



# The FLAG working group: making lattice results accessible to phenomenologists

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(on behalf of the FLAG)

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BERN

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# Outline

Introduction

FLAG activities

Aims and criteria

Examples

$f_+(0)$  and  $f_K/f_\pi$

Low energy constants

Outlook

# What/Who is FLAG?

FLAG = FLAVIAnet Lattice Averaging Group



European network  
on Flavour Physics  
Start: 1.10.2006  
End: 30.09.2010



“Entering the high-precision era of flavour physics through the alliance of lattice simulations, effective field theories and experiment”

# What/Who is FLAG?

**FLAG** = FLAVIAnet Lattice Averaging Group

## Members:

Gilberto Colangelo (Bern)

Stephan Dürr (Jülich, BMW)

Andreas Jüttner (Mainz, RBC/UKQCD)

Laurent Lellouch (Marseille, BMW)

Heiri Leutwyler (Bern)

Vittorio Lubicz (Rome 3, ETM)

Silvia Necco (CERN, Alpha)

Chris Sachrajda (Southampton, RBC/UKQCD)

Silvano Simula (Rome 3, ETM)

Tassos Vladikas (Rome 2, Alpha and ETM)

Urs Wenger (Bern, ETM)

Hartmut Wittig (Mainz, Alpha)

# What/Who is FLAG?

**FLAG** = FLAVIAnet Lattice Averaging Group

## History and status:

- ▶ FLAG start: Orsay, November 2007
- ▶ Meetings: Bern, March 2008 and April 2009
- ▶ draft and webpage are being finalized
- ▶ will be made public in summer 2009\*

\* Statements and numbers in the present talk are *preliminary*

# What exactly will FLAG offer?

An answer to the questions

- ▶ what is the best lattice value for quantity  $X$ ?
- ▶ what is a reliable estimate of the uncertainty?

in a way easily accessible to non-experts

Quantities considered in the first edition:

- ▶ light quark masses
- ▶ LEC's
- ▶ decay constants
- ▶ form factors
- ▶  $B_K$

# What exactly will FLAG offer?

For each quantity we provide:

- ▶ complete list of references
- ▶ summary of relevant formulae and notation
- ▶ summary of the essential aspects of each calculation:
  - ▶ lattice action
  - ▶ number of dynamical quarks ( $N_f$ )
  - ▶ minimal value and range of quark masses
  - ▶ minimal value and range of lattice spacing
  - ▶ maximal value and range of lattice volumes
  - ▶ renormalization method (where applicable)

in a unified and easy to read (color coding) manner

- ▶ averages (if sensible)
- ▶ and a “lattice dictionary” for non-experts  
(details of lattice actions, etc.)

# Color coding – our present definition

- ▶ chiral extrapolation
  - ★  $M_{\pi,\min} < 250 \text{ MeV}$
  - $250 \text{ MeV} \leq M_{\pi,\min} \leq 400 \text{ MeV}$
  - $M_{\pi,\min} > 400 \text{ MeV}$

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- ▶ continuum extrapolation
  - ★ 3 or more lattice spacings, at least 2 points below 0.1 fm
  - 2 or more lattice spacings, at least 1 point below 0.1 fm
  - otherwise

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- ▶ finite volume effects
  - ★  $(M_{\pi}L)_{\min} > 4$  or at least 3 volumes
  - $(M_{\pi}L)_{\min} > 3$  and at least 2 volumes
  - otherwise

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  - otherwise
- ▶ finite volume effects
  - ★  $(M_{\pi} L)_{\min} > 4$  or at least 3 volumes
  - $(M_{\pi} L)_{\min} > 3$  and at least 2 volumes
  - otherwise
- ▶ renormalization (where applicable)
  - ★ non-perturbative
  - 2-loop perturbation theory (converging series)
  - otherwise

# Averages

Different lattice results will be averaged *if*

- ▶ published
  - [lattice proceedings not enough]
- ▶ no red tags
- ▶ same  $N_f$ 
  - [no average of  $N_f = 2$  and  $N_f = 3$  calculations]

Final FLAG number:

- ▶ average or single *no-red-tag*  $N_f = 3$  number (if available)
- ▶ average or single *no-red-tag*  $N_f = 2$  number (if available)

If *both*  $N_f = 3$  *and*  $N_f = 2$  numbers available:

agreement  $\Rightarrow$  more confidence in the final number

# Experiment + unitarity

Unitarity + experiment:

PDG (08)

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$$\left[ |V_{ub}| = 3.39(36) \cdot 10^{-3} \right]$$

Experiment:

FLAVIAnet Kaon WG (08)

$$|V_{us}f_+(0)| = 0.21661(47)$$

$$\left| \frac{V_{us}f_K}{V_{ud}f_\pi} \right| = 0.27599(59)$$

3 relations and 4 unknowns

determine anyone of  $V_{ud}$ ,  $V_{us}$ ,  $f_+(0)$  or  $f_K/f_\pi$

⇒ get the other three

# Lattice calculations of $f_+(0)$ and $f_K/f_\pi$

| $f_+(0)$                   | $N_f$ |              | Publication status | chiral extrap. | finite volume | continuum extrapol. | action      |
|----------------------------|-------|--------------|--------------------|----------------|---------------|---------------------|-------------|
| 0.9644(33)(34)(14)         | 2+1   | RBC/UKQCD 08 | A                  | ●              | ★             | ■                   | DWF         |
| 0.956(3)(5)                | 2     | ETM 08       | C                  | ●              | ●             | ●                   | max. tmQCD  |
| 0.9647(15) <sub>stat</sub> | 2     | QCDSF 07     | C                  | ■              | ★             | ■                   | clover (NP) |
| 0.968(9)(6)                | 2     | RBC 06       | A                  | ■              | ★             | ■                   | DWF         |
| 0.967(6)                   | 2     | JLQCD 05     | C                  | ■              | ★             | ■                   | clover (NP) |

Legend for publication status:

A = published article

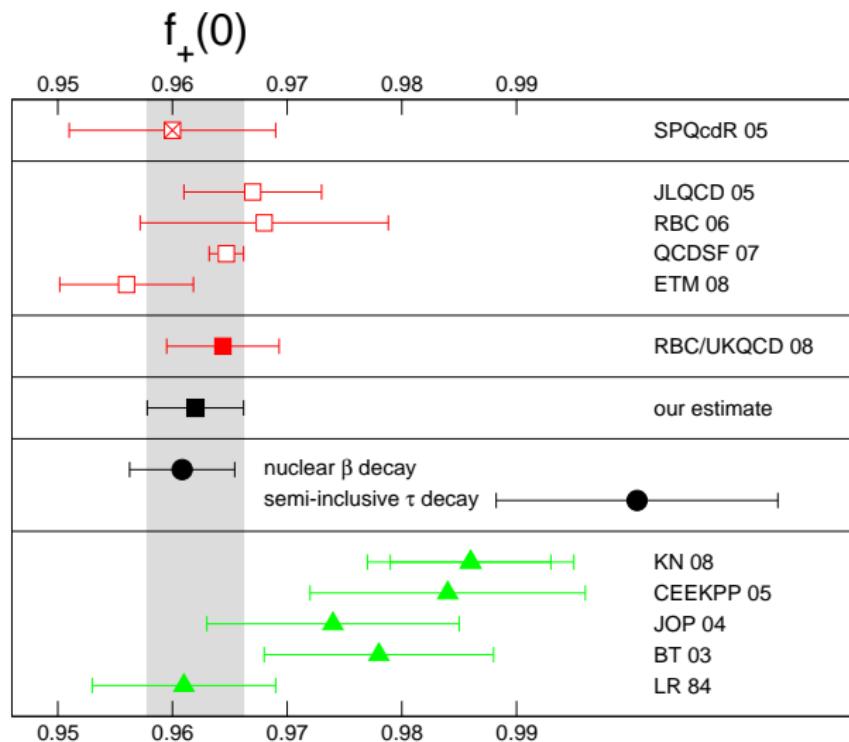
P = preprint

C = conference proceedings

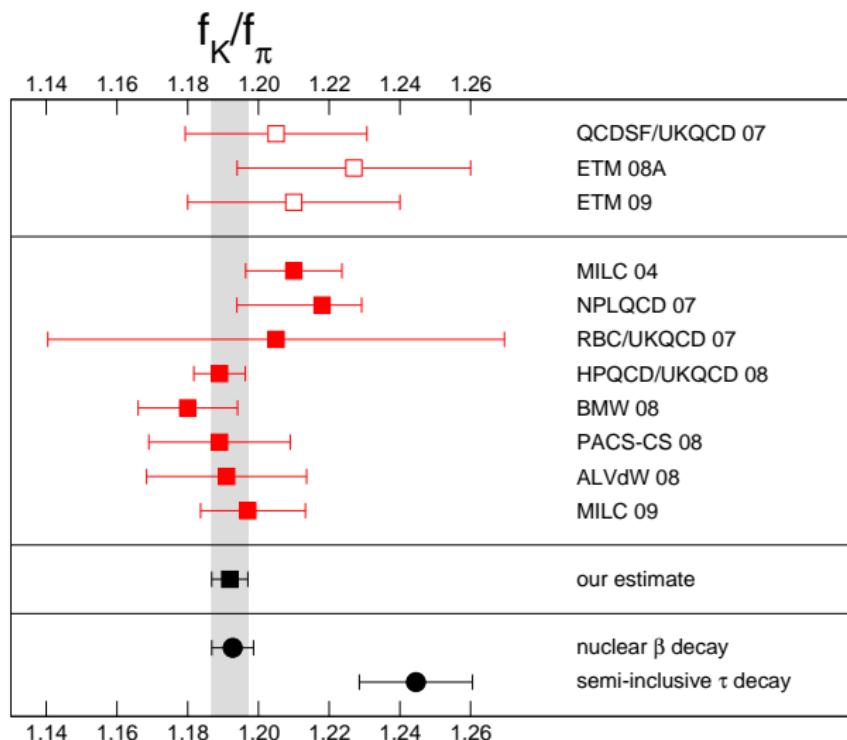
# Lattice calculations of $f_+(0)$ and $f_K/f_\pi$

