratios” employed (hep-ph/0403217)

e.g.

$$\frac{\langle K(\vec{0})|V_0|\pi(\vec{0})\rangle\langle K(\vec{0})|V_0|\pi(\vec{0})\rangle}{\langle K(\vec{0})|V_0|K(\vec{0})\rangle\langle \pi(\vec{0})|V_0|\pi(\vec{0})\rangle} = \frac{(m_K + m_\pi)^2}{4 m_K m_\pi}|f_0(q^2_{\rm max})|^2$$

Lattice methods undergone two related improvements

- Twisted boundary conditions enable simulation directly at $q^2 = 0$
  Previously constrained to discrete Fourier momenta
  Model dependent interpolation to $q^2 = 0$ can be eliminated
- L^3 volume average can be taken improving statistical precision



K_{l3}

Recent results for $f_+^{K\pi}$ (again from Lellouch)

Not aware of any updates in literature

Ref.	N_f	action	$a[\,\mathrm{fm}]$	$L[\,\mathrm{fm}]$	$M_\pi^{min}[\,\mathrm{MeV}]$ typ/val	$f_+(0)$
JLQCD	2	NP SW	0.09	1.8	550/550	0.967(6)
RBC	2	DWF	0.12	2.5	490/490	0.968(9)(6)
ETM	2	tmQCD	0.11	2.7	260/260	0.957(5)
FNAL/MILC	2+1	KS+Wil				0.962(6)(9)
RBC/UKQCD	2+1	DWF	0.11	1.8, 2.8	290/240	0.9644(33)(34)(14)

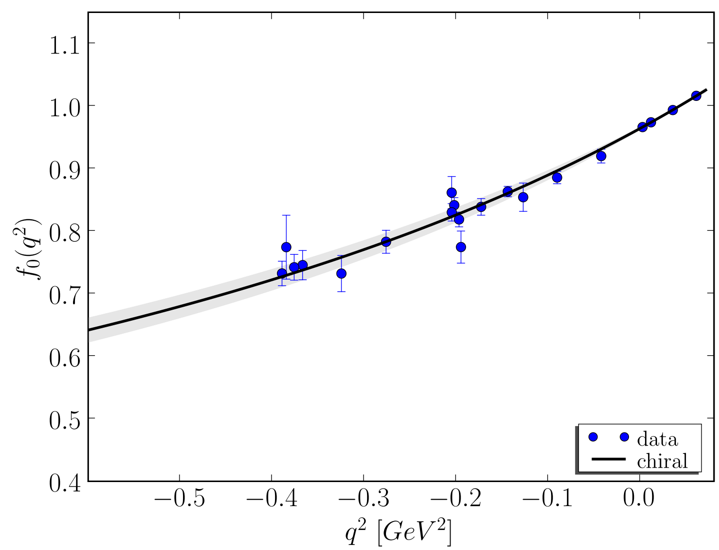
Best constraint is from RBC-UKQCD

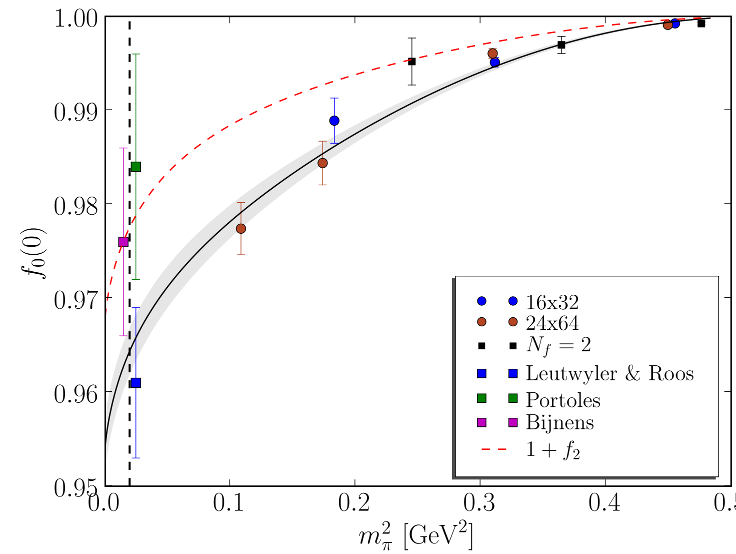
RBC-UKQCD K_{l3}

Phys.Rev.Lett.100:141601,2008.

Combined q^2 and chiral extrapolation w. hybrid pole dominance/Ademollo-Gatto constrained model.

$$f_0(q^2, m_\pi^2, m_K^2) = \frac{1 + f_2 + (m_K^2 - m_\pi^2)^2(A_0 + A_1(m_K^2 + m_\pi^2))}{1 - q^2/(M_0 + M_1(m_K^2 + m_\pi^2))^2}$$

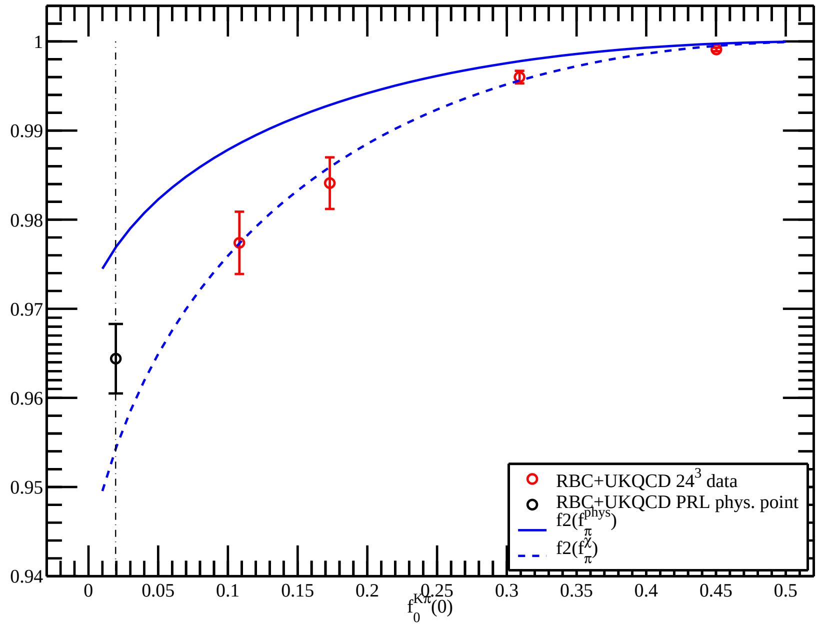




$$f_+(0) = 0.9644(33)^{\text{stat}}(34)^{\text{extrapolation}}(14)^{\text{disc}}$$

- model dependence in q^2 interpolation & chiral extrapolation & strange mass adjust
- no continuum limit; budget 4% of $1 - f_+(0)$
- Is difference from f_2 poor convergence of χ^{PT}???

