ETMC results for Kaon Physics:

dynamical twisted mass lattice QCD

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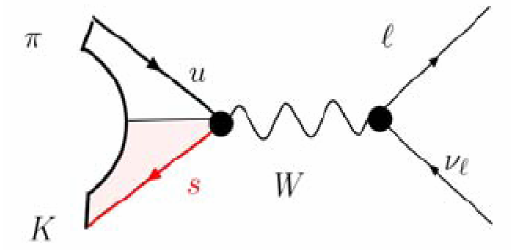
KAON 2009, June 7th-12th, Tsukuba, Japan

- **<u>Simplest Kaon Observables on the lattice:</u>**

<u>1) Tree-level mediated decays</u> **->** ***CKM unitarity, SM tests***

$K \to \pi l \nu$



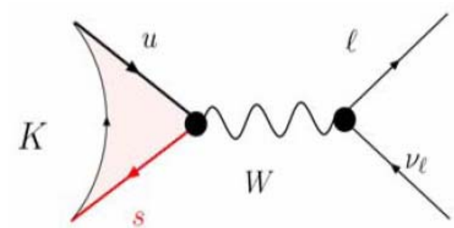
Hadronic uncertainties from

$$\langle \pi | \bar{s} \gamma^\mu u | K \rangle \Leftrightarrow f_{+,0}(q^2)$$

$f_+(0)$, λ_{+0}, c_{+0} ?

2) <u>Helicity suppressed decays</u> -> ***sensitivity to the Higgs sector/r.h. quarks***

$K \to \mu \nu$



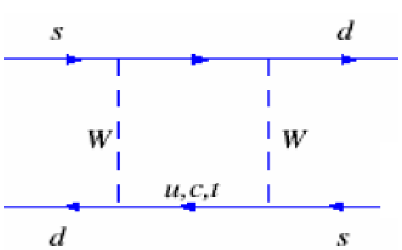
Hadronic uncertainties from

$$\langle 0 | \bar{s} \gamma^\mu \gamma_5 u | K \rangle = p^\mu f_K$$

f_K, f_K / f_π ?

3) <u>FCNC processes</u> -> SUSY, Little Higgs ....

$+\varepsilon_K$



Hadronic uncertainties from

$$\langle \bar{K} | \bar{s} \gamma_L^\mu d \bar{s} \gamma_L^\mu d | K \rangle = 8/3 \left(f_K m_K \right)^2 B_K$$

B_K ?

Warning: *sub-leading effects, no longer negligible in* ε_K

A.Buras & D.Guadagnoli '08,'09

- Lattice simulations with dynamical quarks are now well established

Already available important results:

K_{l2} → ***HPQCD et al, 2005*** PRL100:062002,2008 — f_K/f_π: $280 \le m_\pi \le 400$ MeV, a->0: *Staggered fermions?*

K_{l3} → ***RBC/UKQCD, 2007*** PRL100:141601,2008 — $f_+(0)$: $330 \le m_\pi \le 500$ MeV, *a=0.1fm: DW fermions!*

ε_K → ***RBC/UKQCD, 2006*** PRD78:114509,2008. ***Aubin et al.*** 0905.3947 — B_K: $330/(240) \le m_\pi \le 500$ MeV, *DWF fermions!*

But, setup not yet optimal:

- **Staggered f. unsatisfactory:**
- **DW f. at potentially large residual mass $m_{res}/m_{sea} \sim 0.6$**
- m_π *range too heavy:* $m_\pi << \sqrt{2} m_K$
- continuum limit & finite volume effects
- Renormalisation in mass-independent scheme $N_F=3$

