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Long-distance effects in Rare and radiative K decays

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- Outline

Introduction

I- $K \to \pi\nu\bar{\nu}$

II- $K_L \to \pi^0 \ell^+ \ell^-$

III- $K_L \to \ell^+ \ell^-$

Conclusion

New round of experiments aiming at very rare K decays

Prime targets because of
- the *cleanness of their SM predictions*,
- their *sensitivity to New Physics*.

But, *long-distance effects are nevertheless present*.

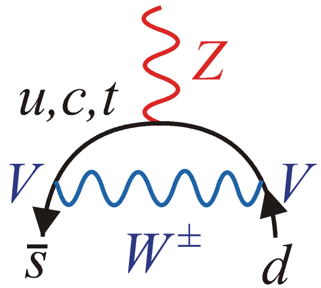
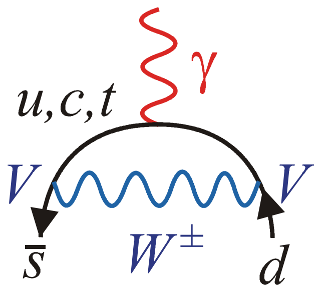
How to deal with these effects?

As usual in ChPT, by relating them to other, well measured observables.

These inputs come essentially from *radiative K decays*.

Needed to learn about the *QCD – EW interplay at low-energy*.

A. Electroweak anatomy of rare & radiative K decays

	CPV: *top dominates*	$K_2 \to \pi^0\nu\bar{\nu}$ $K_1 \to \ell^+\ell^-, K_2 \to \pi^0\ell^+\ell^-$
	CPC: *top & charm* (+ small correction from up)	$K_1 \to \pi^0\nu\bar{\nu}$ $K_2 \to \ell^+\ell^-, K_1 \to \pi^0\ell^+\ell^-$
	CPC: *up dominates*	$K_1 \to \pi^0\ell^+\ell^-$
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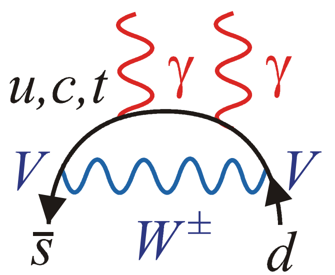	CPC: *up dominates*	$K_{1,2} \to \gamma\gamma,\ K_{1,2} \to \pi^0\gamma\gamma$
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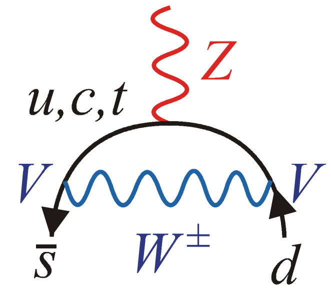
Mass states are combinations of CP states: $K_L \sim K_2 + \varepsilon K_1,\ K_S \sim K_1 + \varepsilon K_2$

→ neutral modes have two contributions: direct and (ε-suppressed) indirect.

A. Electroweak anatomy of rare & radiative K decays

	CPV: *top dominates*	$K_2 \to \pi^0 \nu\bar{\nu}$ $K_1 \to \ell^+\ell^-, K_2 \to \pi^0 \ell^+\ell^-$
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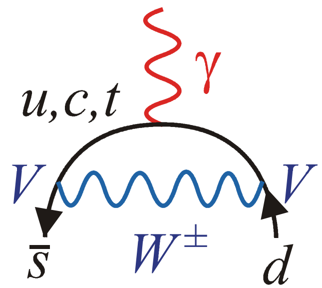	CPC: *up dominates*	$K_1 \to \pi^0 \ell^+\ell^-$
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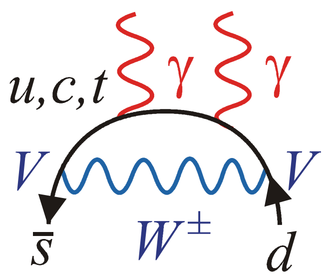	CPC: *up dominates*	$K_{1,2} \to \gamma\gamma,\ K_{1,2} \to \pi^0 \gamma\gamma$
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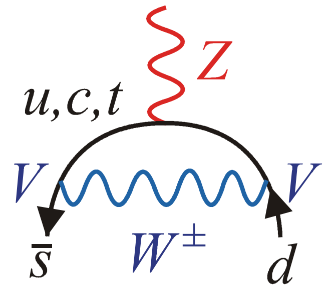
Decays of the K^+ proceed through both the “CPC” and “CPV” contributions.

Except for $K^+ \to \pi^+ \nu\bar{\nu}$, there is always a dominant up-quark contribution.

A. Electroweak anatomy of rare & radiative K decays

	**CPV:** *top dominates*	$K_2 \to \pi^0 \nu\bar{\nu}$ $K_1 \to \ell^+\ell^-, K_2 \to \pi^0 \ell^+\ell^-$
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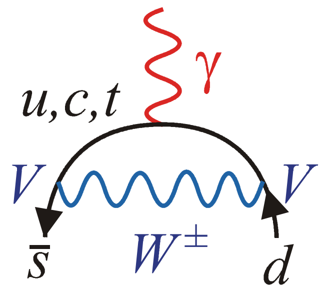	CPC: *up dominates*	$K_1 \to \pi^0 \ell^+\ell^-$ $K \to \pi\pi\gamma$
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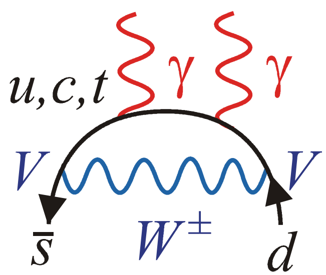	CPC: *up dominates*	$K_{1,2} \to \gamma\gamma,\ K_{1,2} \to \pi^0\gamma\gamma$ $K_{1,2} \to \pi^0 \ell^+\ell^-$
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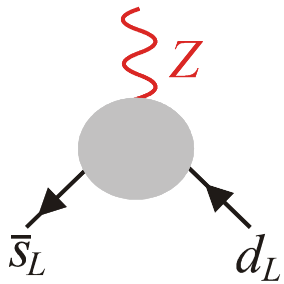
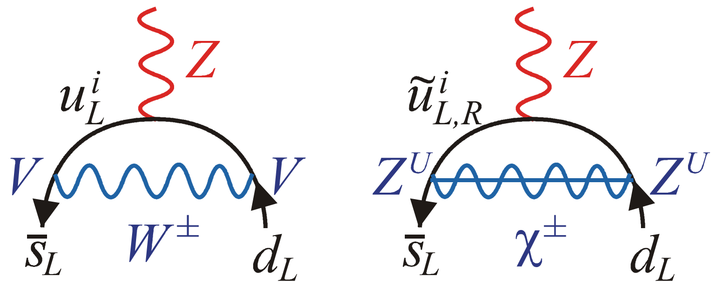
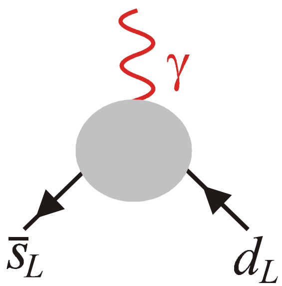
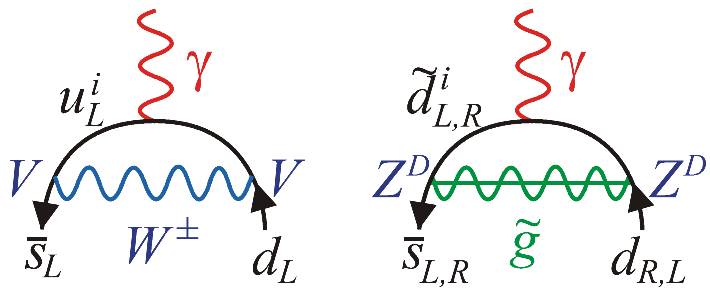
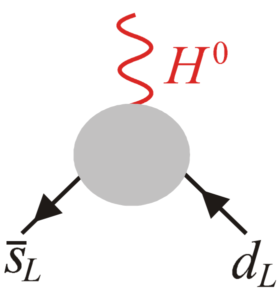
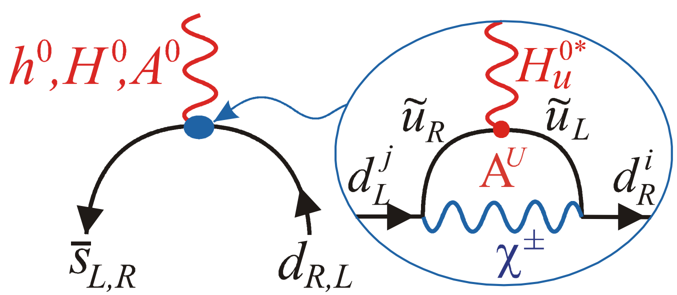
When there are direct LD contributions, they usually dominate.