RBC-UKQCD K_{l3}



- Hang on... SU(3) breaking is *only* 4%
- Replace f_π with $F_0 = 93.5$ in f_2
- Looks much more convergent & commensurate with degree of breaking
- Favours positive Δ_f

Independent determination of F_0?

FLAG have SM determination of $f_+(0)$ from: (Colangelo & Sachrajda, private communication)

- CKM unitarity
- V_{ud} from Towner & Hardy arXiv:0812.1202
- $f_K/f_\pi V_{us}/V_{ud}$
- $f_+(0)V_{us}$

$$f_+(0) = 0.9620(20)(37)$$

assume convergence: f_2 is dominant contribution to $1 - f_+(0)$ &

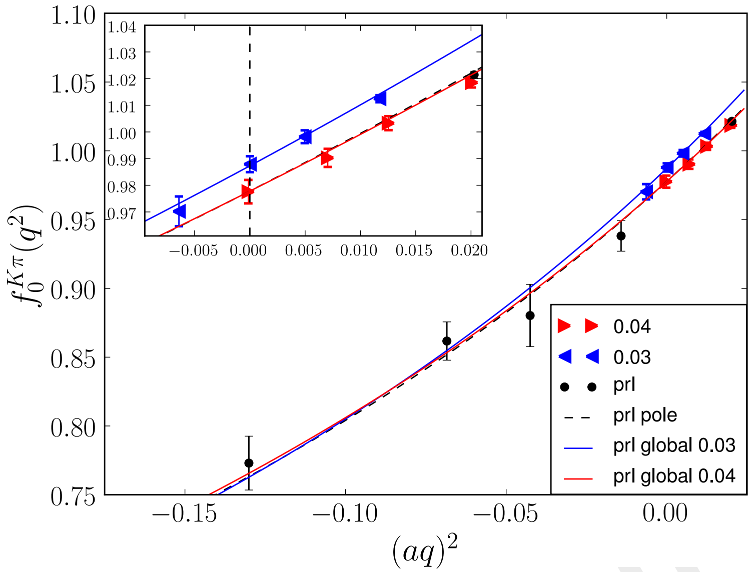
$$F_0 = 101.9(2.6)(4.8)\mathrm{MeV}$$

- interesting given spread of lattice determinations
- Including positive NNLO Δ_f predictions would lead to even slightly lower F_0
- may explain preference of lattice $f_+(0)$ for Leutwyler-Roos
  $\longrightarrow f_2$ should have been lower in the first place

RBC-UKQCD preliminary

Thanks to Zanotti, Juettner, De Lima

Recalculation for our lightest point ($m_l = 0.005$) using twisted BC approach (Z_2 wall noise sources; color-spin trace performed stochastically)
$\rightarrow$ should better constrain chiral limit & address model dep. & stat errors

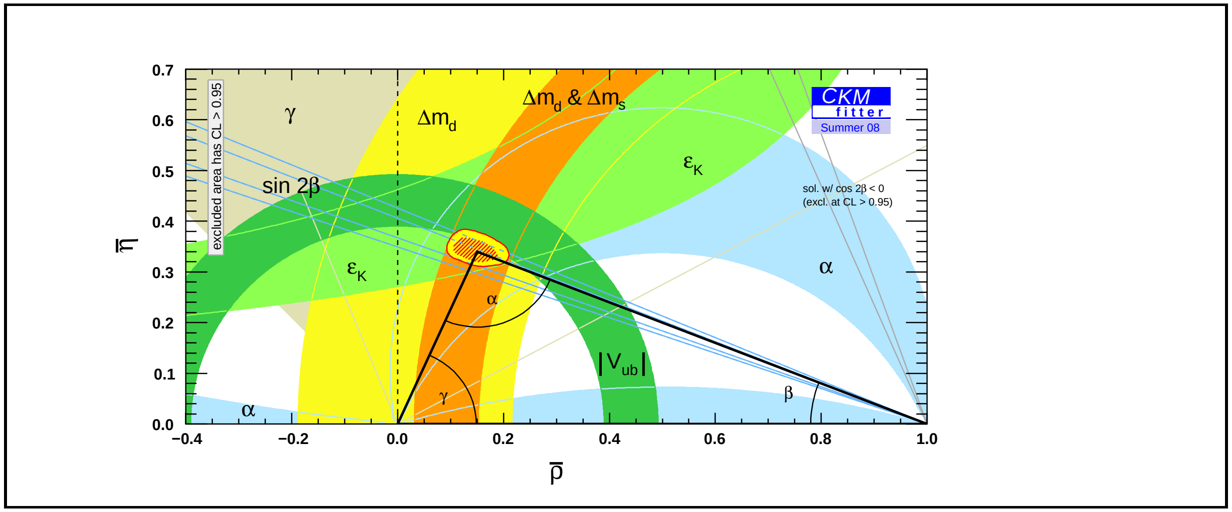


- Lines are prediction from global fit in PRL (black data)
  not a fit to blue and red
- Global fit does correctly describe strange mass dependence
- Remove model dependence by direct $q^2 = 0$
- Improved statistical error with volume average (compare blue/red & black)

32^3 data complete and under analysis. Will address the sub-dominant discretisation error.