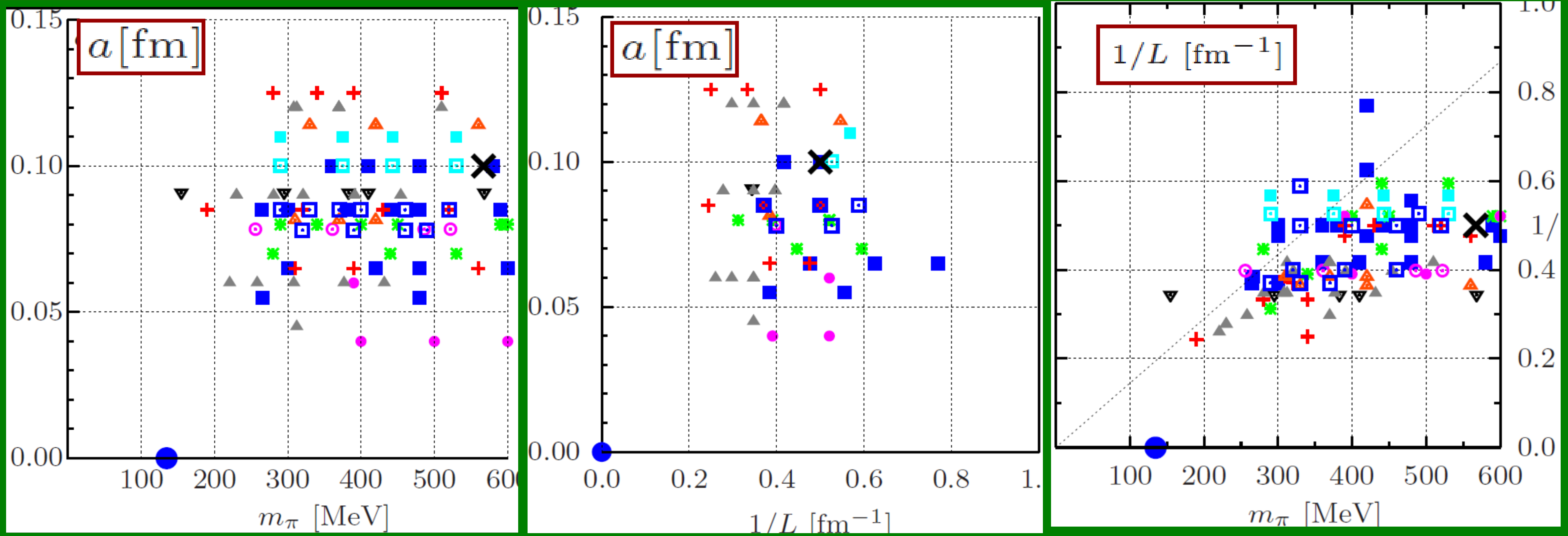
Better Unquenched setup is being generated:

RBC: DW f. at smaller lattice spacing m_{res}/m_{sea}~0.02, mπ=300 MEV → Not cheap

Thanks to recent theoretical developments, very light regime $260 \le m_\pi \le 400$ MeV have been reached by ***Wilson-like*** and ***Overlap*** fermions

cheaper

BMW, ETMC, PACS-CS & JLQCD/TWQCD



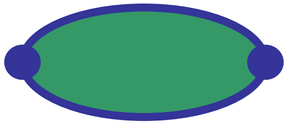
Plots by G. Herdoiza

Recent outstanding achievements:

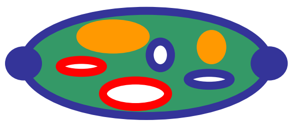
- **Wilson:** ***CERN-TOV.'05,*** $m_\pi \sim 260$ MeV ($N_F=2$)
- **Twisted-mass:** ***ETMC'07,*** $m_\pi \sim 260$ MeV ($N_F=2 \to N_F=2+1+1$)
- **"Clover":** ***BMW'08,*** $m_\pi \sim 190$ MEV ($N_F=2+1$)
- **Clover:** ***PACS-CS'08,*** $m_\pi \sim 156$ MEV ($N_F=2+1$)
- **Overlap:** ***JLQCD/TW'08,*** $m_\pi \sim 290$ MEV ($N_F=2$ & $2+1$)

- **Domain Wall:** ***RBC-'06,*** $m_\pi \sim 330$ MEV ($N_F=2+1$) !!
- **Staggered:** ***MILC-2002,*** $m_\pi \sim 280$ MeV ($N_F=2+1$) ??