New Physics can be significant when SD is significant (exception: *asymmetries*!).

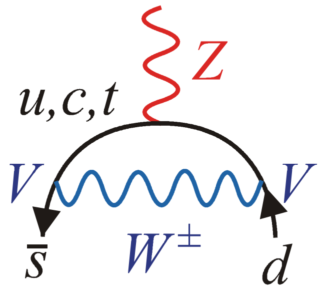
B. Probing EW structures with rare K decays

EW Penguin	SM and/or example of SUSY diagram	Contributes to
		$K \to \pi\nu\bar{\nu}$ $K_L \to \pi^0 \ell^+ \ell^-$ $K_L \to \ell^+ \ell^-$
		$K_L \to \pi^0 \ell^+ \ell^-$ $K \to \pi\pi\gamma$
		$K_L \to \pi^0 \mu^+ \mu^-$ $K_L \to \mu^+ \mu^-$ (helicity-suppressed)

New Physics to be identified by looking at *patterns of deviations*!

$$K \to \pi\nu\bar{\nu}$$

A. Where are the long-distance effects?

	CPV: *top dominates*	$\boxed{K_2 \to \pi^0 \nu\bar{\nu}}$ $K_1 \to \ell^+\ell^-, K_2 \to \pi^0 \ell^+\ell^-$
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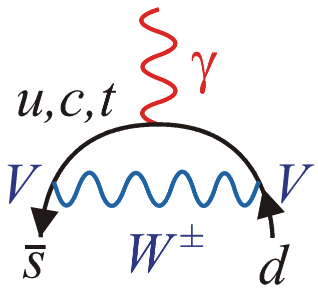	CPC: *up dominates*	$K_1 \to \pi^0 \ell^+\ell^-$ $K \to \pi\pi\gamma$
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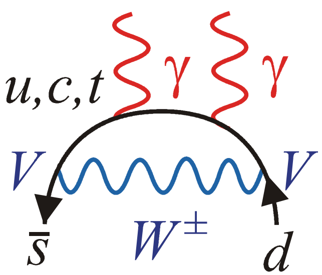
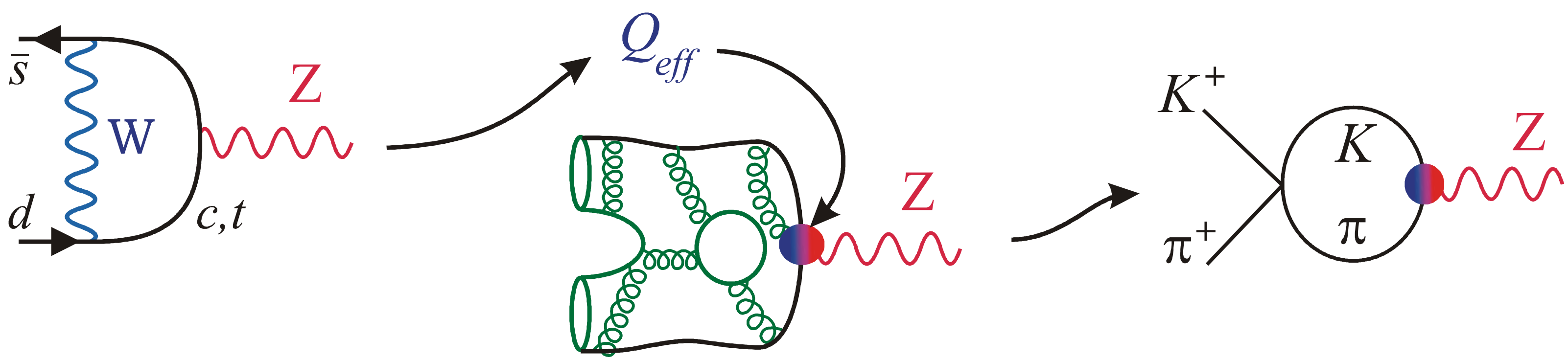
	CPC: *up dominates*	$K_{1,2} \to \gamma\gamma,\ K_{1,2} \to \pi^0 \gamma\gamma$ $K_{1,2} \to \pi^0 \ell^+\ell^-$ $K_{1,2} \to \ell^+\ell^-$
	CPV: *only top & charm* (suppressed ~ $1/m_{c,t}$)	

These modes probe exclusively the Z penguin (and W box).
Dominated by short-distance physics, but…

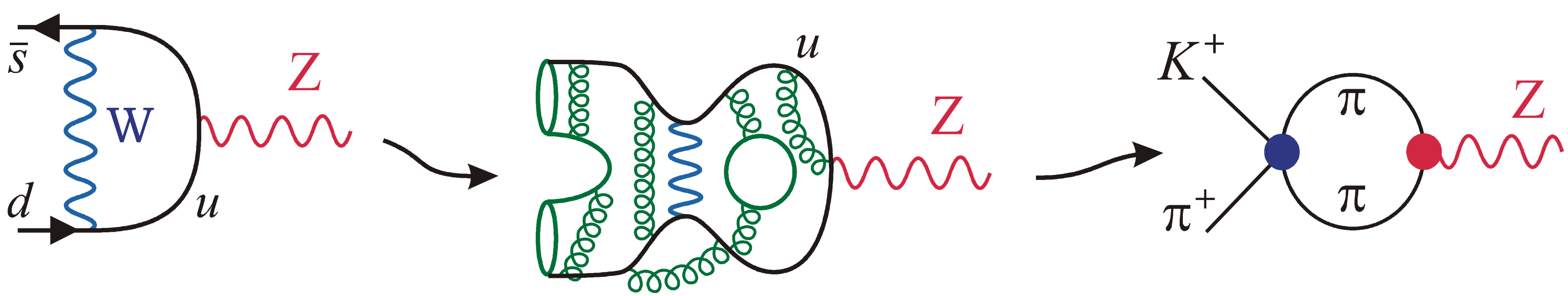
A. Where are the long-distance effects?