| $f_K/f_\pi$                               | $N_f$ |                | Publication status | chiral extrapol. | finite volume | continuum extrap. | action                             |
|-------------------------------------------|-------|----------------|--------------------|------------------|---------------|-------------------|------------------------------------|
| 1.210(6)(15)(9)                           | 2     | ETM 09         | P                  | ●                | ●             | ★                 | max. tmQCD                         |
| 1.197(3)( <sup>+6</sup> <sub>-13</sub> )  | 2+1   | MILC 09        | A                  | ★                | ●             | ★                 | KS <sub>MILC</sub>                 |
| 1.191(16)(16)                             | 2+1   | AUBIN 08       | C                  | ★                | ■             | ●                 | KS <sub>MILC</sub> /DWF            |
| 1.189(20)                                 | 2+1   | PACS-CS 08     | P                  | ★                | ■             | ■                 | clover (NP)                        |
| 1.227(9)(24)                              | 2     | ETM 08A        | A                  | ●                | ●             | ■                 | max. tmQCD                         |
| 1.18(1)(1)                                | 2+1   | BMW 08         | C                  | ★                | ★             | ★                 | impr. Wilson                       |
| 1.189(2)(7)                               | 2+1   | HPQCD/UKQCD 08 | A                  | ★                | ●             | ★                 | KS <sub>MILC</sub> <sup>HISQ</sup> |
| 1.205(18)(62)                             | 2+1   | RBC/UKQCD 07   | A                  | ●                | ★             | ■                 | DWF                                |
| 1.21(3)                                   | 2     | QCDSF/UKQCD 07 | C                  | ●                | ★             | ●                 | clover (NP)                        |
| 1.218(2)( <sup>+11</sup> <sub>-24</sub> ) | 2+1   | NPLQCD 07      | A                  | ●                | ■             | ■                 | KS <sub>MILC</sub> /DWF            |
| 1.210(4)(13)                              | 2+1   | MILC 04        | A                  | ★                | ●             | ●                 | KS <sub>MILC</sub>                 |

## Lattice



## Lattice



# Lattice calculations of $f_+(0)$ and $f_K/f_\pi$

According to our policy only three results are relevant

$$f_+(0) = 0.964(3)(4) \quad \text{RBC/UKQCD 08}$$

$$f_K/f_\pi = 1.197(3)(^{+6}_{-13}) \quad \text{MILC 09}$$

$$f_K/f_\pi = 1.189(2)(7) \quad \text{HPQCD/UKQCD 08}$$

all three can be translated into a value for  $V_{us}$ :

$$|V_{us}| = 0.2246(9)(9) \quad \text{RBC/UKQCD 08}$$

$$|V_{us}| = 0.2247(7)(^{+23}_{-11}) \quad \text{MILC 09}$$

$$|V_{us}| = 0.2261(6)(13) \quad \text{HPQCD/UKQCD 08}$$

— and averaged —

$$|V_{us}| = 0.2253(4)(9) \quad \text{our average}$$

# Lattice calculations of $f_+(0)$ and $f_K/f_\pi$

According to our policy only three results are relevant

|                                     |                |
|-------------------------------------|----------------|
| $f_+(0) = 0.964(3)(4)$              | RBC/UKQCD 08   |
| $f_K/f_\pi = 1.197(3)(^{+6}_{-13})$ | MILC 09        |
| $f_K/f_\pi = 1.189(2)(7)$           | HPQCD/UKQCD 08 |

or of any other of the four unknowns

|                              |             |
|------------------------------|-------------|
| $ V_{ud}  = 0.97427(9)(20)$  | our average |
| $f_+(0) = 0.9620(20)(37)$    | our average |
| $f_K/f_\pi = 1.1919(16)(48)$ | our average |

