Theoretical strategies for ε'/ε

Kaon09

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RBC and UKQCD Collaborations

Outline

- Introduction
- Challenges for lattice QCD
  - Operator renormalization
  - Quadratic divergences
  - $\pi - \pi$ final states
    [2008 RBC/UKQCD results using ChPT]
  - Disconnected graphs
- Outlook
  - $\Delta I = 3/2$
  - $\Delta I = 1/2$

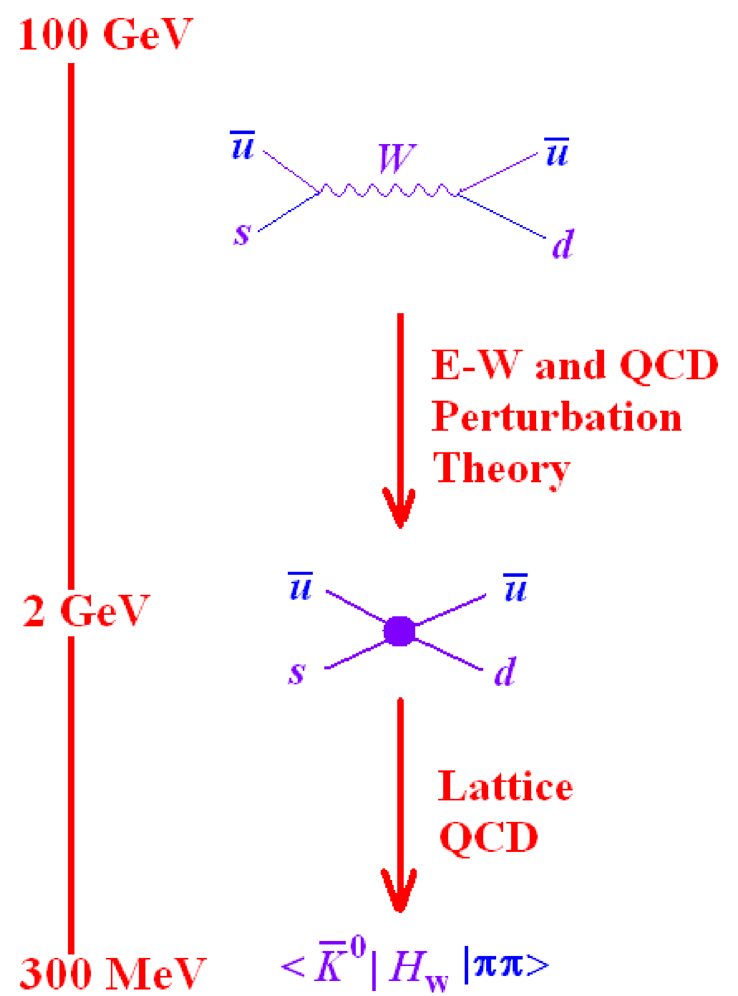
Introduction

Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

$$\mathcal{H}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) - \frac{V_{td}}{V_{ts}^*}\frac{V_{us}^*}{V_{ud}} y_i(\mu) \right] Q_i \right\}$$

- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



Four quark operators

- **Current-current operators**

$$Q_1 \equiv (\bar{s}_\alpha d_\alpha)_{V-A}(\bar{u}_\beta u_\beta)_{V-A}$$
$$Q_2 \equiv (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}$$

- **QCD Penguins**

$$Q_3 \equiv (\bar{s}_\alpha d_\alpha)_{V-A}\sum_{q=u,d,s}(\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_4 \equiv (\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s}(\bar{q}_\beta q_\alpha)_{V-A}$$
$$Q_5 \equiv (\bar{s}_\alpha d_\alpha)_{V-A}\sum_{q=u,d,s}(\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_6 \equiv (\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s}(\bar{q}_\beta q_\alpha)_{V+A}$$

- **Electro-Weak Penguins**

$$Q_7 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A}\sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V+A}$$
$$Q_8 \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V+A}$$
$$Q_9 \equiv \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A}\sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V-A}$$
$$Q_{10} \equiv \frac{3}{2}(\bar{s}_\alpha d_\beta)_{V-A}\sum_{q=u,d,s} e_q(\bar{q}_\beta q_\alpha)_{V-A}$$

Status

- The $\Delta I = ½$ rule and ε'/ε are long-standing problems in particle physics.
- Accurate experimental result allows test of standard model CP violation.

$$\mathrm{re}(\varepsilon'/\varepsilon) = 16.8\ (1.4) \times 10^{-4}$$

- Natural target for lattice QCD.
- Even 10-20% errors would be of great value.

Challenges

- Match lattice and continuum operators
- Eye diagrams contain quadratic divergences

} Ultraviolet

- Difficult $\pi - \pi$ final state
  - SU(3) x SU(3) ChPT fails
  - Physical decay: p ~ 205 MeV
  - Euclidean, large time limit: p ~ 0 MeV

} Infrared

- $\Delta I = 1/2$ amplitudes require disconnected graphs

Computational Challenges

Operator Normalization

Operator Renormalization

- RI/MOM scheme, gauge-fixed off-shell Green's functions.
- Earlier quenched and recent 2+1 flavor calculation demonstrate errors ~few % errors are feasible.
- Sub-percent statistical errors possible from 5-10 configurations (Dirk Broemmel, Chris Kelley, Jan Wennekers)
- Non-exceptional kinematics gives sub-percent infrared effects at $\mu = 1.7$ GeV.
- Largest uncertainty comes from $\mu = 2$ GeV QCD perturbation theory. Remove by step-scaling
  - Compare RI/MOM Green's functions or Schrodinger functional amplitudes on a sequence of ensembles with small physical volumes, L ~ $1/2^N$
  - Match with continuum perturbation theory at $\mu = 1.7 \cdot 2^N$ GeV → error $\sim 1/N$