1. LD effects for the top/charm "pure" SD contribution = *matrix elements*

$$Q_{eff} = (\bar{s}d)_V \otimes (\bar{\nu}\nu)_{V-A} \;\to \langle \pi | (\bar{s}d)_V | K \rangle$$



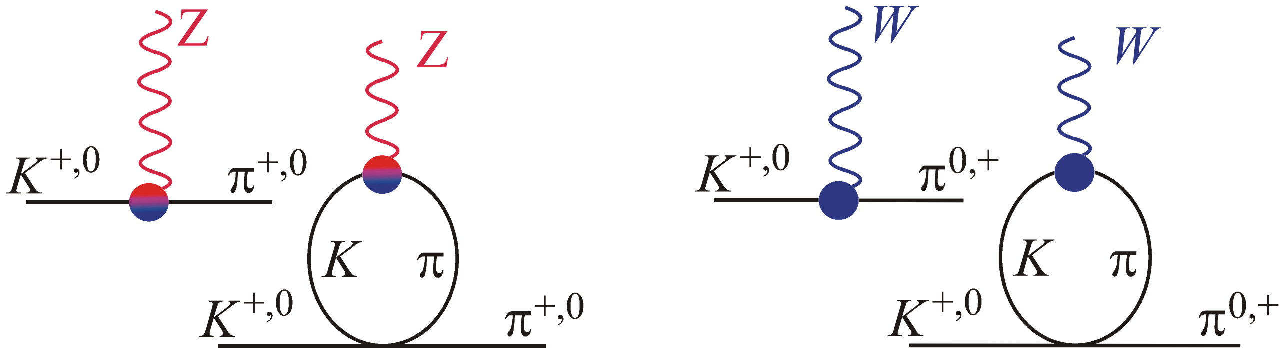
2. The up-quark pure LD contribution (*CP-conserving*)



B. Matrix elements of the dimension-six operator

Mescia, C.S. '06

The "mesonic dressings" of Q_{eff} is very similar to those for the Fermi operator:



The *vector and scalar form-factors* are needed (values at zero and slopes).

Isospin-breaking effects, $\varepsilon^{(2)} \sim m_d - m_u \sim 1\%$, must be included!

For that, two *very clean ratios* can be used:

$$r(q^2) = \frac{f_+^{K^+\pi^0}(q^2)\, f_+^{K^0\pi^0}(q^2)}{f_+^{K^+\pi^+}(q^2)\, f_+^{K^0\pi^+}(q^2)} = 1 + \mathcal{O}((\varepsilon^{(2)})^2) = \boxed{1.0000(2)}$$

$$r_K = \frac{f_+^{K^+\pi^+}(0)}{f_+^{K^0\pi^+}(0)} = 1.00027(8) + \varepsilon^{(2)}\, 0.12(7) = \boxed{1.0015(7)}$$

$\varepsilon^{(2)}\delta_{LR}$

(NLO + partial NNLO)

Mescia,C.S. '06

For the slopes: $$\frac{\lambda_+^{FCNC}}{\lambda_+^{CC}} = \frac{M^2(K^{*+})}{M^2(K^{*0})} = 0.990\ (\pm 0.005)$$

The Flavianet fit to $K_{\ell 3}$ *form-factors & slopes* (2008) leads to

$$\kappa_\nu \sim \int d\Phi_3 \,|\langle \pi\nu\bar{\nu}|Q_{eff}|K\rangle|^2$$

		τ_+	$f(0)$	$slopes$	r_K	r	Future?
		Exp.			*Th.*		
κ_ν^+	0.5168(25)	19%	43%	21%	17%	-	±0.0023
κ_ν^0	2.190(18)	-	77%	12%	9%	2%	±0.013

$\delta_{SU(2)}$

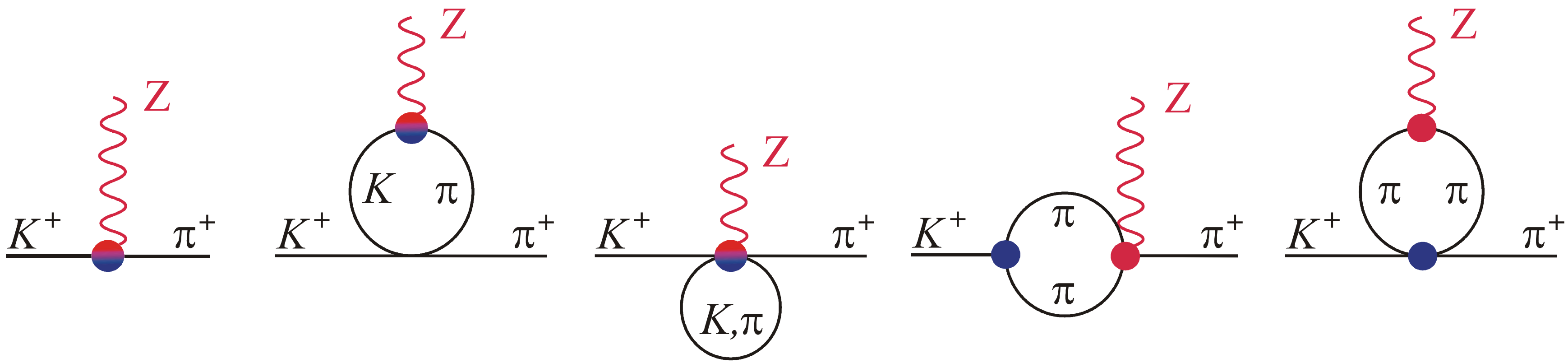
$$\frac{\kappa_\nu^+}{\kappa_\nu^0} = 0.2359(17) \qquad (\text{Future? } \pm 0.0008)$$

Still room for improvement on the experimental side.

C. Long-distance up-quark contribution

Isidori,Mescia,C.S. '05

Naïve inclusion of the Z through the covariant derivative in ChPT produces



How to *disentangle the genuine up-quark contribution*?

Remove from the Z coupling any Q_{eff} structure.

Ask that the Z coupling does not induce a local $K_L \to Z$ coupling.

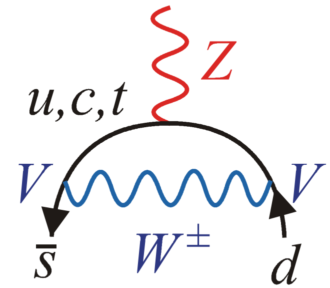
Many unknown counterterms, part of them occuring in $K^+ \to \pi^+\gamma^* \to \pi^+\ell^+\ell^-$.

Overall, these contributions are small, about 10% of the charm contribution.