Outline

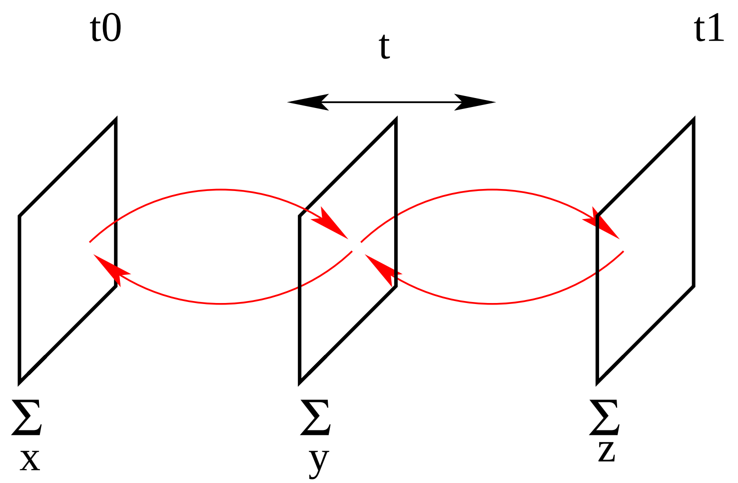
- Introduction
- Mass extrapolation & chiral lagrangian
- V_{us}
  - f_K/f_π
  - K_{l3}
- Kaon bag parameters
  - B_K
  - BSM
- Other topics



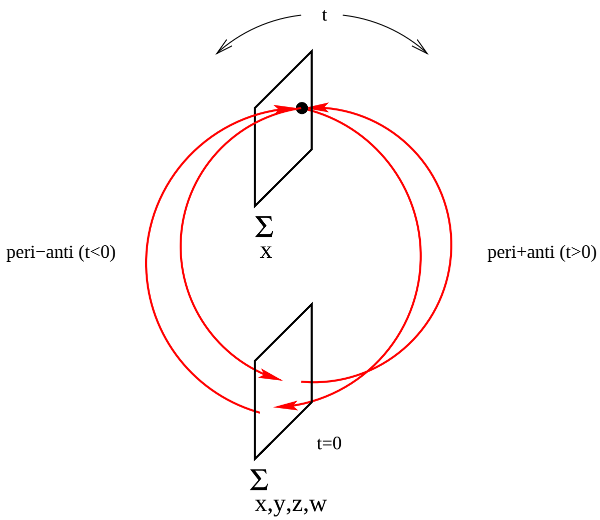
B_K

$$B_K = \frac{\langle K^0|\mathcal{O}_{\mathrm{VV+AA}}|\bar{K}^0\rangle}{\frac{8}{3}\langle K^0|A_0\rangle\langle A_0|K^0\rangle}$$

- Chiral symmetry gives multiplicatively renormalised calculation
  $\longrightarrow$ non-symmetric actions deal with unphysical taste/chirality mixings which inflate errors
  $\longrightarrow$ *mixed actions* sometimes used (e.g. ALV, ETMC)



OR



Recent B_K results

Lellouch 2008 table

Ref.	N_f	action	$a[\,\mathrm{fm}]$	$L[\,\mathrm{fm}]$	$M_\pi^{min}[\,\mathrm{MeV}]$ typ/val	$\hat{B}_K$
JLQCD'08	2	Overlap	0.12	1.9	290/290	0.734(5)(55)
ETM'08	2	OS/tmQCD	0.07,0.09	2.1,2.7	300/300	0.78(3)
HPQCD/ UKQCD'06	2+1	$\mathrm{KS}^{\mathrm{HYP}}_{\mathrm{MILC}}$	0.125	2.5	460/360	0.85(2)(18)
RBC/UKQCD '07-08	2+1	DWF	0.11	1.8,2.8	290/240	0.720(13)(37)
Bae et al '08	2+1	$\mathrm{KS}^{\mathrm{HYP}}_{\mathrm{MILC}}$	≥ 0.06	4	$300/240$	$\delta\hat{B}_K \to 3\%$

Lellouch average was weighted combination of RBC/UKQCD and HPQCD

$$\hat{B}_K = 0.723(11)(35)$$

Recent B_K results

Update Lellouch 2008 table to include recent **ALV arXiv:0905.3947** DWF valence on MILC configurations.

Ref.	N_f	action	$a[\,\mathrm{fm}]$	$L[\,\mathrm{fm}]$	$M_\pi^{min}[\,\mathrm{MeV}]$ typ/val	$\hat{B}_K$
JLQCD'08	2	Overlap	0.12	1.9	290/290	0.734(5)(55)
ETM'08	2	OS/tmQCD	0.07,0.09	2.1,2.7	300/300	0.78(3)
HPQCD/ UKQCD'06	2+1	$\mathrm{KS}^{\mathrm{HYP}}_{\mathrm{MILC}}$	0.125	2.5	460/360	0.85(2)(18)
RBC/UKQCD '07-08	2+1	DWF	0.11	1.8,2.8	290/240	0.720(13)(37)
Bae et al '08	2+1	$\mathrm{KS}^{\mathrm{HYP}}_{\mathrm{MILC}}$	≥ 0.06	4	$300/240$	$\delta\hat{B}_K \to 3\%$
Aubin et al '09	2+1	$\mathrm{DWF/KS}$	0.12,0.09	2.5-3.6	300/230	0.724(8)(28)

Recent B_K results

Update precise 0f DWF continuum limit from **CP-PACS arXiv:0803.2569**

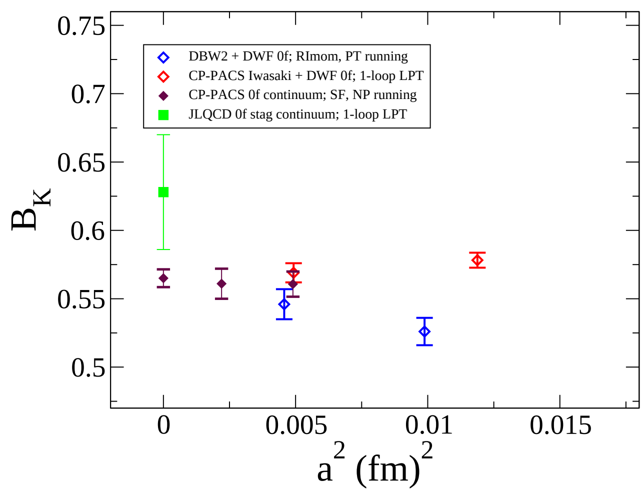
Ref.	N_f	action	$a[\,\mathrm{fm}]$	$L[\,\mathrm{fm}]$	$M_\pi^{min}[\,\mathrm{MeV}]$ typ/val	$\hat{B}_K$
JLQCD'08	2	Overlap	0.12	1.9	290/290	0.734(5)(55)
ETM'08	2	OS/tmQCD	0.07,0.09	2.1,2.7	300/300	0.78(3)
HPQCD/ UKQCD'06	2+1	$\mathrm{KS}^{\mathrm{HYP}}_{\mathrm{MILC}}$	0.125	2.5	460/360	0.85(2)(18)
RBC/UKQCD '07-08	2+1	DWF	0.11	1.8,2.8	290/240	0.720(13)(37)
Bae et al '08	2+1	$\mathrm{KS}^{\mathrm{HYP}}_{\mathrm{MILC}}$	≥ 0.06	4	$300/240$	$\delta\hat{B}_K \to 3\%$
Aubin et al '09	2+1	DWF/KS	0.12,0.09	2.5-3.6	300/230	0.724(8)(28)
JLQCD '08	0f	DWF	0.05-0.10	2.3-2.6	$\infty/340$	0.782(5)(7))