# Other sources of information on $V_{ud}$ and $V_{us}$

Super-allowed nuclear  $\beta$  decays

$$|V_{ud}| = 0.97425(22)$$

Hardy & Towner 08

$$\Rightarrow |V_{us}| = 0.22544(95) \quad f_+(0) = 0.9608(46) \quad f_K/f_\pi = 1.1927(59)$$

$\tau \rightarrow [\text{hadrons}(S=1)] + \nu$  decays

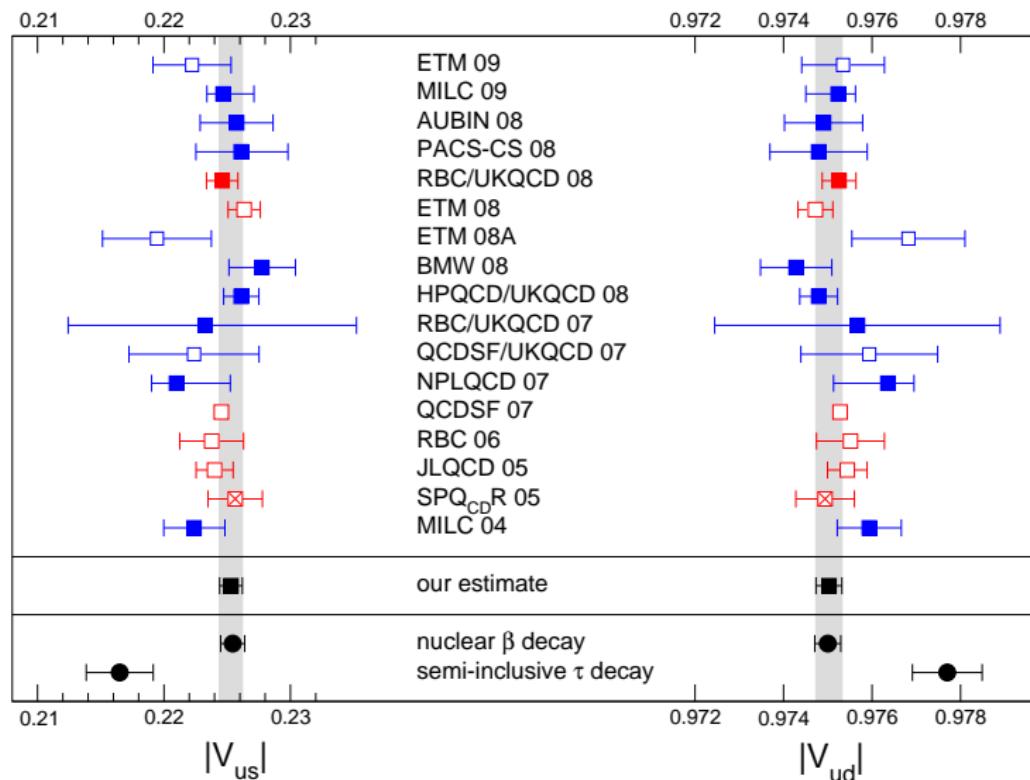
$$|V_{us}| = 0.2165(26)_{\text{exp}}(5)_{\text{th}}$$

Gamiz et al. 07

$$\Rightarrow |V_{ud}| = 0.9763(6) \quad f_+(0) = 1.001(12) \quad f_K/f_\pi = 1.245(16)$$

Problematic data:  $\sum$  exclusive channels  $\neq$  inclusive

## Comparison between lattice and other determinations



# $SU(3)$ low-energy constants

Determination based on masses and decay constants

|                  | $N_f$ | Publication status | chiral extrapol. | finite volume | continuum extrapol. | renormalization | action |
|------------------|-------|--------------------|------------------|---------------|---------------------|-----------------|--------|
| (a) RBC/UKQCD 08 | 2+1   | A                  | ●                | ★             | ■                   | ★               | DW     |
| (b) PACS-CS 08   | 2+1   | P                  | ★                | ■             | ■                   | ■               | clover |
| (c) MILC 09      | 2+1   | P                  | ★ *              | ★             | ★                   | ●               | KS     |

\* Based on value of lowest pion mass. Average of tastes would be more appropriate, but cannot be reconstructed from the paper.