(expected from the behavior of the Z penguin ~ m_q^2).

$$K_L \to \pi^0 \ell^+ \ell^-$$

A. Where are the long-distance effects?

	CPV: *top dominates*	$K_2 \to \pi^0\nu\bar{\nu}$ $K_1 \to \ell^+\ell^-$, $K_2 \to \pi^0\ell^+\ell^-$
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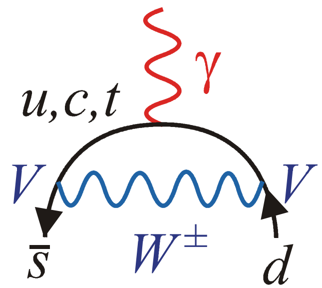	CPC: *up dominates*	$K_1 \to \pi^0\ell^+\ell^-$ $K \to \pi\pi\gamma$
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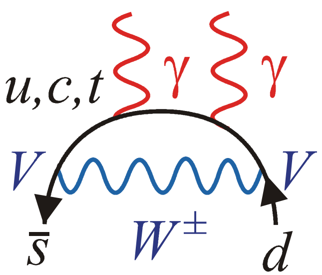	CPC: *up dominates*	$K_{1,2} \to \gamma\gamma$, $K_{1,2} \to \pi^0\gamma\gamma$ $K_{1,2} \to \pi^0\ell^+\ell^-$
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Direct CPV (Short-distance)

Indirect CPV (Long-distance)

CPC (Long-distance)

B. Direct CPV: Matrix elements of the dimension-six operators

Mescia,C.S. '06

LD effects for the top/charm "pure" SD contribution = *matrix elements*

$$Q_{eff}^{V} = (\bar{s}d)_V \otimes (\bar{\ell}\ell)_V,\ Q_{eff}^{A} = (\bar{s}d)_V \otimes (\bar{\ell}\ell)_A$$

As for $K \to \pi\nu\bar{\nu}$, those are extracted from $K_{\ell 3}$ *decays*:

		Exp.			Th.		
		τ_+	$f(0)$	*slopes*	r_K	r	Future?
$\kappa_e^{V,A}$	0.7691(64)	-	77%	12%	9%	2%	±0.0046
κ_μ^{V}	0.1805(16)	-	73%	16%	8%	2%	±0.0011
κ_μ^{A}	0.4132(51)	-	54%	38%	6%	2%	±0.0031

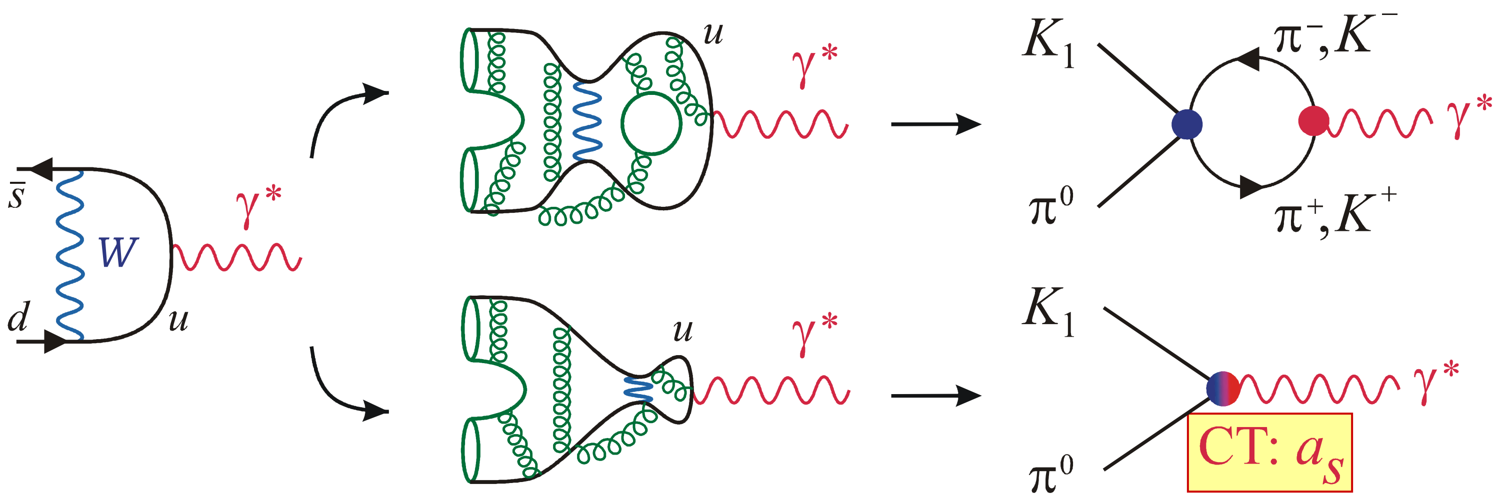
$$\kappa_\ell^{V,A} \sim \int d\Phi_3 \, |\langle \pi^0 \ell\bar{\ell} | Q_{eff}^{V,A} | K_L \rangle|^2$$

Already *very precise compared the other contributions*.

C. Indirect CPV: Long-distance photon penguin

D'Ambrosio et al. '98

Indirect CP-violation is $K_L \to \varepsilon K_1 \to \pi^0 \ell^+ \ell^-$, related to $K_S \to K_1 \to \pi^0 \ell^+ \ell^-$:



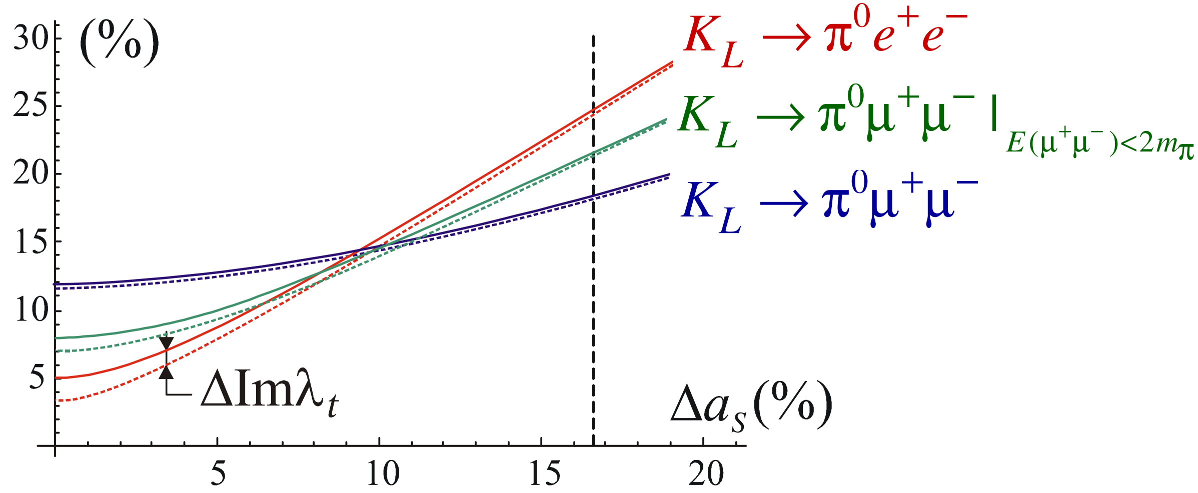
Loops are rather small, a single counterterm a_S dominates.