QUENCHED QCD
$N_F=0$ $m_q^{sea} \to \infty$



$N_F=2+1$
$m_u=m_d < m_s$ $m_c^{sea} \to \infty$

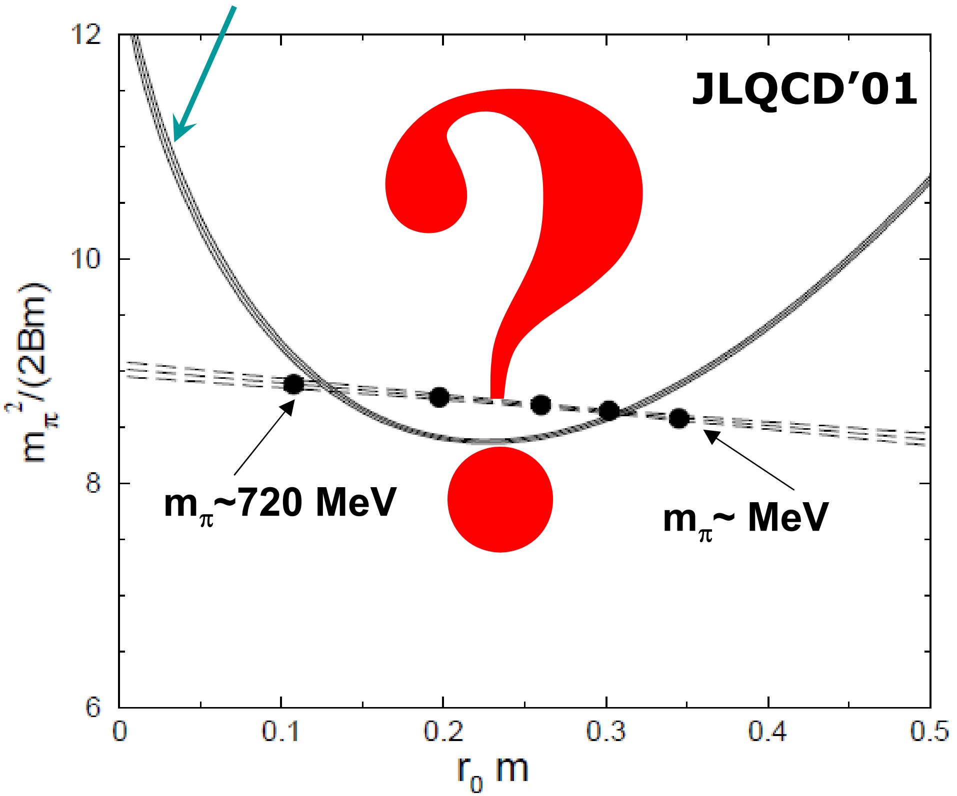


Chiral Regime of QCD

A problem with (old) lattice simulations:
Incompatibility with ChPT?

Chiral regime of QCD

$$m_\pi^2 = M^2 \left[1 + \frac{M^2}{32\pi^2 F^2} \log\left(M^2/\Lambda_3^2\right) + \ldots\right], \quad M^2 = 2Bm$$