Beautiful result with non-perturbative SF renormalisation and running three lattice spacing extrapolation continuum limit.

Averaging B_K

Lellouch estimated B_K as the weighted average of HPQCD and RBC-UKQCD 2+1f results, while noting discrepancy between staggered $n_f = 0$ and $n_f = 2$ DWF surprisingly large.

- *Already clarified by CP-PACS 0f DWF continuum limit*
- In quenched approximation JLQCD got a similar high staggered result
  Has simply not agreed with simpler formulations in continuum limit.
- one loop LPT staggered operator mixing is fragile
  $\mathrm{N^nLO}$ lattice taste mixing *easily* underestimated & amplified through mixing matrix cancellation



Tension recent lattice unquenched B_K induces between SM and $|\varepsilon|$ is created by the theoretically better actions.

Lessons ?

- **FLAG should include a Universality "star"**

  The most important "star" for dynamical simulation may be passing a robust universality test.

  A universality test could be to obtain a true, agreed, quenched continuum result with comparable simulation parameters.

  Simple sociology $\Rightarrow$ lower quality 2+1f preferred to careful quenched continuum limit. Particularly important for b-quarks to verify the various effective methods.

- **nf=2?**

  Several (JLQCD, ETMC) high quality $N_f = 0$ and $N_f = 2$ calculations should have created a strong bias against the high B_K result.

- **sys-errors are subjective estimates**

Average B_K

We now have two well renormalised 2+1f B_K calculations in remarkable agreement

RBC-UKQCD '07, '08

$$\hat{B}_K = 0.720(13)(37)$$

Discretisation	4%
NP renormalisation	2%
Chiral extrapolation	2%
Statistical	1.8%
Sea strange mass adjust	1%
Finite volume	1%
total	5.4%

Pros:

- SU(2)
- Controlled sea content

Cons:

- Single lattice spacing

discretisation estimated from quenched scaling

Aubin, Laiho, Van de Water '09

$$\hat{B}_K = 0.724(8)(28)$$

NP renormalisation	3.3%
Chiral/cont extrapolation	1.9%
statistical	1.2%
Scale & masses	0.8%
Finite volume	0.6%
total (quad)	4.1%

Pros:

- Two lattice spacings

Cons:

- SU(3) removal of taste unitarity violations

$\Delta_I = (460\text{MeV})^2 \Rightarrow \leq 670$ MeV pi-taste-ons & even heavier taste-η terms
Only NLO Taste mixing despite partial NNLO fit.

Average B_K

We now have two well renormalised 2+1f B_K calculations in remarkable agreement

Averaging difficult due to subjective nature of error estimates

- **extrapolation**
  RBC-UKQCD SU(2) approach more convergent but estimate larger chiral extrap. error than ALV combined chiral/continuum rs-χ_3^{PT}extrap. error
- **volume**
  Volumes similar for most data points; ALV use CDH model to adjust data and estimate remnant error, RBC directly fit data and estimate FV.
- **renormalisation**
  Same renormalisation method but again different errors;
  ALV estimate larger error mainly from using difference between NPR & LPT.

Average B_K

We now have two well renormalised 2+1f B_K calculations in remarkable agreement

Averaging difficult due to subjective nature of error estimates

- **extrapolation**
  RBC-UKQCD SU(2) approach more convergent but estimate larger chiral extrap. error than ALV combined chiral/continuum rs-$\chi_3^{\rm PT}$extrap. error
- **volume**
  Volumes similar for most data points; ALV use CDH model to adjust data and estimate remnant error, RBC directly fit data and estimate FV.
- **renormalisation**
  Same renormalisation method but again different errors;
  ALV estimate larger error mainly from using difference between NPR & LPT.

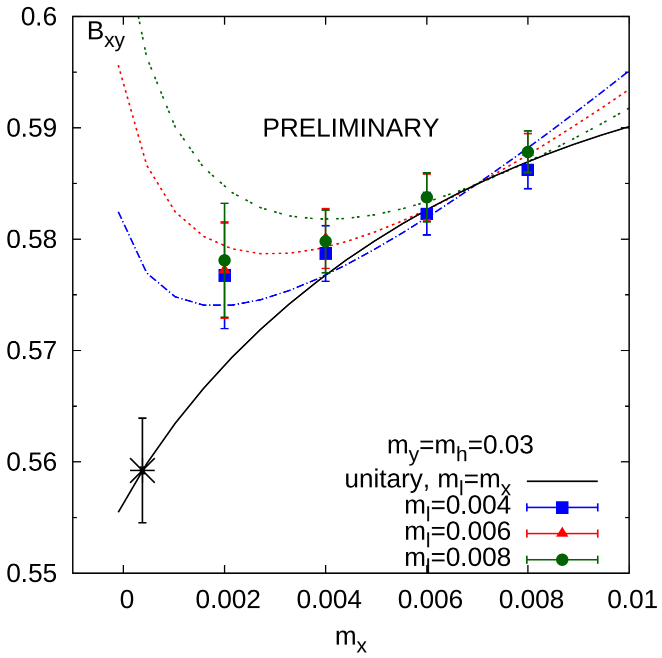
Propose

$$\hat{B}_K = 0.722(40)$$

and emphasize that most components of the errors are subjective

What's next for B_K

RBC-UKQCD second lattice spacing (thanks to Scholz, Kelly)