It is fixed from $K_S \to \pi^0 \ell^+ \ell^-$ (up to its sign) measured by NA48:

$$\left.\begin{aligned} Br(K_S \to \pi^0 e^+ e^-)_{m_{ee}>165\mathrm{MeV}} &= (3.0^{+1.5}_{-1.2} \pm 0.2)\times 10^{-9} \\ Br(K_S \to \pi^0 \mu^+ \mu^-) &= (2.9^{+1.4}_{-1.2} \pm 0.2)\times 10^{-9} \end{aligned}\right\} \to |a_S| = 1.2 \pm 0.2$$

C. Indirect CPV: Long-distance photon penguin

This CT is the main source of error for



Besides $K_S \to \pi^0 \ell^+ \ell^-$, the paths to constrain or measure a_S are:

- The decay $K^+ \to \pi^+ \ell^+ \ell^-$ is similar, dominated by a_+, theory can approximately relate the two ($a_S \sim 2N_{14} + N_{15}$, $a_+ \sim N_{14} - N_{15}$).

e.g. Buchalla,D'Ambrosio,Isidori '03, Greynat,Friot,de Rafael '04; see also Bruno,Prades '03

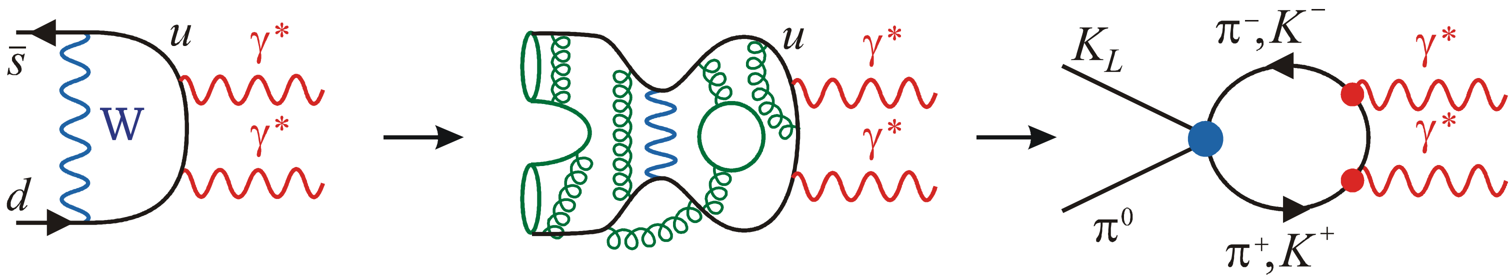
- $K_L \to \pi^0 \pi^0 \ell^+ \ell^-$ depends on the same a_S and is sensitive to its sign. However, its branching is $\leq 10^{-9}$ for $\ell = e$ (KTeV limit: $< 6.6 \times 10^{-9}$).

Funck,Kambor '93

- FB asymmetries for $K_L \to \pi^0 \mu^+ \mu^-$ could fix the sign.

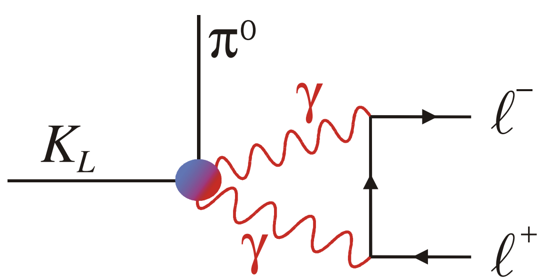
Mescia,Trine,C.S. '06

D. CPC: Long-distance double photon penguin



LO (p^4) is finite, produces $\ell^+\ell^-$ in a scalar state only (helicity-suppressed),

Higher order estimated using the $K_L \to \pi^0\gamma\gamma$ rate and spectrum:



$\ell = \mu\,,\, \ell = e$

$(0^{++}, 2^{++})$

- Production of $(\mu^+\mu^-)_{0^{++}}$ under control within 30%. *Isidori,Unterdorfer,C.S. '04*

- No signal of $(\gamma\gamma)_{2^{++}}$ implies $(e^+e^-)_{2^{++}}$ is negligible. *Buchalla,D'Ambrosio, Isidori '03*

  ($K_S \to \gamma\gamma$ is also useful to constrain the p^6 CT structure)

E. Indirect accesses to the photon penguin

1. Direct CP-asymmetry $A_{CP} = \dfrac{\Gamma(K^+ \to \pi^+\ell^+\ell^-) - \Gamma(K^- \to \pi^-\ell^+\ell^-)}{\Gamma(K^+ \to \pi^+\ell^+\ell^-) + \Gamma(K^- \to \pi^-\ell^+\ell^-)}$

Sensitive to the interference between the up γ penguin and charm, top contributions. Expected to be in the 10^{-5} range in the SM.

e.g. D'Ambrosio et al. '98

2. Direct CP-asymmetry $A_{CP} = \dfrac{\Gamma(K^+ \to \pi^+\pi^0\gamma) - \Gamma(K^- \to \pi^-\pi^0\gamma)}{\Gamma(K^+ \to \pi^+\pi^0\gamma) + \Gamma(K^- \to \pi^-\pi^0\gamma)}$

Sensitive to EM operator, again expected to be small in the SM (10^{-5}).

e.g. D'Ambrosio,Isidori. '95

3. Phase-space asymmetries for $K_L \to \pi^+\pi^-\gamma^*$

Large, but dominated by indirect CPV effects ($K_L \to \varepsilon K_1 \to \pi^+\pi^-$)

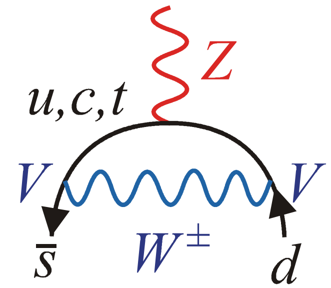
e.g. D'Ambrosio,Isidori. '95

4. BUT: $K_L \to \pi^0\ell^+\ell^-$ *is richer since it probes also the Higgs penguins*.

Mescia,Trine,C.S. '06

$$K_L \to \ell^+ \ell^-$$

A. Where are the long-distance effects?

	CPV: *top dominates*	$K_2 \to \pi^0\nu\bar{\nu}$ $K_1 \to \ell^+\ell^-$, $K_2 \to \pi^0\ell^+\ell^-$
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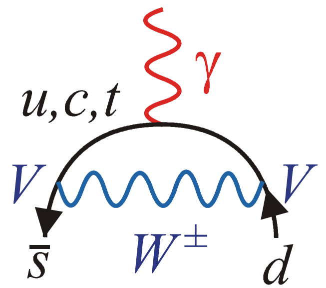
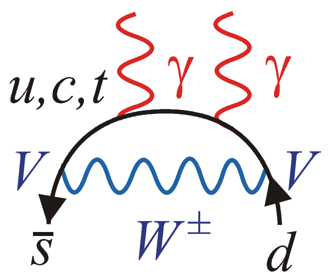
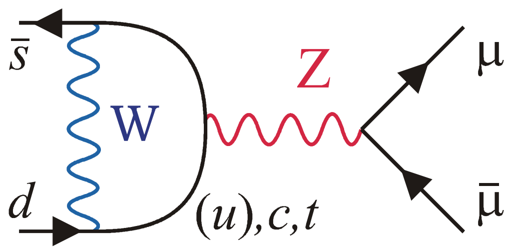
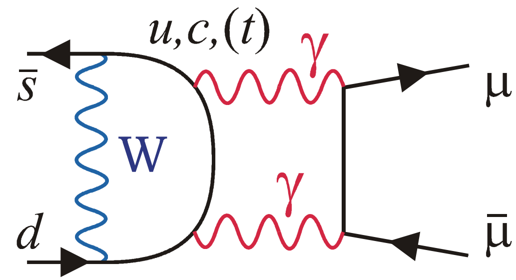
	CPC: *up dominates*	$K_1 \to \pi^0\ell^+\ell^-$ $K \to \pi\pi\gamma$
	CPV: *only top & charm* $(\mathrm{Im}\, V_{ud}V_{us}^{\dagger} = 0)$	$K \to \pi\pi\gamma$ $K_2 \to \pi^0\ell^+\ell^-$
	CPC: *up dominates*	$K_{1,2} \to \gamma\gamma$, $K_{1,2} \to \pi^0\gamma\gamma$ $K_{1,2} \to \pi^0\ell^+\ell^-$
	CPV: *only top & charm* (suppressed ~ $1/m_{c,t}$)	$K_{1,2} \to \ell^+\ell^-$