Operator Renormalization (con't)

- Seven $\Delta S = 1$ operators divide into three groups which mix:
  - $O_{(27,1)}$
  - O_7 and O_8
  - O_2, O_3, O_5, O_6
- Accurately handled by RI/MOM (Chris Dawson, Shu Li)
- Mixing with lower dimension operators is a small effect and easily treated.
- Effects of a single gluonic operator not yet included.

Operator Renormalization (con't)

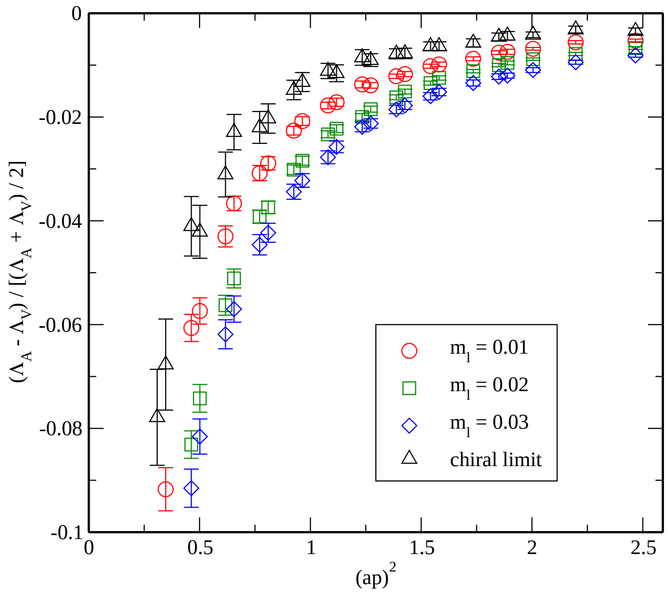
	1	2	3	5	6	7	8
1	1.218(33)	0.0(0.0)	0.0(0.0)	0.0(0.0)	0.0(0.0)	0.0033(53)	-0.0063(33)
2	0.0(0.0)	1.062(84)	0.076(77)	0.001(33)	0.016(29)	0.0026(80)	0.0026(68)
3	0.0(0.0)	0.13(20)	1.30(27)	-0.180(89)	0.120(99)	0.044(22)	-0.037(26)
5	0.0(0.0)	-0.08(24)	-0.03(21)	1.00(12)	0.269(93)	-0.016(23)	-0.034(24)
6	0.0(0.0)	-0.64(72)	-0.31(92)	-0.67(37)	1.97(38)	0.130(93)	-0.14(10)
7	-0.00030(89)	0.006(20)	0.024(25)	-0.0012(75)	-0.0074(92)	1.084(26)	0.294(29)
8	0.0002(14)	0.052(55)	0.138(76)	0.007(29)	-0.010(21)	0.060(22)	1.711(97)

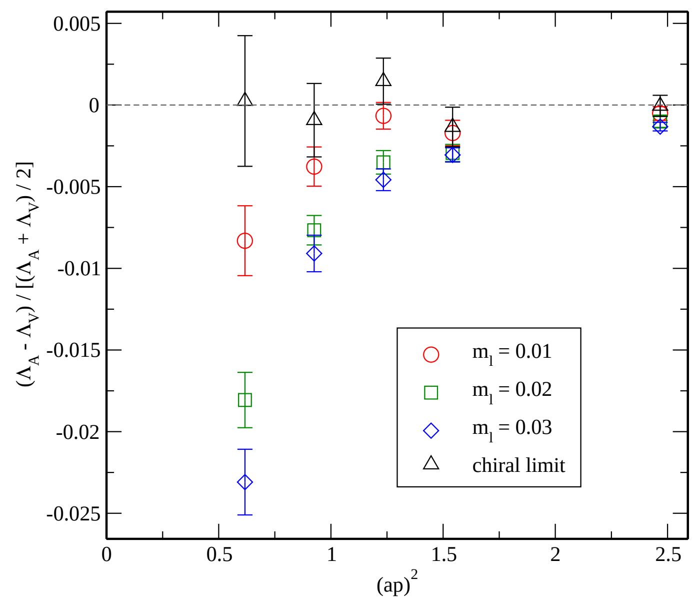
[Shu Li]

Inverse of renormalization matrix in 7 operator basis for unitary mass 0.01 and μ = 1.92 GeV. Done with 75 configurations, 16^3 x 32 1/a=1.73 GeV

Operator Renormalization (con't)

$(\Lambda_V - \Lambda_A)/\big((\Lambda_V + \Lambda_A)/2\big)$





Exceptional momenta

Nonexceptional momenta

Y. Aoki, et al

Phys.Rev.D78:054510,2008,

arXiv:0712.1061 [hep-lat]