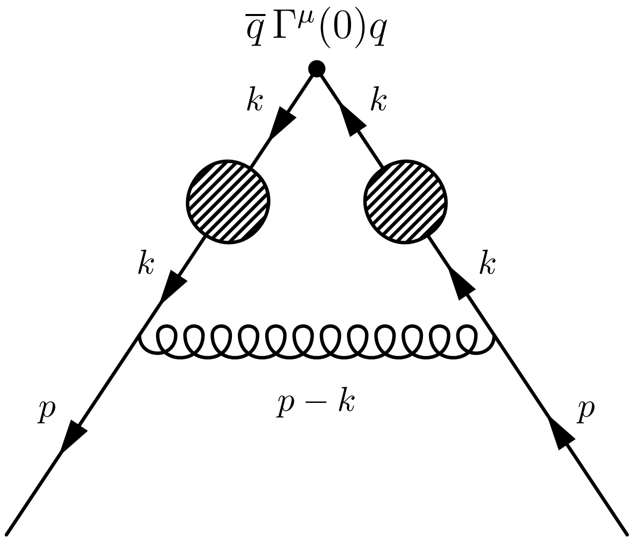
- bare, unrenormalised B_K extrapolation for $a^{-1} = 2.4$ GeV DWF
- Preliminary, and will change with *strange mass reweighting*

- Soon (if not already) error dominated by renormalisation
- Statistical & extrapolation uncertainties will be sub-leading

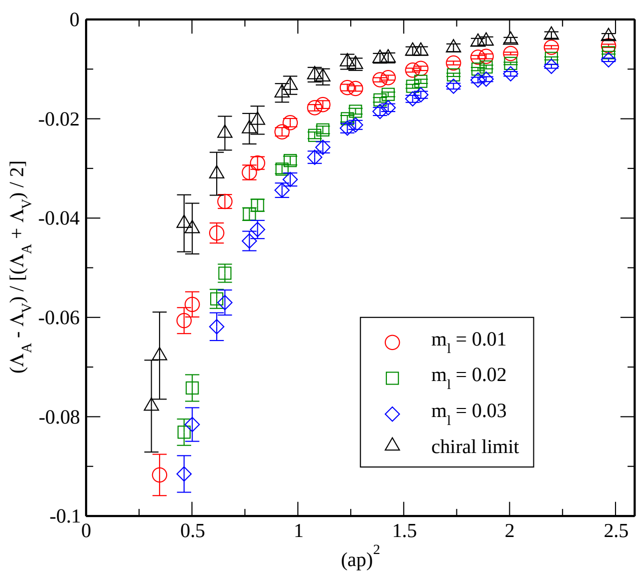
Non-perturbative renormalisation

RBC-UKQCD arXiv:0712.1061

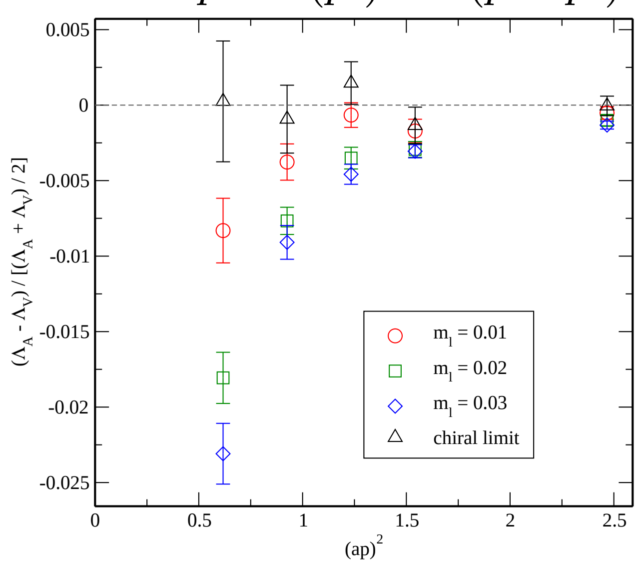
- RI-mom scheme traditionally uses $p = p'$
- Chiral symmetry $\Rightarrow \Lambda_A = \Lambda_V$
- Spontaneous χ^{SB} at low p^2
- 2% systematic introduced ;
  obscures good chiral properties of DWF



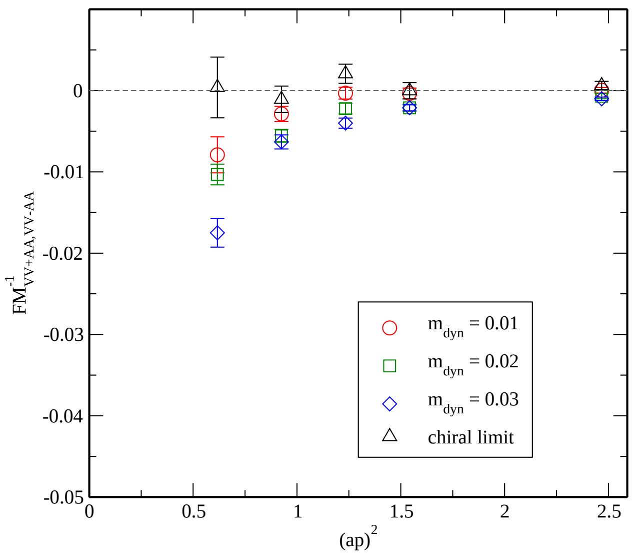
$\Lambda_A - \Lambda_V$ with $p = p'$



$\Lambda_A - \Lambda_V$ with non exceptional momenta $p^2 = (p')^2 = (p - p')^2$



NPR with non-exceptional momenta

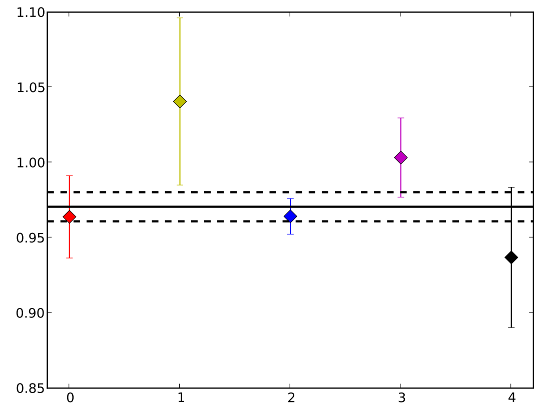


- Non-exceptional B_K chirality mixing consistent with zero
- New: 1-loop conversion known for Z_m, $\mathcal{O}_{\mathrm{VV+AA}}$
  Sturm, Sachrajda arXiv:0901.2599 & preliminary
- *Higher order perturbative conversion to $\overline{MS}$ is needed*

Jan Wennekers, RBC-UKQCD, preliminary

Five 1-loop mom schemes to estimate systematics

- Improved renormalisation may shift central value to around 0.54 from 0.52
- Expect good scaling from RBC-UKQCD data
- Two loop MOM scheme matching for non-exceptional kinematics required to reduce dominant error in the next few years.
- Who will do this?