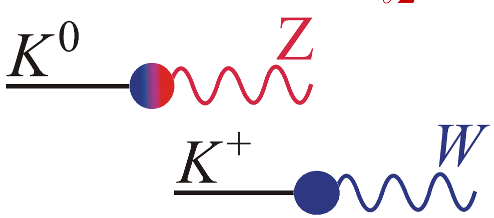
Direct CPC (Short-distance) | Indirect CPV (Negligible) | CPC (Long-distance)

B. Detailed structure of the $K_L \to \ell^+\ell^-$ process

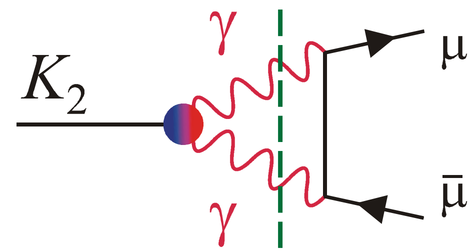




Matrix element from $K_{\ell 2}$:



$Abs(\gamma\gamma) =$



$Disp(\gamma\gamma)$

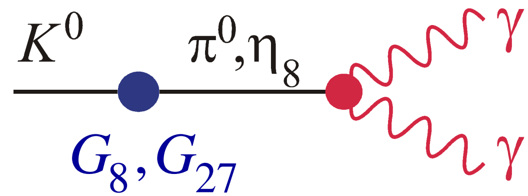
$$Br(K_L \to \mu^+\mu^-) \approx ((-0.95 \pm ???)^2 + 6.7) \cdot 10^{-9}$$

- Nearly saturated by $Abs(\gamma\gamma)$ since $B^{\exp} = 6.87(11) \cdot 10^{-9}$ (*smaller exp. error* ?)
- Short-distance is CPC, and interfere with the $\gamma\gamma$ contribution (*sign*?)
- The dispersive part $Disp(\gamma\gamma)$ diverges (*how to estimate it reliably*?)

C. The two-photon decay $K_L \to \gamma\gamma$

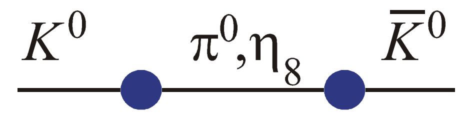
Gérard, Trine, C.S '05

The *SU(3) pole amplitude vanishes*:



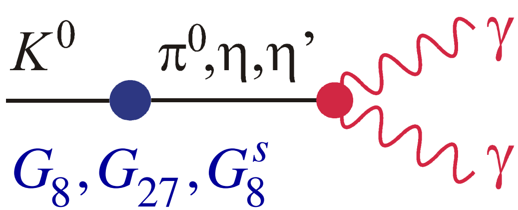
The decay is driven by $Q_1^u = (\bar{s}d)\otimes(\bar{u}u)$, but there is no linear combinations such that $\alpha\pi^0 + \beta\eta_8 = \bar{u}u$!

Same mechanism at play in $K_L \to \pi^+\pi^-\gamma$ & ΔM_K :



To *consistently* account for NLO corrections (unknown CTs), *go first to U(3)*.

Leading N_c SU(3)-$\mathcal{O}(p^6)$ CTs all collapse to a single parameter G_8^s .



$$\approx \overbrace{\left(G_8^s + \tfrac{2}{3}G_{27}\right)}^{\sim z_1(\mu_{hadr.})}\Big((0.46)_\pi - (1.83)_\eta - (0.12)_{\eta'}\Big)$$

Using the experimental value $B(K_L \to \gamma\gamma)^{\exp} \Rightarrow G_8^s / G_8 \approx \pm\frac{1}{3}$.

D. The SD-LD interference sign in $K_L \to \ell^+\ell^-$

Gérard, Trine, C.S '05

Requires the sign of the $K_L \to \gamma\gamma$ amplitude $\Leftrightarrow$ Sign of G_8^s.

1- Theoretical clues:

$$H_{eff}(\mu > 1\,\mathrm{GeV}) = z_1 Q_1^u + z_2 Q_2^u + z_6 Q_6^u + \ldots$$

$$H_{eff}(\mu_{hadr.}) = -(G_8^s + \tfrac{2}{3}G_{27})\tilde{Q}_1 + (G_8^s - G_{27})\tilde{Q}_2 - (G_8 + G_8^s - \tfrac{1}{3}G_{27})\tilde{Q}_6 + \ldots$$

If the non-perturbative evolution of Q_1^u & Q_2^u is ~smooth (no sign change):

$$(z_1 + z_2)^2(z_2 - z_1) = 1.0 \pm 0.3 \;\Rightarrow\; G_8^s / G_8 = -0.38(12)$$

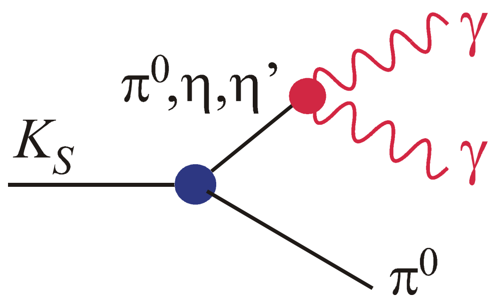
One can then resolve the current-current vs. penguin fraction in $K \to \pi\pi$:

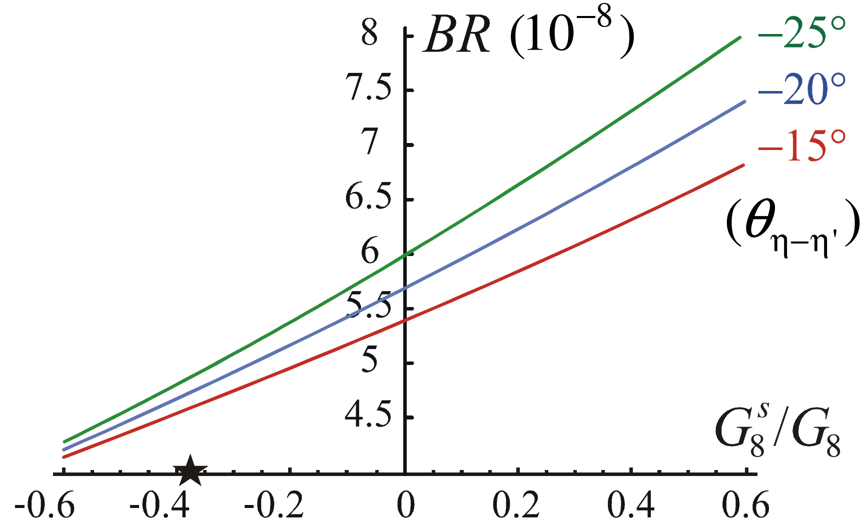
$$\tilde{Q}_{1,2} : 35\% \quad \leftrightarrow \quad \tilde{Q}_6 : 65\%$$

Penguins account for ~2/3 of the $\Delta I = ½$ rule (at the hadronic scale, not at m_c!).