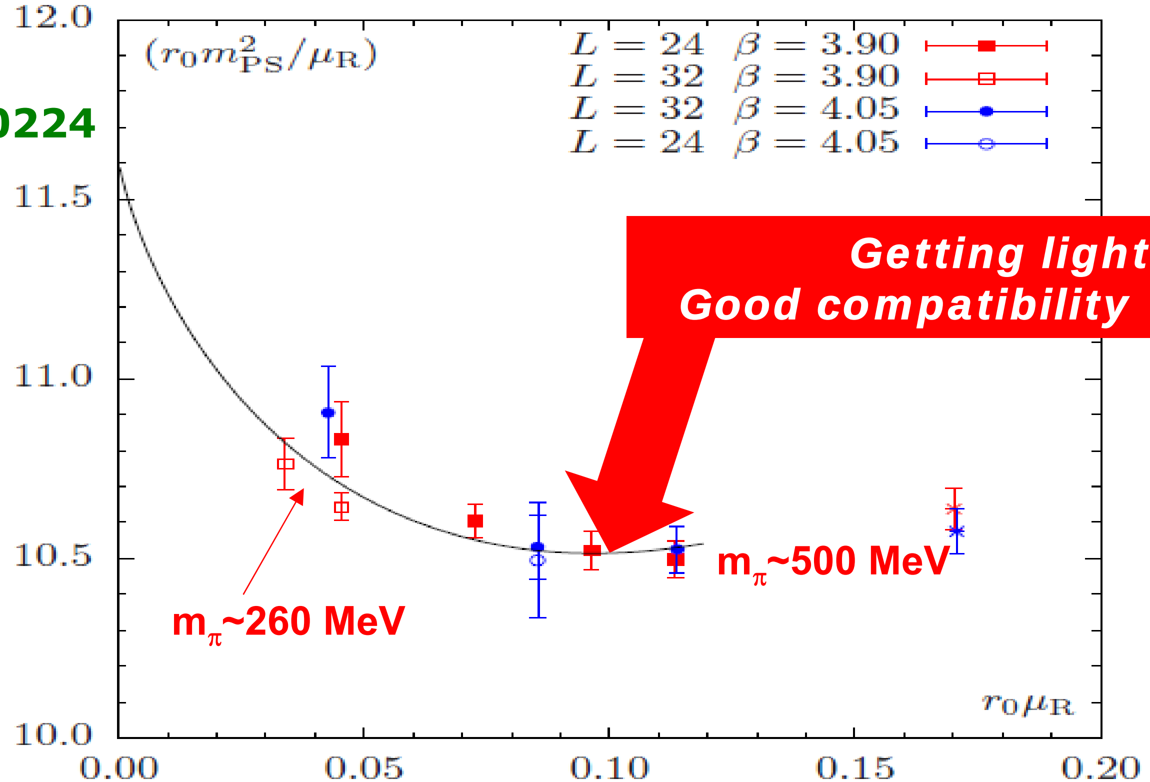


Chiral Regime of QCD

No more problems with (new) lattice simulations:
Good compatibility with ChPT

ETMC arXiV:0803.0224

$(am_\pi)^2/am_q$



Getting lighter: Good compatibility with ChPT

am_q

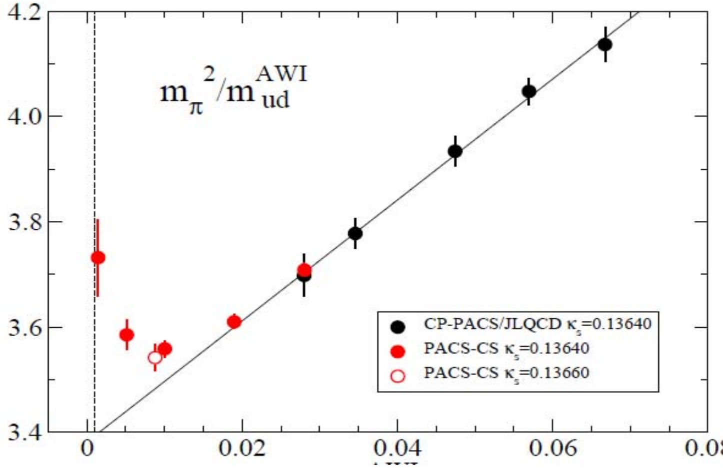
MareNostrum **BlueGene** **BlueGene** **QCDOC** **APENEXT**

Similar plots by other collaborations (using different approaches)