Outline

- Introduction
- Mass extrapolation & chiral lagrangian
- V_{us}
  - f_K/f_π
  - K_{l3}
- Kaon bag parameters
  - B_K
  - BSM
- Other topics

Matrix elements of BSM $\Delta_S = 2$ effective operators

$$\langle K_0 | Q_i | \overline{K_0} \rangle$$

$$\begin{aligned}
Q_1 &= \bar{s}^a \gamma_\mu P_L d^a \bar{s}^b \gamma_\mu P_L d^b \\
Q_2 &= \bar{s}^a \gamma_\mu P_L d^a \bar{s}^b \gamma_\mu P_R d^b \\
Q_3 &= \bar{s}^a P_L d^a \bar{s}^b P_R d^b \\
Q_4 &= \bar{s}^a P_L d^a \bar{s}^b P_L d^b \\
Q_5 &= \bar{s}^a P_L d^a \bar{s}^b P_L d^b
\end{aligned}$$

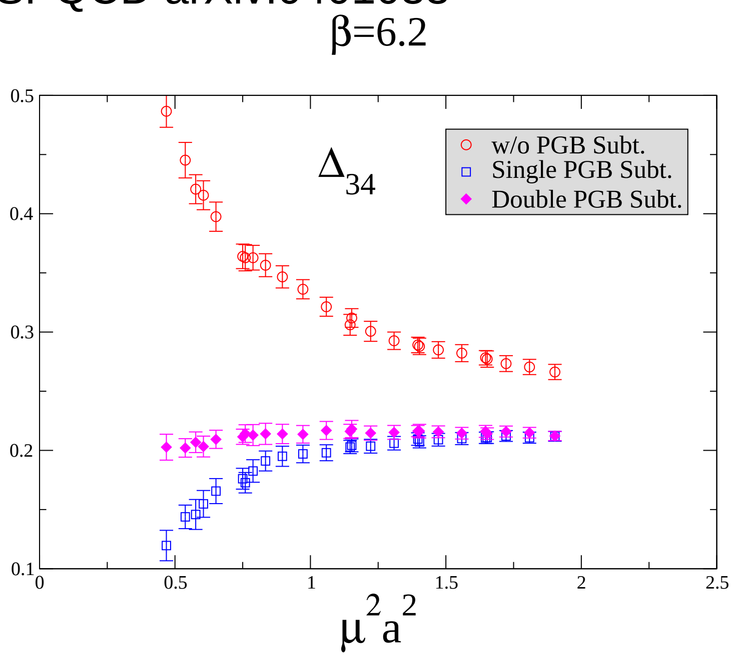
BSM Kaon Bag parameters

Matrix elements of $Q_2, \ldots, Q_5$ allow ϵ_K to constrain certain BSM $\Delta_S = 2$ processes

Mixing matrix becomes 5×5 in absence of chiral symmetry.
$\longrightarrow$ Overlap and DWF actions required for control.

Current best study remains quenched overlap **hep-lat/0605016** Boston/Marseille/Wuppertal

SPQCD arXiv:0401033

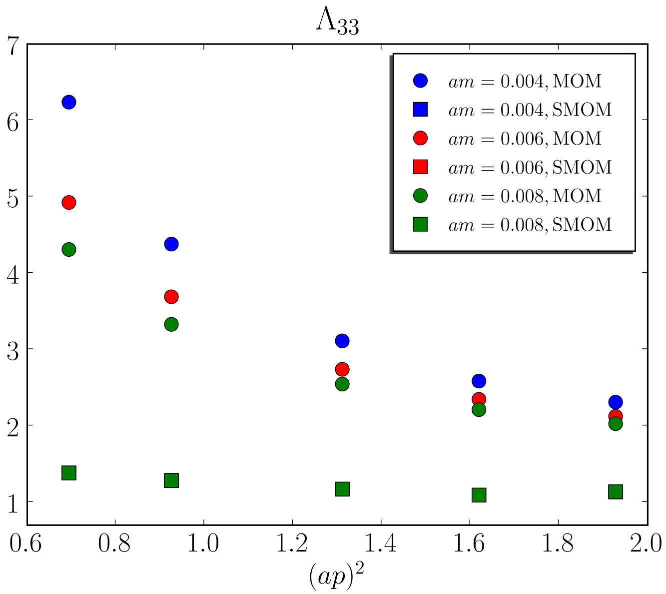


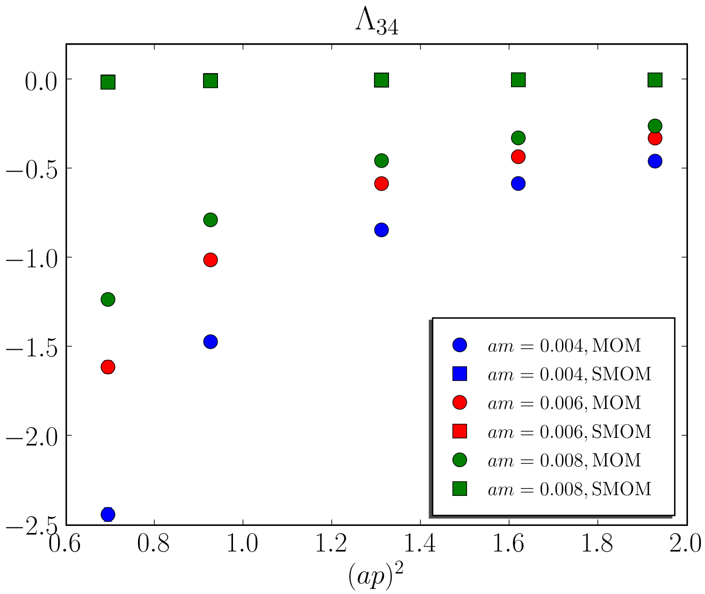
- Standard RI-mom renormalisation has $\frac{1}{mp^2}$ infra-red pole.
- residue in p^2 diverges as $m \longrightarrow 0$
- Difficult to subtract precisely; affects *all* existing calculations