Gérard, Trine, C.S '05

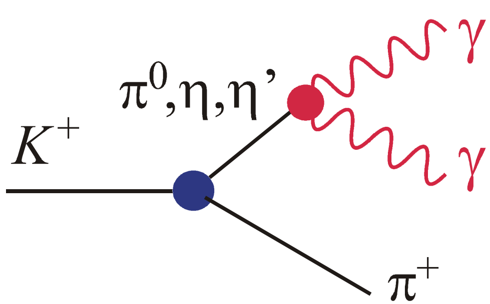
2- *Experimentally*, G_8^s could be fixed from $K_S \to \pi^0\gamma\gamma$:

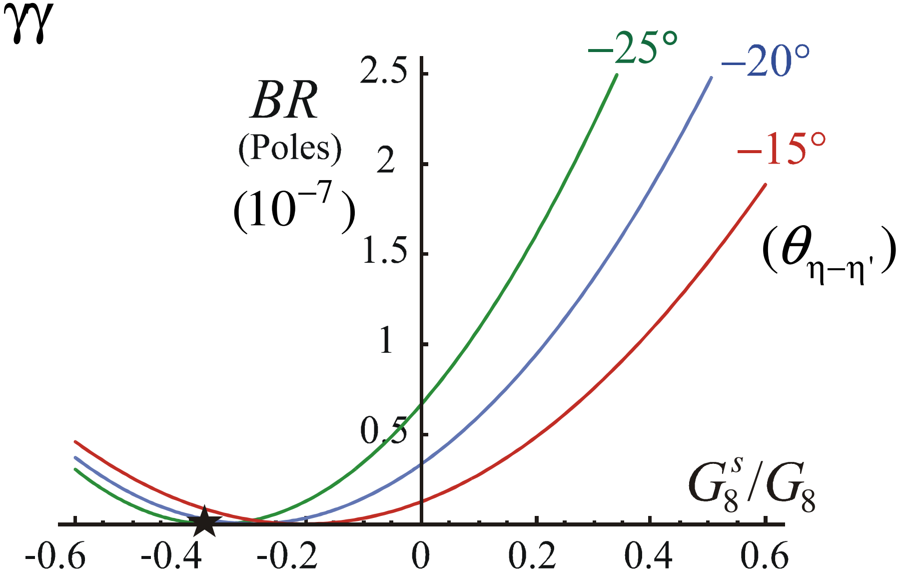




$$B(K_S \to \pi^0\gamma\gamma)^{\text{exp}}_{z>0.2} = (4.9 \pm 1.8)\cdot 10^{-8}$$

or from pole contributions to $K^+ \to \pi^+\gamma\gamma$



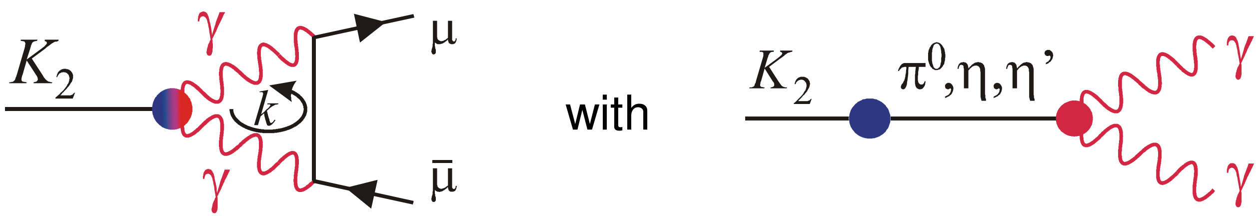


(even more constraining at the low-energy end of the γ spectrum)

E. The dispersive two-photon contribution to $K_L \to \ell^+\ell^-$

Isidori,Unterdorfer '03

The $\gamma\gamma$ loop diverges (requires *unknown CTs*) for a constant vertex:



CTs estimated by accounting for the momentum-dependence of the vertex as

$$f(q_1^2, q_2^2) = \sum_i \left(1 + \alpha_i \left(\frac{q_1^2}{q_1^2 - m_i^2} + \frac{q_2^2}{q_2^2 - m_i^2} \right) + \beta_i \frac{q_1^2 q_2^2}{(q_1^2 - m_i^2)(q_2^2 - m_i^2)} \right)$$

With two resonances: ρ + one around J/ψ.

Low-energy contraints from the $K_L \to \gamma e^+ e^-, \gamma\mu^+\mu^-, e^+e^-\mu^+\mu^-$ linear slope.

(We would need also the quadratic slope, and other modes like $\mu^+\mu^-\mu^+\mu^-$!)

High-energy constraints from the perturbative up & charm-quark $\gamma\gamma$ penguin.

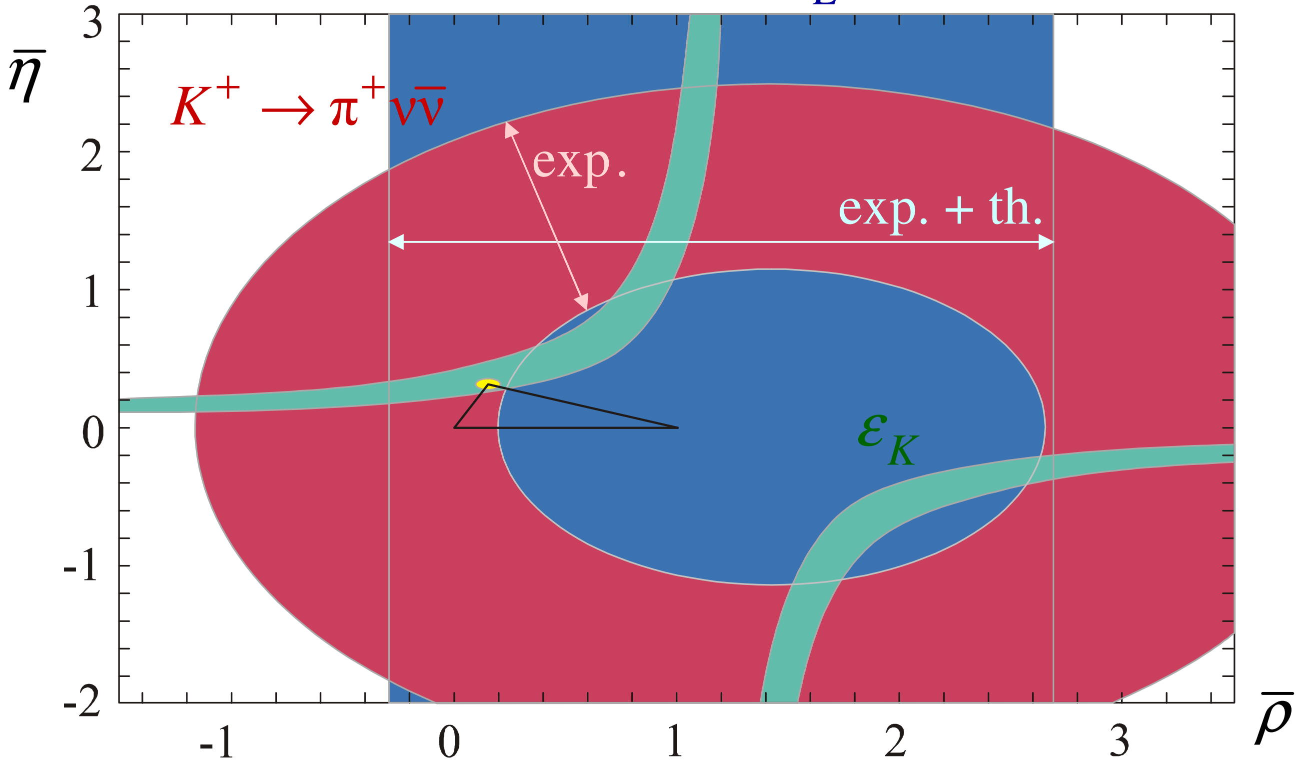
F. $K_L \to \mu^+\mu^-$ *summary* --- *Preliminary* ---