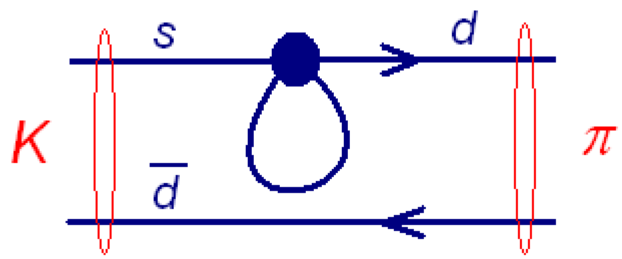
Quadratic Divergences

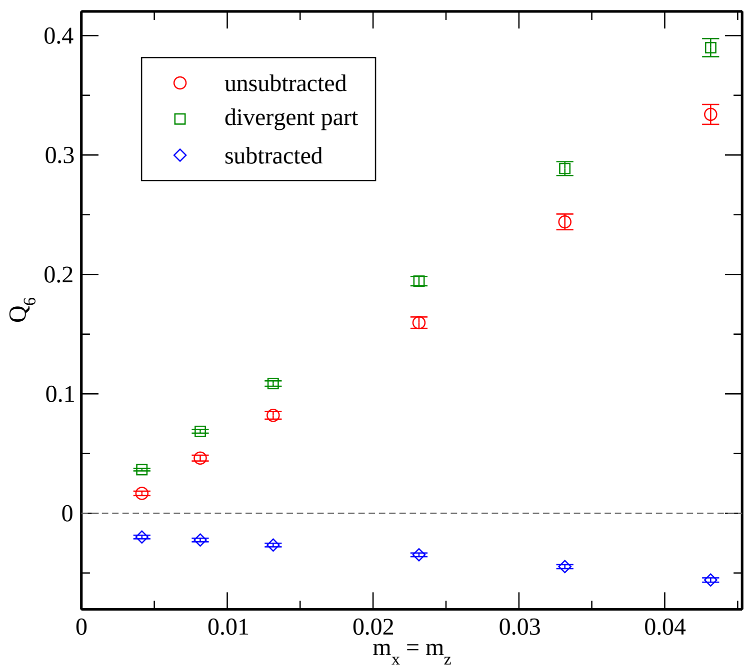
Quadratic Divergences

- Penguin diagrams have quadratically divergent part ~ $1/a^2$
- Easily determined and subtracted with sub percent errors.

  RBC:

  CP-PACS:
- Easily controlled at the percent level!





Two pion final state

SU(3) x SU(3) Chiral Perturbation Theory

- Use "soft-pion" methods to related $K \rightarrow \pi\pi$ to $K \rightarrow \pi$ and $K \rightarrow vac$.
- Earlier 2001 quenched calculations suggested this was promising (but gave $\varepsilon'/\varepsilon = ??$).
- However, quenched ChPT highly unphysical (**Golterman and Pallante**).
- Quenched calculation now repeated in 2+1 flavor QCD again using chiral (domain wall) fermions.

RBC and UKQCD Collaboration

First description presented by Bob Mawhinney at Kaon07

Tom Blum (Connecticut)

Norman Christ (Columbia)

Chris Dawson (Virginia)

Shu Li (Columbia)

Robert Mawhinney (Columbia)

Enno Scholz (BNL)

Amarjit Soni (BNL)

2+1 Flavor partially quenched chiral perturbation theory

Christopher Aubin (W&M)

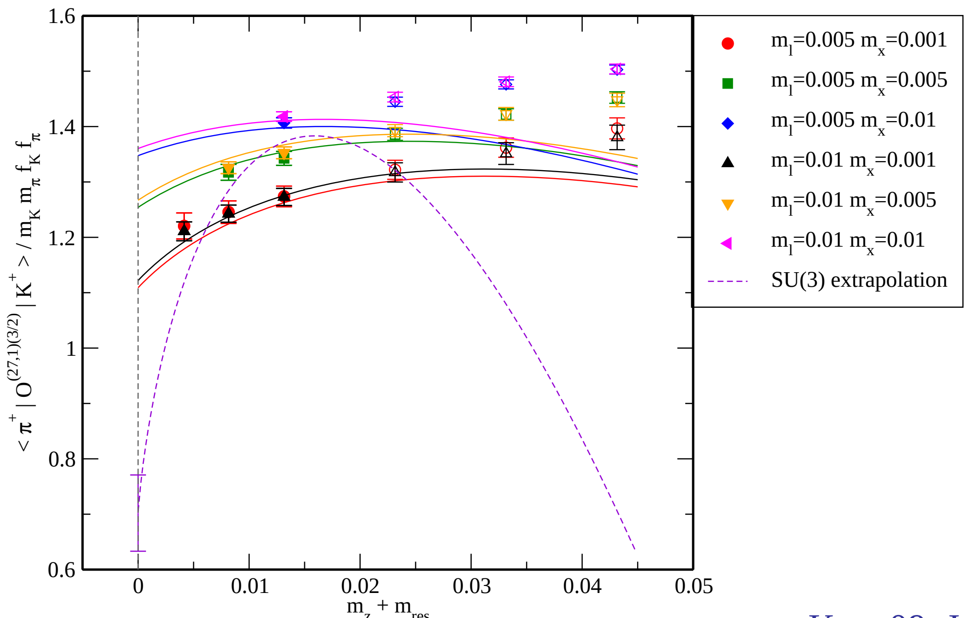
Jack Laiho (St Louis)

Shu Li (Columbia)

Meifeng Lin (Columbia)