BSM bag parameters in 2+1f DWF

RBC-UKQCD (Wennekers) arXiv:0810.1841

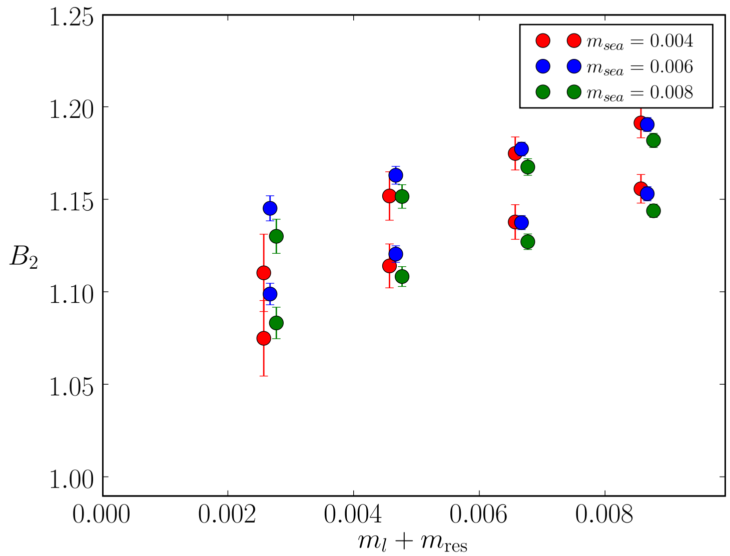
- First calculation of SUSY bag parameters with dynamical chiral fermions
- Non-exceptional momentum eliminates pion pole
- Includes all lattice components of NPR
- Perturbative conversion to MS in progress (1-loop, Sachrajda & Sturm)

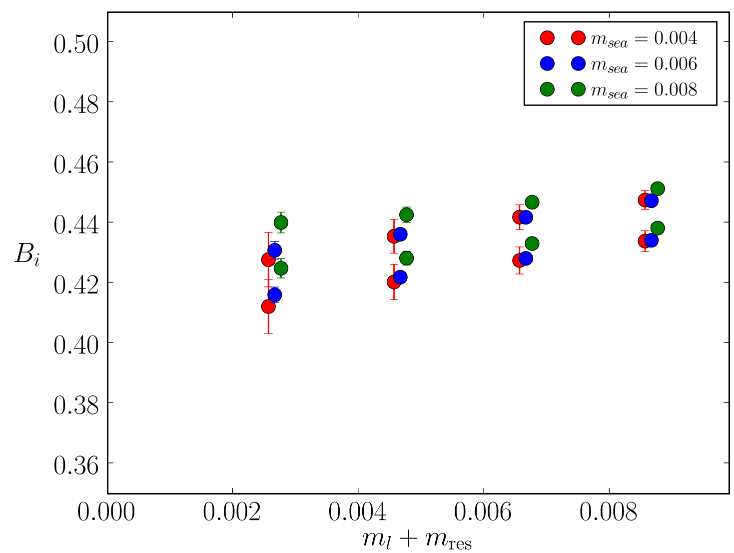




BSM bag parameters in 2+1f DWF

RBC-UKQCD (Wennekers) arXiv:0810.1841





Bare matrix elements of Q_2 (left) and Q_5 (right) as function of mass

When 1-loop calculation is complete expect:

- matrix elements to 2% – comparable accuracy to RBC-UKQCD B_K
- Dramatic systematic improvement in renormalisation

Outline

- Introduction
- Mass extrapolation & chiral lagrangian
- V_{us}
  - f_K/f_π
  - K_{l3}
- Kaon bag parameters
  - B_K
  - BSM
- Other topics

Other topics

Unfortunately there was insufficient time to cover :

- $K \to \pi\pi$
  Norman Christ's talk this conference
- Electromagnetic effects
  Taku Izubuchi's talk this conference
- $\bar{m}_s$ RI-mom approach is best so far used in 2+1f simulations
  Suffers poor perturbative convergence despite 3-loops
  Yasumichi Aoki's forthcoming talk at Lattice 2009
- Kaon distribution amplitudes
  RBC-UKQCD & QCDSF

Thanks

I wish to thank:

- Kaon 2009 organising committee
- KEK & Tsukuba University for their support and hospitality

BACKUP

Non-perturbative renormalisation

Matrix elements of non-conserved operators renormalise between any lattice simulation and $\overline{\mathrm{MS}}$.

Recent progress on Z_m and Z_{VV+AA} using the gauge-fixed off-shell Rome-Southampton RImom scheme.

Bilinear amputated vertex function

$$\Lambda(p,p')$$

in some scheme (Lattice or $\overline{\mathrm{MS}}$) is related to the RI mom scheme defined at this momentum point by

$$Z^{RI}_{\mathrm{scheme}}\Lambda_{\mathrm{scheme}}(p,p') = \Lambda_{\mathrm{tree}}(p,p')$$

Multi-loop calculations long available for $p = p'$ for bilinears

$\mathcal{O}_{\mathrm{VV+AA}}$ known only to 1-loop.