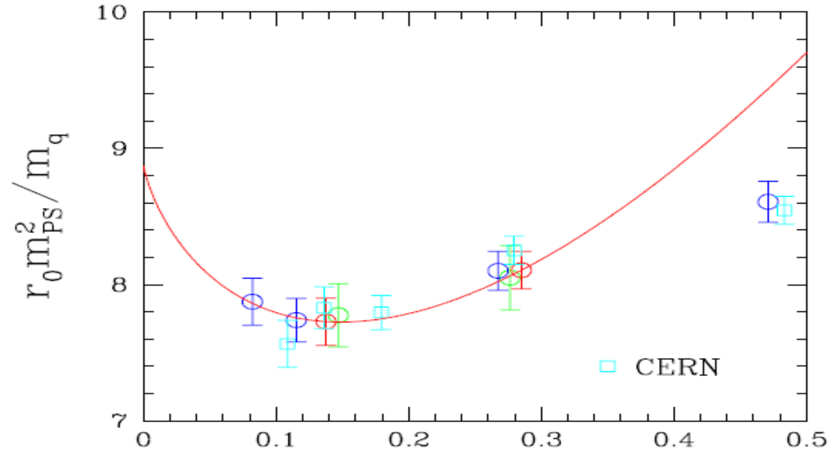
Chiral regime of QCD

$(am_\pi)^2/am_q$

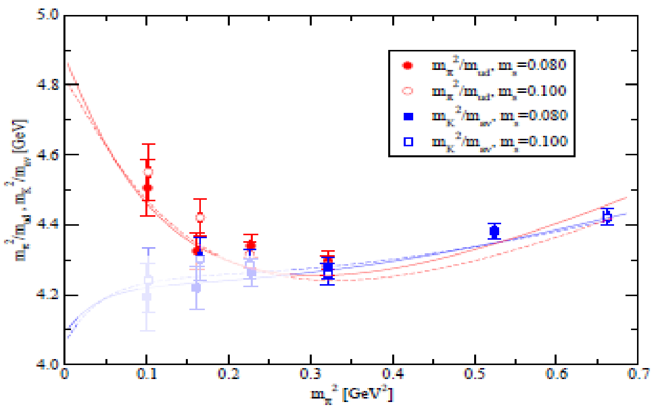
Hints of chiral Logs



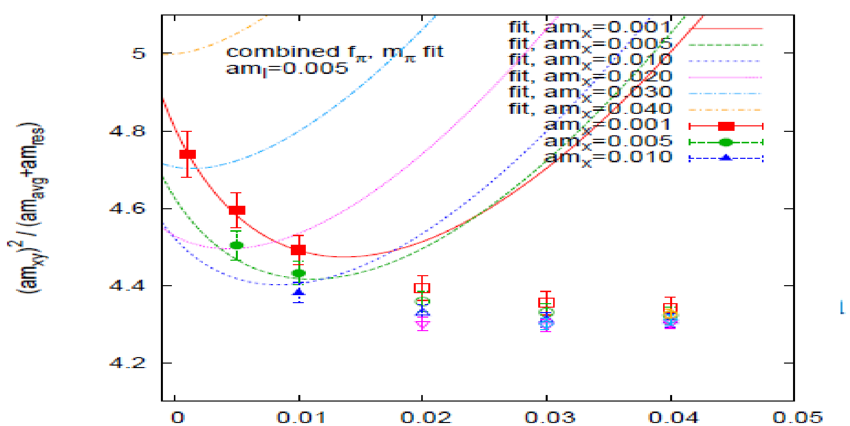
PACS-CS'08, N_F=2+1, Clover:
a=0.09 fm, M_π=156 *MeV*, $L\,M_\pi$ ~2.3!!



QCDSF'08, N_F=2, Clover:
a=0.07 fm, M_π=313 *MeV*, LM_π ~4



JLQCD/TWQCD'08, N_F=2+1, Overlap:
a=0.11 fm, M_π=300 *MeV*, $L\,M_\pi$ ~2.6!!



RBC/UKQCD'06, N_F=2+1, DWF
a=0.114 fm, M_π=310, LM_π ~3-4

Twisted Mass Fermions (tmLQCD): Properties

Frezzotti, Grassi, Sint & Weisz 1999

$$S_{sea}^{TW} = \sum_x \overline{q}_f \left(\underline{\widehat{\nabla}^\mu \gamma^\mu + m_{q_f}} + i a \gamma_5 r_f W_{term} \right) \cdot q_f$$

- ***Discretized QCD Action* + *a "generalized" Wilson term*, W_{term}:** **Wilson 1974**
  - *up and down flavors as doublets in the r-Wilson parameter space: $r_u=1$, $r_d=-1$*

- ***Successful studies for both $N_F=2$ and $N_F=2+1+1$***

  - ☺ *protection* against unphysical small eigenvalues:
    stable simulations for light pions
  - ☺ ***low computation cost***
  - ☺ ***only $\mathcal{O}(a^2)$* lattice discretization errors** **Frezzotti & Rossi 2003**
  - ☺ ***simple renormalization pattern => $f_{\pi,K}$, B_K*** **Frezzotti & Rossi 2005**

  - ☹ ***explicit breaking of parity and isospin:***
    however, this breaking is an *$\mathcal{O}(a^2)$ effect in physical quantities*

V_{us} *determinations and the CKM Unitarity*

- *Axial* **Weak Universality** $\Rightarrow$

$$\underbrace{V_{us}/V_{ud}\, f_K/f_\pi = 0.2760(6)}_{\text{exp:0.21\%}} \Rightarrow \begin{array}{l} V_{us}/V_{ud} = 0.2281(5)\underbrace{(35)}_{\text{th:1.5\%}} \\ V_{us} = 0.2222(34) \end{array}$$

exp:0.21%

th:1.5%

$f_K/f_\pi = 1.210(6)_{fit}(17)_{syst}$ **ETMC arXiV:0904.0954**

- *Vector* **Weak Universality** $\Rightarrow$

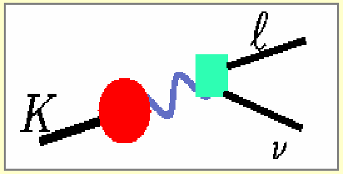
$$V_{us}\underbrace{f_+(0) = 0.21668(45)}_{\text{exp:0.21\%}} \Rightarrow V_{us}^{Kl3} = 0.2267(5)\underbrace{(22)}_{\text{th:0.9\%!!}}$$

exp:0.21%

th:0.9%!!

$$|V_{ud}|^2 + |V_{us}|^2 = 1.0004(15)$$

$f_+(0) = 0.956(6)_{fit}(6)_{syst}$ **ETMC in preparation**



Determination of f_K/f_π

$$\frac{\Gamma(K_{\ell 2})}{\Gamma(\pi_{\ell 2})} \propto \left|\frac{V_{us}}{V_{ud}}\right|^2 \left(\frac{f_K}{f_\pi}\right)^2$$