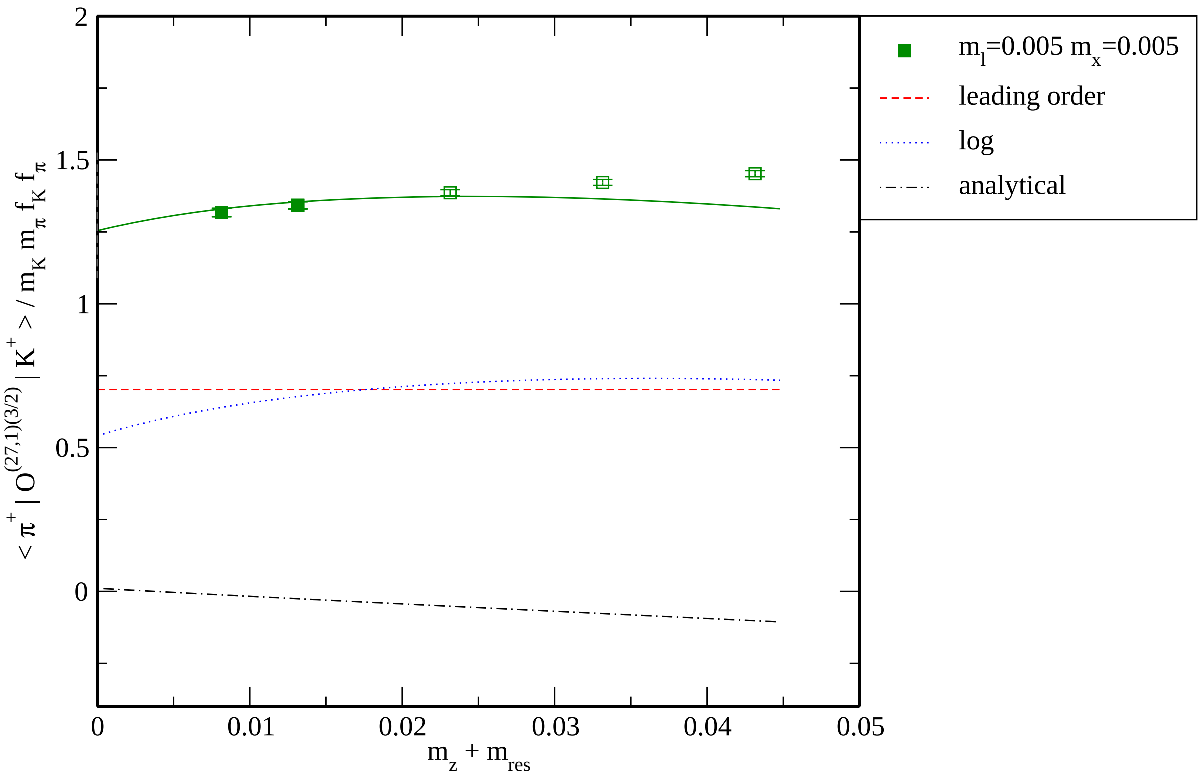
Determination of α_{27}

- Fit to points with $(m_{val} + m_{res})_{avg} \leq 0.013$
- PQChPT describes this data
- Large, ~100% correction!?
- Same large ChPT corrections as RBC/UQKCD, arXiv:0804.0473
- **Fit does not work without $m_K\, m_\pi\, f_K\, f_\pi$ division.**