Conversion between Lattice and $\overline{\mathrm{MS}}$ is then

$$Z^{\overline{\mathrm{MS}}}_{\mathrm{Lat}} = Z^{\overline{\mathrm{RI}}}_{\mathrm{Lat}} / Z^{\overline{\mathrm{RI}}}_{\overline{\mathrm{MS}}} = \Lambda_{\overline{\mathrm{MS}}}(p,p')/\Lambda_{\mathrm{Lat}}(p,p')$$

For simultaneous discretisation and perturbative accuracy we require

$$\Lambda^2_{\mathrm{QCD}} \ll p^2 \ll \left(\frac{\pi}{a}\right)^2$$

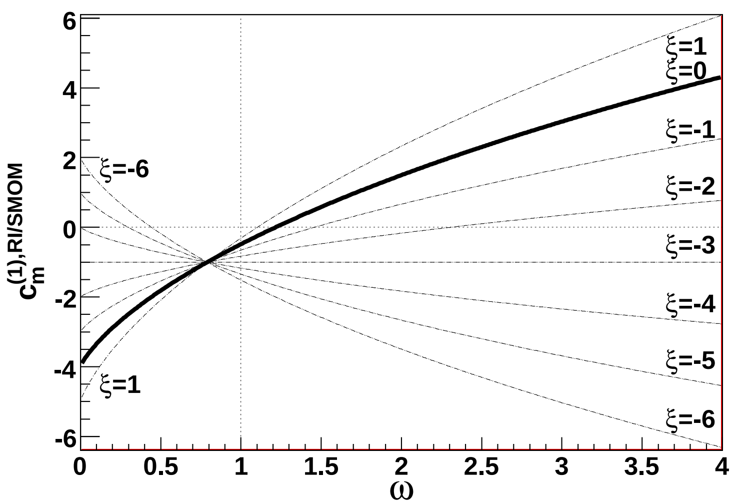
Strange mass

arXiv:0901.2599

From vertex function $\Lambda_S(p_1, p_2)$; $p_1^2 = p_2^2 = -\mu^2$ and $q^2 = -\omega\mu^2$ obtain

$$C_m = \frac{Z_m^{\overline{\mathrm{MS}}}}{Z_m^{\mathrm{RI}-\omega}} = 1 - \frac{\alpha_s}{4\pi} C_F c_m^{(1)}(\omega)$$

$\omega = 1$ is the symmetric momentum point (smom)

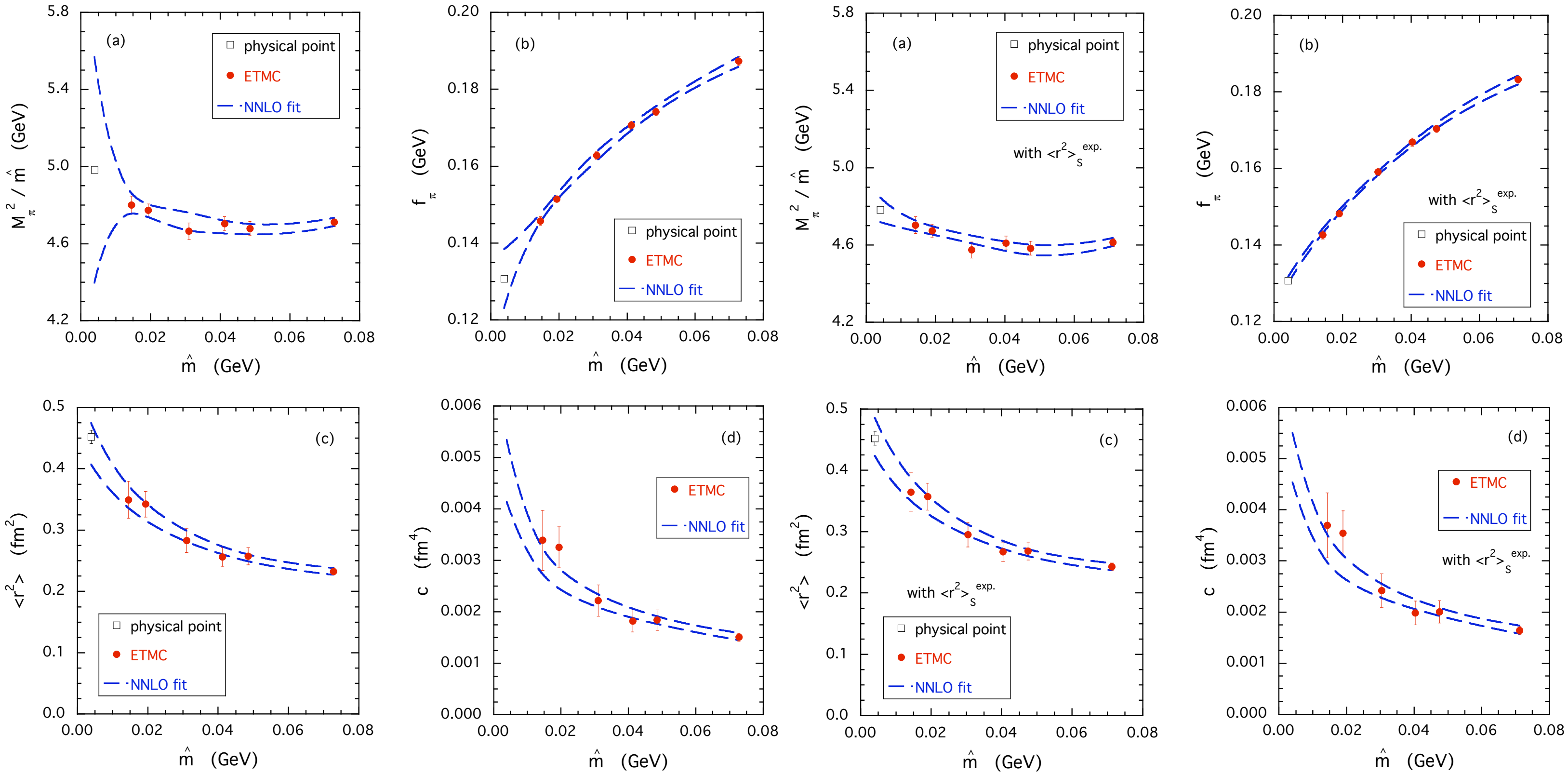


$$C_m^{\mathrm{RI}} = 1 - \frac{\alpha_s}{4\pi} C_F 4 + \mathcal{O}\left(\alpha_s^2\right)$$
$$C_m^{\mathrm{RIsmom}} = 1 - \frac{\alpha_s}{4\pi} C_F 0.4841391 + \mathcal{O}\left(\alpha_s^2\right)$$

- Is this reduction preserved at higher orders?
- 5% perturbative error from RImom approach to m_s could be greatly reduced
- Step-scaling alternative has not been performed with 2+1f to date

ETMC NNLO

ETMC NNLO fits poorly constrained where there is no data due to many parameters



Both ETMC include FULL NNLO but need additional constraint on parameters (from scalar pion form factor).

Does this mean anything for partial NNLO analyses?