|     | $F_0$ [MeV] | $\Sigma_0^{1/3}$ [MeV]   | $F/F_0$                | $B/B_0$   | $\Sigma/\Sigma_0$        | $F_\pi/F$              |
|-----|-------------|--------------------------|------------------------|-----------|--------------------------|------------------------|
| (a) | 66.1(5.2)   |                          | 1.229(59)              | 1.03(05)  | 1.55(21)                 |                        |
| (b) | 83.8(6.4)   | 290(15)                  | 1.078(44)              | 1.089(15) | 1.245(10)                | 1.062(8)               |
| (c) |             | $242(9)(^{+5}_{-17})(4)$ | $1.15(5)(^{+13}_{-3})$ |           | $1.52(17)(^{+38}_{-15})$ | $1.052(2)(^{+6}_{-3})$ |

# $SU(3)$ low-energy constants

Determination based on masses and decay constants

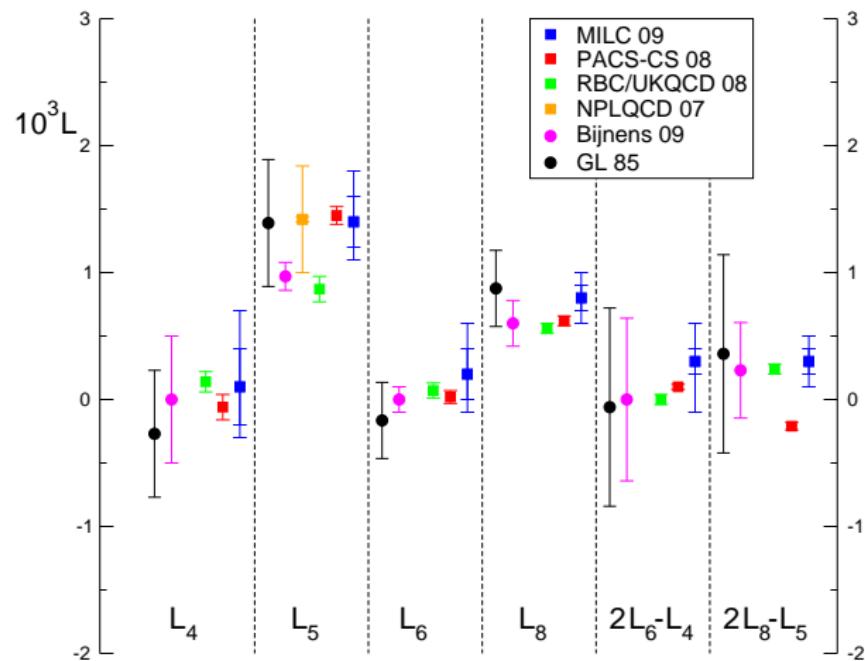
|                  | $N_f$ | Publication status | chiral extrapol. | finite volume | continuum extrapol. | renormalization | action |
|------------------|-------|--------------------|------------------|---------------|---------------------|-----------------|--------|
| (a) RBC/UKQCD 08 | 2+1   | A                  | ●                | ★             | ■                   | ★               | DW     |
| (b) PACS-CS 08   | 2+1   | P                  | ★                | ■             | ■                   | ■               | clover |
| (c) MILC 09      | 2+1   | P                  | ★ *              | ★             | ★                   | ●               | KS     |

\* Based on value of lowest pion mass. Average of tastes would be more appropriate, but cannot be reconstructed from the paper.

|     | $10^3 L_4$             | $10^3 L_5$             | $10^3 L_6$             | $10^3 L_8$ | $10^3 (2L_6 - L_4)$    | $10^3 (2L_8 - L_5)$ |
|-----|------------------------|------------------------|------------------------|------------|------------------------|---------------------|
| (a) | 0.14(8)(-)             | 0.87(10)(-)            | 0.07(6)(-)             | 0.56(4)(-) | 0.00(4)(-)             | 0.24(4)(-)          |
| (b) | -0.06(10)(-)           | 1.45(7)(-)             | 0.02(5)(-)             | 0.62(4)(-) | 0.10(2)(-)             | -0.21(3)(-)         |
| (c) | 0.1(3)( $^{+3}_{-1}$ ) | 1.4(2)( $^{+2}_{-1}$ ) | 0.2(2)( $^{+2}_{-1}$ ) | 0.8(1)(1)  | 0.3(1)( $^{+2}_{-3}$ ) | 0.3(1)(1)           |