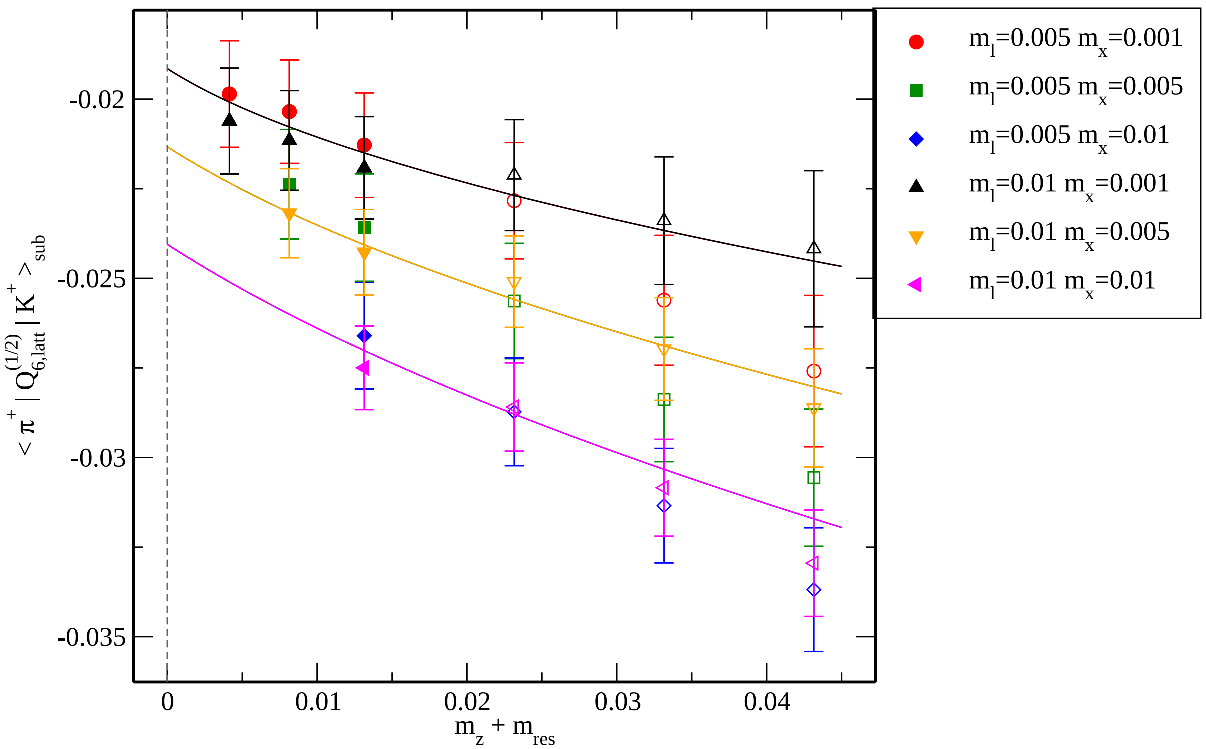
Relative size of LO and NLO terms

- LO and NLO log terms are the same size.
- Consistent results if we divide by $m_K m_\pi (f_K f_\pi)^2$
- Double the difference between two fits to estimate systematic error.



Determination of α_6

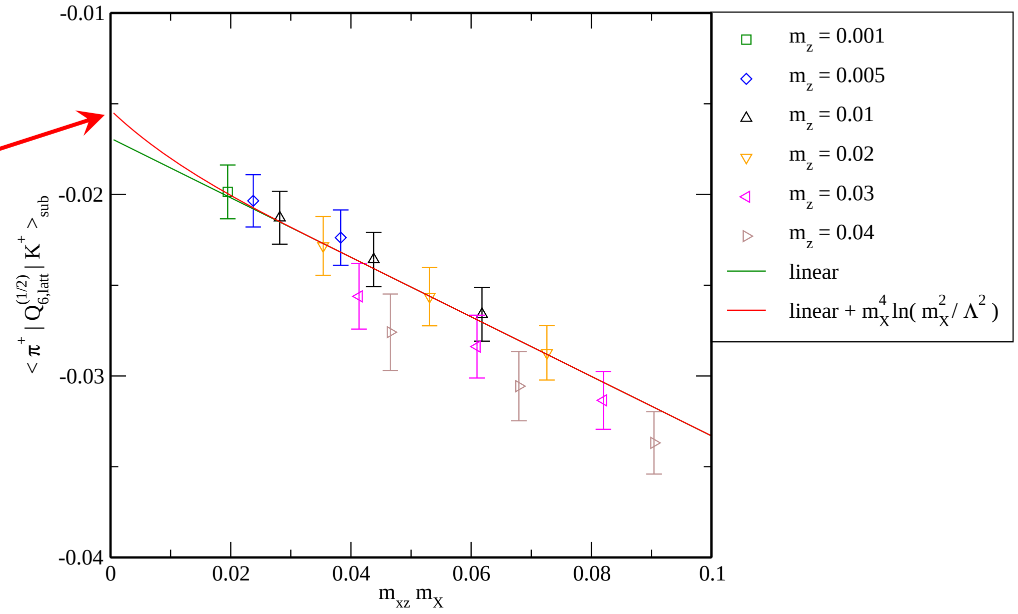
- NLO fit not possible, insufficient data to determine 8 LEC's.
- LO fit works well for large mass range.
- Omitted NLO logs are important!



Effect of NLO logs on α_6

- Chose $m_{max} = 0.005$.
- Use linear fit for $m_{max} \le m$
- Use chiral log for $m \le m_{max}$
- Match value, slope and curvature at $m = m_{max}$

Slope doubled by including NLO chiral log



Results for LEC's

Q_i	$\alpha_{i,\text{ren}}^{(1/2)}$	$\alpha_{i,\text{ren}}^{(3/2)}$
1	$-6.6(15)(66) \times 10^{-5}$	$-2.48(24)(39) \times 10^{-6}$
2	$9.9(21)(99) \times 10^{-5}$	$-2.47(24)(39) \times 10^{-6}$
3	$-0.8(31)(21) \times 10^{-5}$	**0.0**
4	$1.62(44)(162) \times 10^{-4}$	**0.0**
5	$-1.52(29)(152) \times 10^{-4}$	**0.0**
6	$-4.1(7)(41) \times 10^{-4}$	**0.0**
7	$-1.11(17)(18) \times 10^{-5}$	$-5.53(85)(91) \times 10^{-6}$
8	$-4.92(72)(75) \times 10^{-5}$	$-2.46(37)(37) \times 10^{-5}$
9	$-9.8(20)(98) \times 10^{-5}$	$-3.72(37)(59) \times 10^{-6}$
10	$6.8(15)(68) \times 10^{-5}$	$-3.69(37)(59) \times 10^{-6}$

- Q_1– Q_6, Q_9, Q_{10} in $(\text{GeV})^4$ Q_7, Q_8 in $(\text{GeV})^6$
- Heroic 7-operator NPR performed.

SU(3) x SU(3) ChPT Critique

- Difficult to extrapolate to chiral limit and extract needed LEC's (240 MeV $\leq m_\pi \leq$ 430 MeV)
- Highly unrealistic to then use those LEC's to reconstruct physical 495 MeV kaon.
- **Soft-pion methods are too unreliable to be used.**
- While not a positive result, this reflects a major RBC/UQKCD effort since Kaon07 and is an important conclusion.