❖ ETMC determination of f_K/f_π

ETMC arXiV:0904.0954

☹ $N_F=2$ → the strange quark is quenched

☺ **3 values of the lattice spacing: a≈0.10fm, a≈0.085fm, a≈0.065fm** ⇒ **Continuum Limit**

☺ **different volumes** ⇒ **FSE studies**

☺ **(several) light sea quark masses**

$M_\pi \leftrightarrow (\mu_l, \mu_l)$ ***(unitarity pions $\mu_l=\mu_{sea}$)***

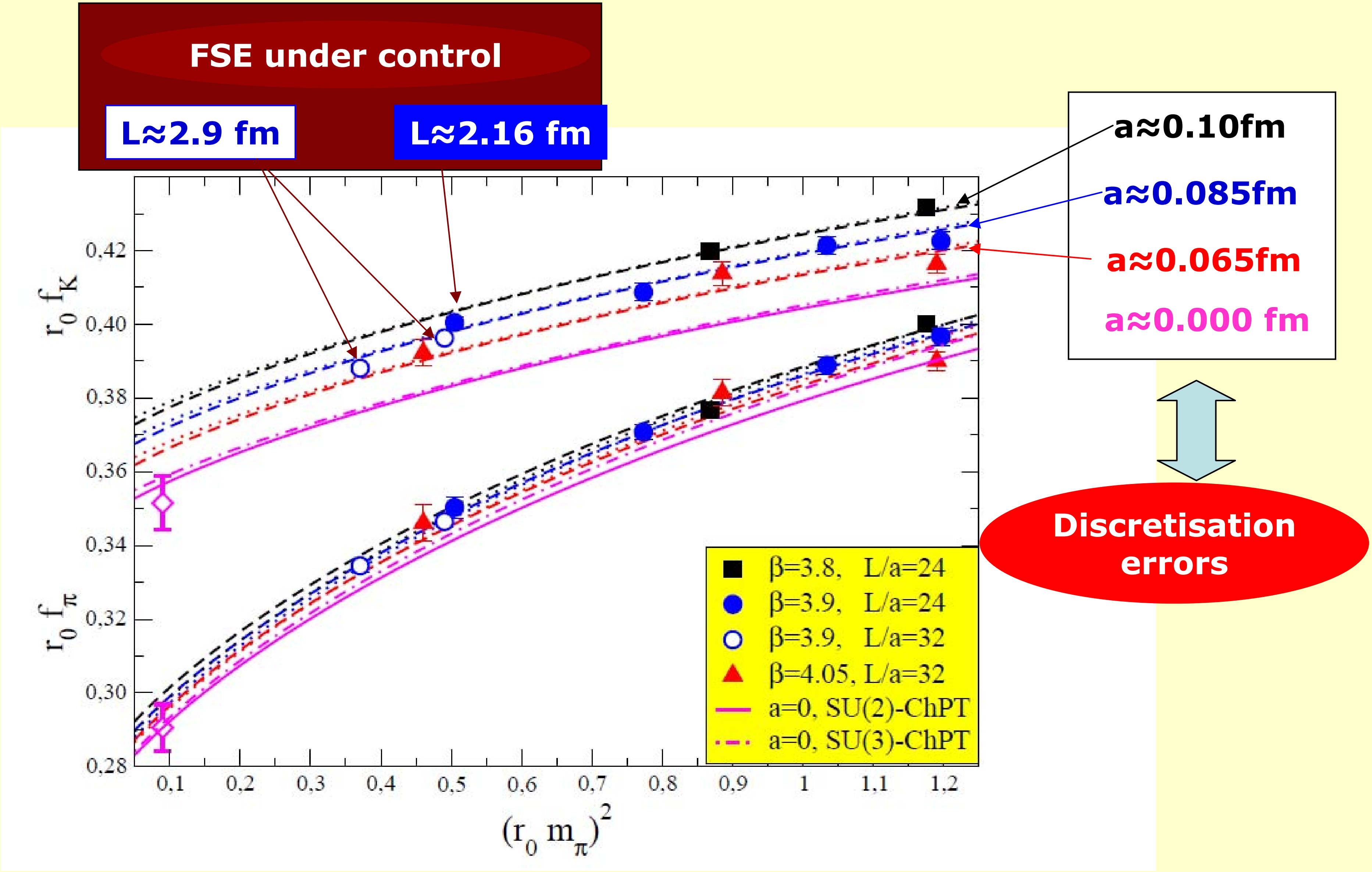
$$0.2 \cdot m_s^{phys} \le \mu_l = \mu_{sea} \le 0.5 \cdot m_s^{phys}$$