# $SU(3)$ low-energy constants

Determination based on masses and decay constants



# $SU(2)$ low-energy constants

| $N_f$                         | <i>Publication status</i> | obs.      | $F[\text{MeV}]$                        | chiral extrapol. | continuum extrapol.  | finite volume extrapol. | renormalization |
|-------------------------------|---------------------------|-----------|----------------------------------------|------------------|----------------------|-------------------------|-----------------|
|                               |                           |           |                                        | ●                | ■ ■                  | ★                       |                 |
| JLQCD/TWQCD 08                | 2                         | A ● ■ ■ ★ | $M_\pi, F_\pi$                         | 79.0(2.5)(0.7)   | ( $^{+4.2}_{-0.0}$ ) |                         |                 |
| RBC/UKQCD 08                  | 2+1                       | A ● ■ ★ ★ | $M_\pi, F_\pi$                         | 81.2(2.9)(5.7)   |                      |                         |                 |
| PACS-CS 08                    | 2+1                       | A ★ ■ ■ ■ | $M_\pi, F_\pi$                         | 89.4(3.3)        |                      |                         |                 |
| ETM 08A                       | 2                         | A ● ○ ○ ○ | " $\langle r_\pi^2 \rangle_V, c_\pi^V$ | 86.6(7)(7)       |                      |                         |                 |
| JLQCD/TWQCD 08                | 2                         | A         | $C_{PP,AA}(\epsilon)$                  | 87.3(5.6)        |                      |                         |                 |
| A. Hasenfratz <i>et al</i> 08 | 2                         | A         | $C_{PP,AA}(\epsilon)$                  | 90(4)            |                      |                         |                 |
| DeGrand-Schaefer 07           | 2                         | A         | RTM ( $\epsilon$ )                     | 84(5)            |                      |                         |                 |

# $SU(2)$ low-energy constants

|                | $N_f$ | publication status<br>chiral extrapol.<br>continuum extrapo.<br>finite volume extrapo.<br>renormalization | obs.                                    | $\bar{\ell}_3$              |
|----------------|-------|-----------------------------------------------------------------------------------------------------------|-----------------------------------------|-----------------------------|
| CERN-TOV 07    | 2     | A ● ● ■ ★                                                                                                 | $M_\pi, F_\pi$                          | 3.0(5)(1)                   |
| ETM 08         | 2     | A ● ● ○ ★                                                                                                 | $M_\pi, F_\pi$                          | 3.42(8)(10)(27)             |
| JLQCD/TWQCD 08 | 2     | A ● ■ ■ ★                                                                                                 | $M_\pi, F_\pi$                          | $3.38(40)(24)(^{+31}_{-0})$ |
| RBC/UKQCD 08   | 2+1   | A ● ■ ★ ★                                                                                                 | $M_\pi, F_\pi$                          | 3.13(33)(24)                |
| PACS-CS 08     | 2+1   | A ★ ■ ■ ■                                                                                                 | $M_\pi, F_\pi$                          | 3.14(23)                    |
| ETM 08A        | 2     | A ● ● ○                                                                                                   | ", $\langle r_\pi^2 \rangle_V, c_\pi^V$ | 3.2(8)(2)                   |