Trine,C.S. 'soon

- $G_8^s / G_8 < 0 \Rightarrow$ constructive interference between SD and LD.
- Updating the analysis, we find $Disp(\gamma\gamma) = -0 \pm 1.5$,

Compared to $Disp(\gamma\gamma) = \pm 0.7 \pm 1.15$ *Isidori & Unterdorfer '03*

$K_L \to \mu^+\mu^-$



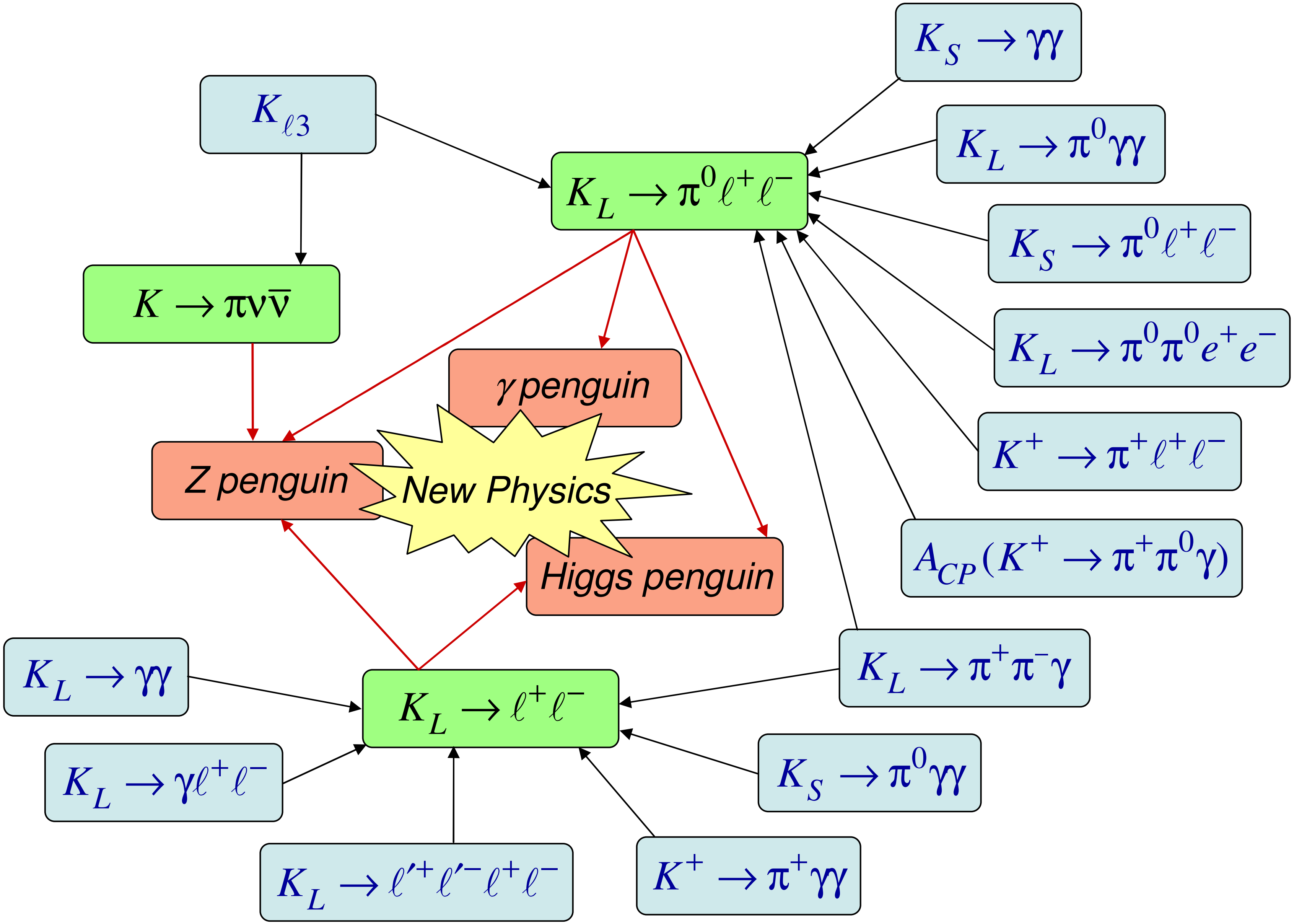
$K_L \to \pi^0 \nu\bar{\nu}$:
$\bar{\eta} < 17$

$K_L \to \pi^0 e^+ e^-$:
$\bar{\eta} < 3.3$

$K_L \to \pi^0 \mu^+ \mu^-$:
$\bar{\eta} < 5.4$

Conclusion

Conclusion



$K_S \to \gamma\gamma$
$K_{\ell 3}$
$K_L \to \pi^0 \gamma\gamma$
$K_L \to \pi^0 \ell^+ \ell^-$
$K_S \to \pi^0 \ell^+ \ell^-$
$K \to \pi\nu\bar{\nu}$
$K_L \to \pi^0 \pi^0 e^+ e^-$
γ penguin
$K^+ \to \pi^+ \ell^+ \ell^-$
Z penguin
New Physics
$A_{CP}(K^+ \to \pi^+ \pi^0 \gamma)$
Higgs penguin
$K_L \to \gamma\gamma$
$K_L \to \pi^+ \pi^- \gamma$
$K_L \to \ell^+ \ell^-$
$K_L \to \gamma \ell^+ \ell^-$
$K_S \to \pi^0 \gamma\gamma$
$K_L \to \ell'^+ \ell'^- \ell^+ \ell^-$
$K^+ \to \pi^+ \gamma\gamma$