$M_K \leftrightarrow (\mu_l, \mu_s)$

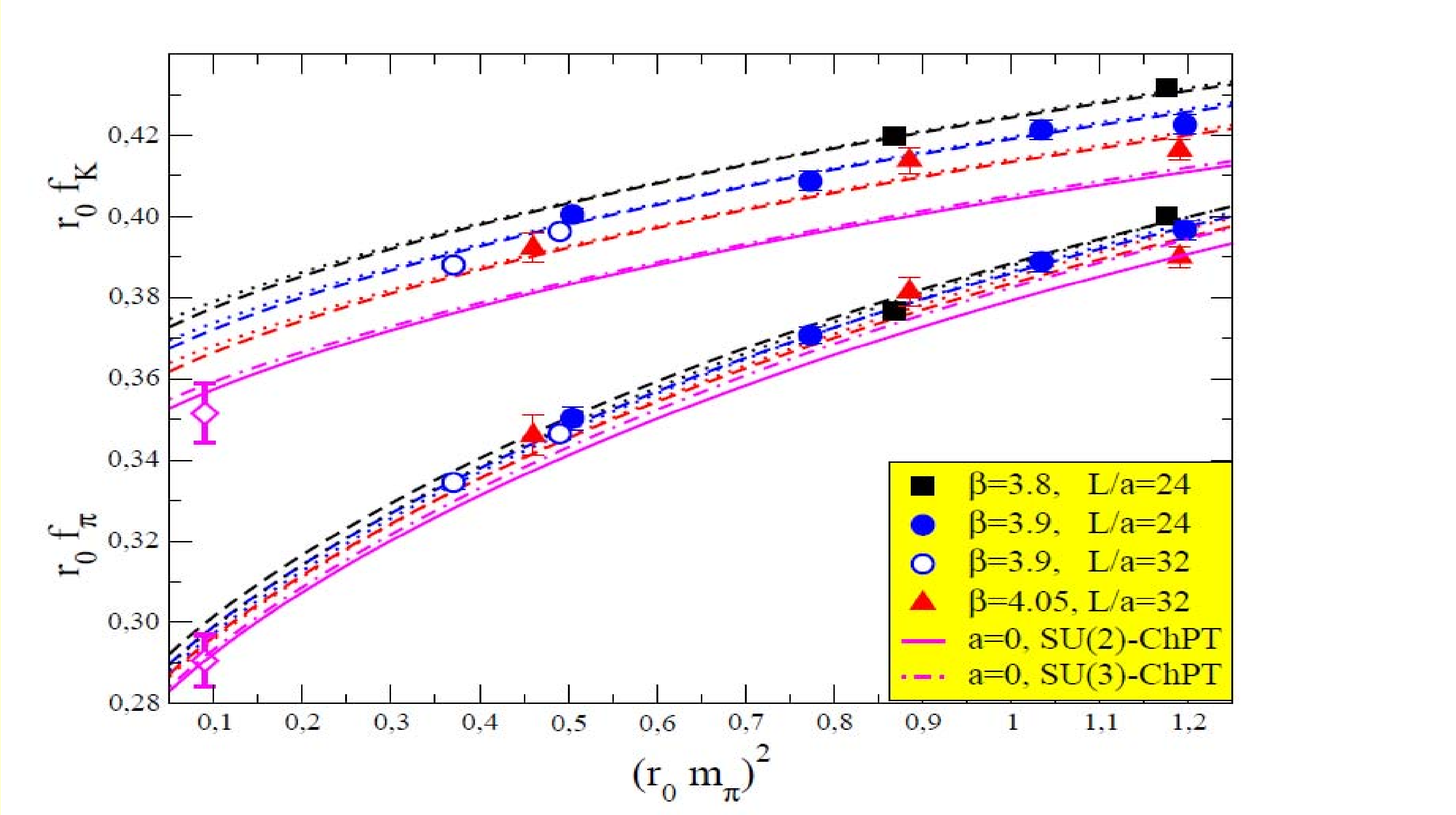
$$0.9 \cdot m_s^{phys} \le \mu_s \le 1.5 \cdot m_s^{phys}$$

a [fm]	V/a^4	$a\mu_{sea}$	m_π [MeV]	$m_\pi L$
0.10	$24^3 \times 48$	0.0080	410	5.0
		0.0110	480	5.8
0.085	$24^3 \times 48$	0.0040	315	3.3
		0.0064	390	4.0
		0.0085	450	4.7
		0.0100	490	5.0
0.085	$32^3 \times 64$	0.0030	270	3.7
		0.0040	310	4.3
0.065	$32^3 \times 64$	0.0030	310	3.3
		0.0060	430	4.6
		0.0080	500	5.3