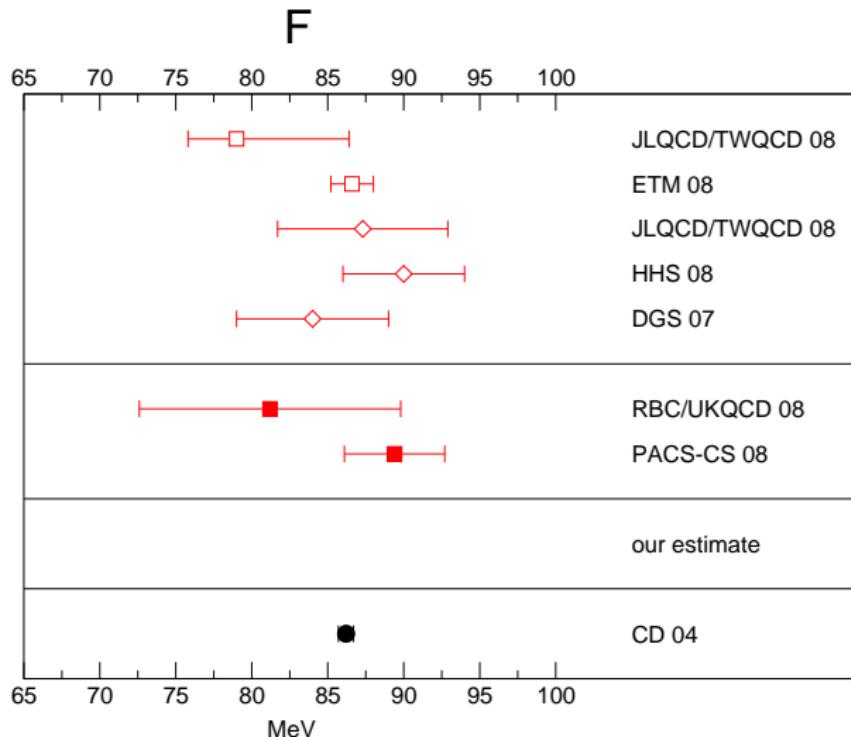
# $SU(2)$ low-energy constants

|                | $N_f$ | publication status | chiral extrapol. | continuum extrapo. | finite volume extrapo.                                          | obs.           | $\bar{\ell}_4$                 |
|----------------|-------|--------------------|------------------|--------------------|-----------------------------------------------------------------|----------------|--------------------------------|
| ETM 08         | 2     | A                  | ●                | ● ●                | ★                                                               | $M_\pi, F_\pi$ | 4.59(4)(2)(13)                 |
| JLQCD/TWQCD 08 | 2     | A                  | ●                | ■ ■                | ★                                                               | $M_\pi, F_\pi$ | 4.12(35)(30) ( $^{+31}_{-0}$ ) |
| RBC/UKQCD 08   | 2+1   | A                  | ●                | ■ ★                | ★                                                               | $M_\pi, F_\pi$ | 4.43(14)(77)                   |
| PACS-CS 08     | 2+1   | A                  | ★                | ■ ■                | ■                                                               | $M_\pi, F_\pi$ | 4.04(19)                       |
| ETM 08A        | 2     | A                  | ●                | ● ●                | " , $\langle r_\pi^2 \rangle_V, c_\pi^V$                        | 4.4(2)(1)      |                                |
| JLQCD/TWQCD 09 | 2     | P                  | ●                | ■ ■                | $\langle r_\pi^2 \rangle_V, \langle r_\pi^2 \rangle_S, c_\pi^V$ | 4.09(50)(52)   |                                |

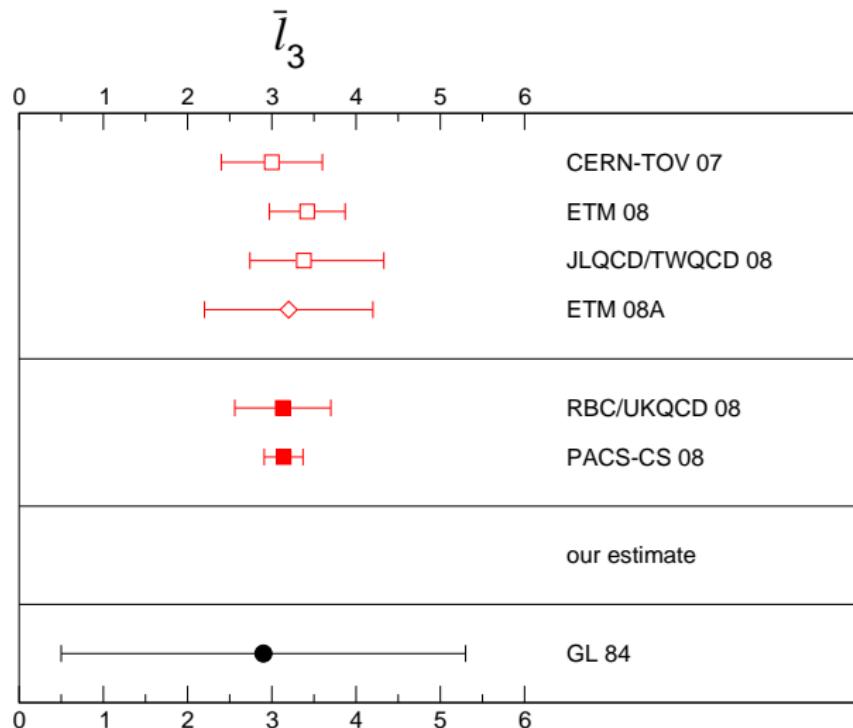
## $SU(2)$ low-energy constants

| $N_f$          | publication status | chiral extrapol. | continuum extrapol. | finite volume | obs. | $\bar{\ell}_6$                                                  |
|----------------|--------------------|------------------|---------------------|---------------|------|-----------------------------------------------------------------|
| ETM 08         | 2                  | A                | ●                   | ●             | ●    | $M_\pi, F_\pi, \langle r_\pi^2 \rangle_V, c_\pi^V$              |
| JLQCD/TWQCD 09 | 2                  | P                | ●                   | ■             | ■    | $\langle r_\pi^2 \rangle_V, \langle r_\pi^2 \rangle_S, c_\pi^V$ |
| RBC/UKQCD 08   | 2+1                | A                | ●                   | ■             | ★    | $F_V^\pi(q^2)$                                                  |