Calculate π - π final state directly

- Lellouch-Luscher method:
  - Correct normalization for mixing of different *l* coming from cubic box.
  - Correctly include π - π interactions and Euclidean space Watson theorem.
  - Defeat Maiani-Testa theorem by tuning finite volume so that 1^{st} or 2^{nd} excited state has physical relative momentum.
- Further refinements:
  - G-parity boundary conditions – force π - π to carry physical 205 MeV momentum. (Changhoan Kim)
  - Non-zero cm mass momentum adjusted to make π - π relative momentum physical. (Takeshi Yamazaki)

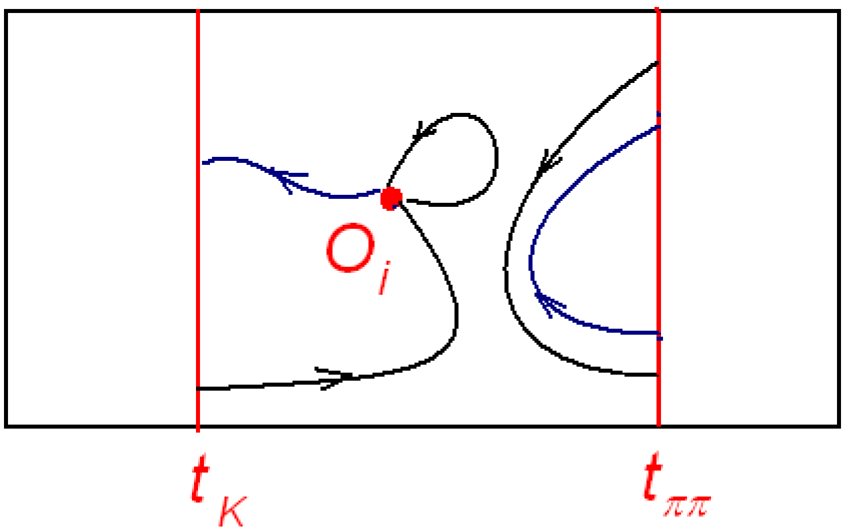
Disconnected diagrams

Disconnected graphs

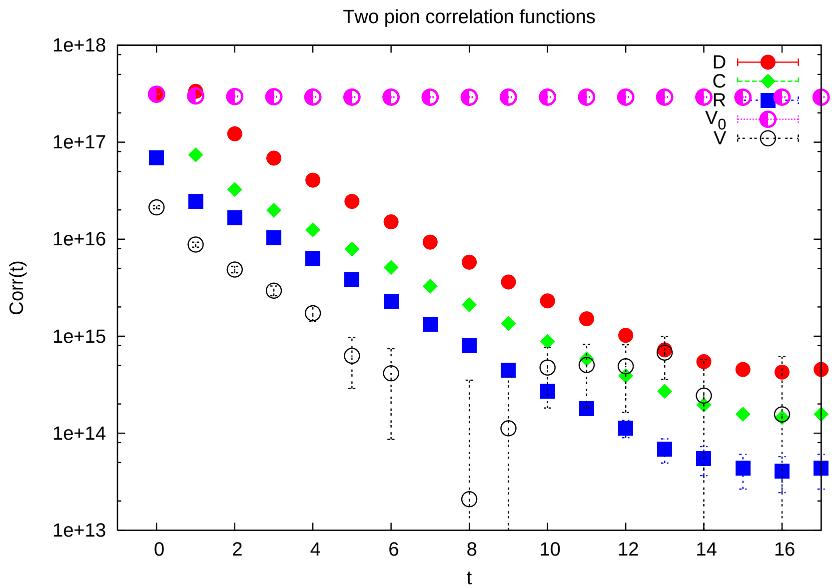
- Exponential e $^{-E_{\pi\pi}t}$ fall off produced by stochastic average rather than explicit quark propagators
- Many-source, high statistics methods needed
- Reliable signals must be extracted from small time separations:
  - Multi-state fits
  - Luscher-Wolff
- A serious challenge for LQCD
- The $\pi-\pi$ system is likely the easiest!

Disconnected graphs

- Current testing:
  - 2+1 flavors
  - 16^3 x 32
  - $m_\pi = 430$ MeV



$I = 0 \;\; \pi - \pi$ scattering



[Qi Liu]

- π - π correlator
- 32 wall sources, one for each t.
- 146 configurations
- 12 hrs/config. at 1/8 Tflops.

Outlook

Direct calculation of $K \rightarrow \pi\pi$ a major RBC/UKQCD project

- Collaborators:

RBC
Tom Blum
Norman Christ
Taku Izubuchi
Changhoan Kim
Matthew Lightman
Qi Liu
Bob Mawhinney
Amarjit Soni

UQKCD
Dirk Broemmel
Jonathan Flynn
Elaine Goode
Chris Sachrajda

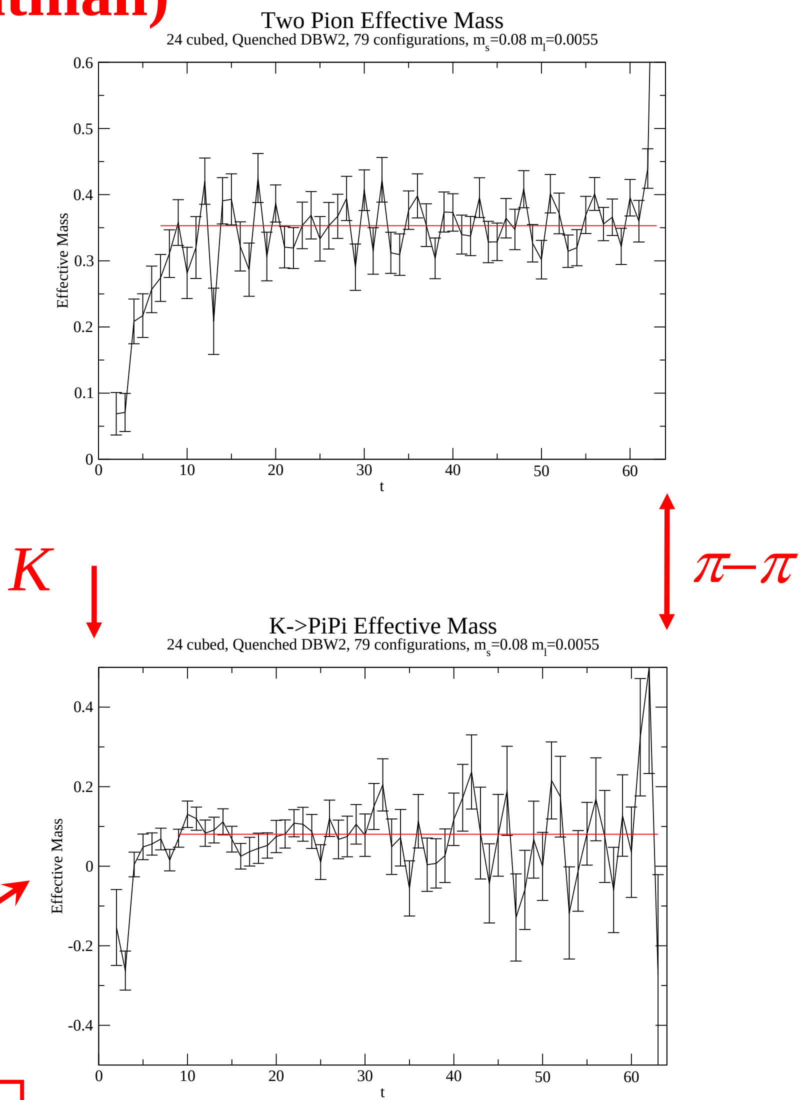
- Ready to start USQCD 80 M core-hour BG/P Argonne Incite allocation:
  - 4.5 fm box, $1/a = 1.4$ GeV, AuxDet action
  - $m_\pi = 240$ and 180 MeV

Outlook $\Delta I = 3/2$

(Matthew Lightman)

- Quenched 24^3 x 64, $1/a$=1.3 GeV, m_π = 228 MeV tests underway.
- Anti-periodic *d* quark.
  - p = 0, 170, 240, 295 MeV.
  - p_{phys} = 205 MeV
  - Only needed for valence *d*'s
- Use AuxDet large volume lattices
  - m_{res} = 0.0018/a ~ 3 MeV
  - 1/a = 1.4 GeV
  - L = 4.5 fm
  - m_π = 180 and 240 MeV
- Computing re A_2 and im A_2
  - ~15% accuracy
  - Practical 2-year goal