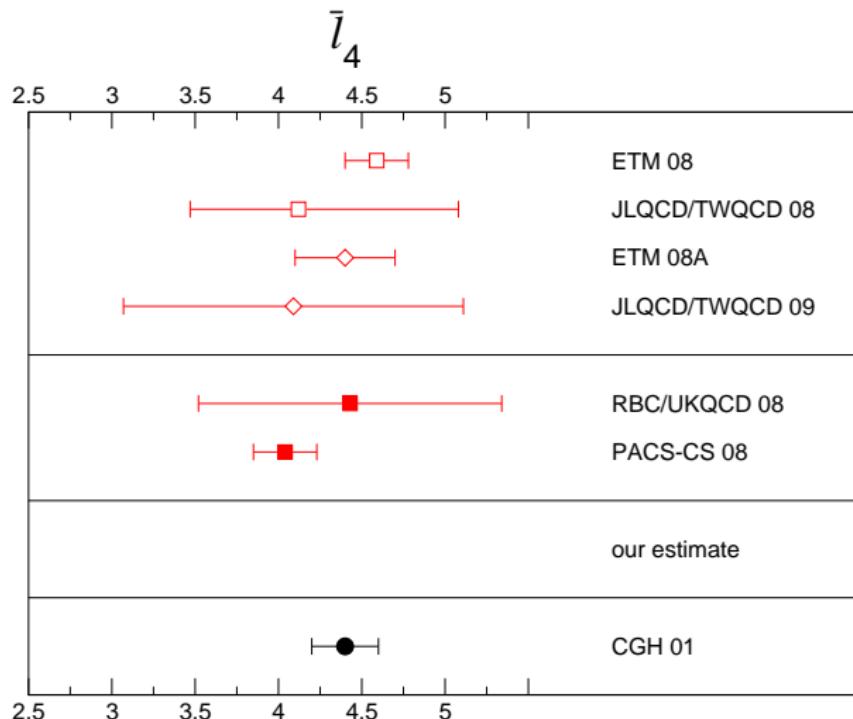
# $SU(2)$ low-energy constants



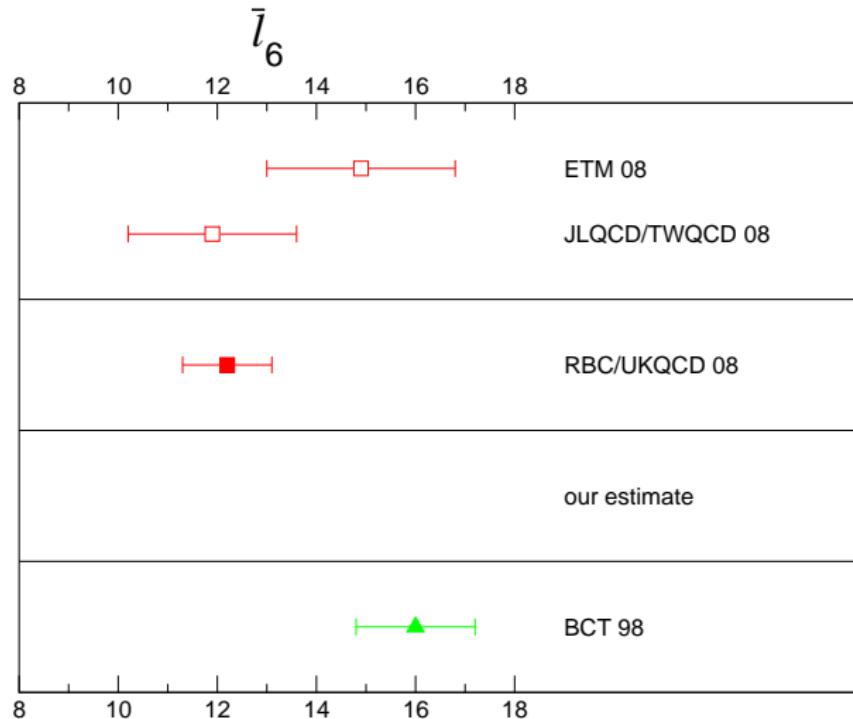
# $SU(2)$ low-energy constants



# $SU(2)$ low-energy constants



# $SU(2)$ low-energy constants



# Outlook

- ▶ FLAG aims to provide a summary of lattice results relevant for the phenomenology accessible to non-experts
- ▶ paper and webpage will become public **this summer**
- ▶ we plan to have **yearly updates**
- ▶ for the future we are open to and encourage contributions from **outside Europe**
- ▶ if you are interested and/or have suggestions or criticisms **you are most welcome**