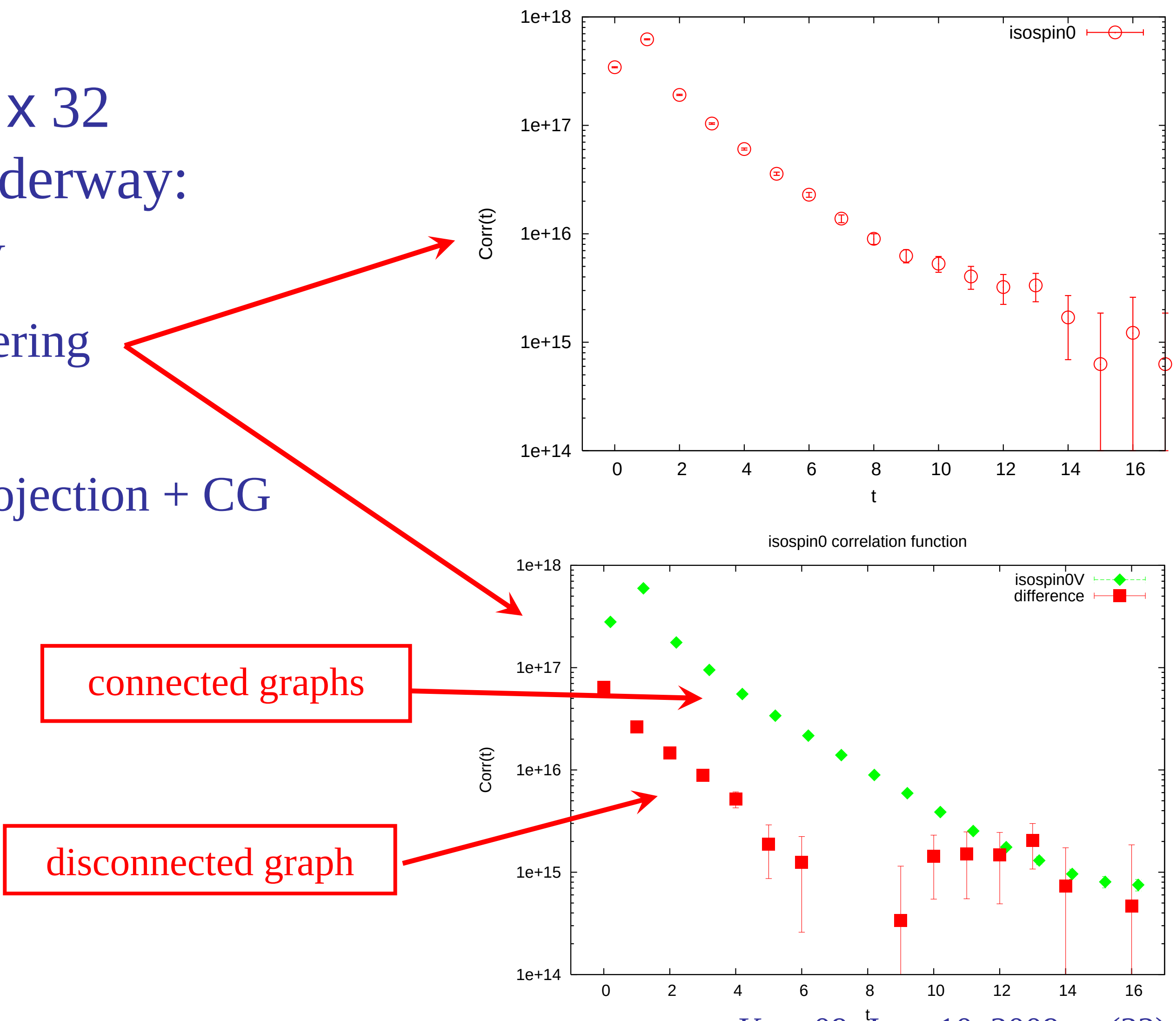
$A_2 = 2.17(12)\ 10^{-8}$ GeV, $p = 0$



Outlook: $\Delta I = ½$

(Qi Liu)

- 2+1 flavor, 16^3 x 32 experiments underway:
  - $m_\pi = 427$ MeV
  - 1st $\pi - \pi$ scattering
  - 2nd $K \rightarrow \pi \pi$
  - Eigenmode projection + CG (Ran Zhou)

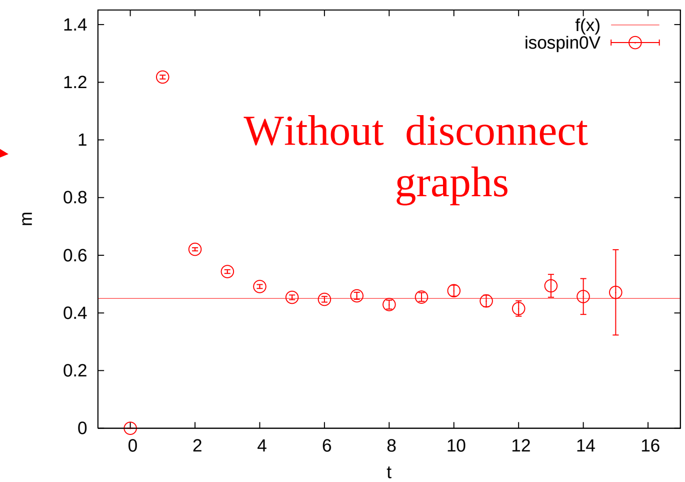


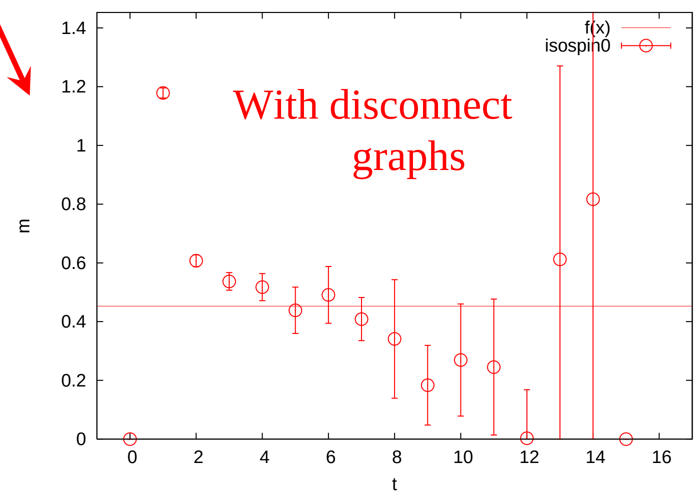
Outlook: $\Delta I = ½$

(Qi Liu)

$\pi - \pi$ effective mass

- Disconnected graphs introduce large errors into $\pi - \pi$ scattering for $t \geq 5$
- Non-zero momentum:
  - Non-zero cm momentum
  - G-parity boundary conditions
- Complete $K \rightarrow \pi\pi$ code written and first 8^3 x 12 calculations underway.
- 16^3 x 32 → 32^3 x 64 requires:
  - Improved short-time resolution
  - More efficient inversions





Conclusion

- Calculation of re A_2 and im A_2 to ~15% a realistic 1 - 2 year goal
- re A_0 and im A_0 more difficult
  - Theoretical issues are resolved.
  - Disconnected diagrams easiest in this $\pi - \pi$ case.
  - Next generation of computer hardware likely needed for definitive results: Next generation IBM BG/? machine should be sufficient!
- Expect 20% result for $\Delta I = ½$ rule and ε'/ε in ~3 years!