Systematics Overview: finite L and a



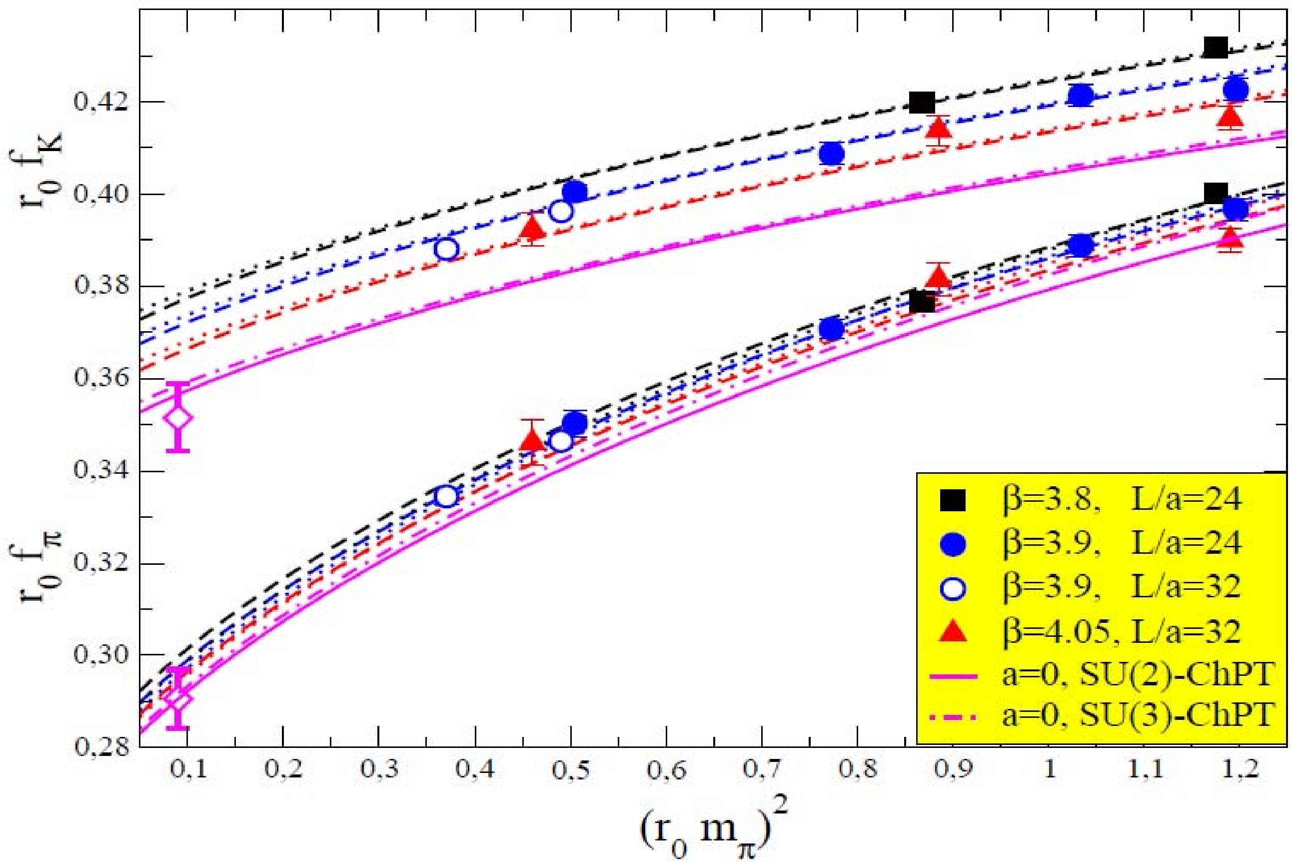
ETMC arXiV:0904.0954



- Combined fit of chiral dependence + a^2 and L corrections
  - *Chiral-fit:* **SU(2) ChPt for μ_l** + *Linear dep. of LECS in "strange quarks"*

$$f_{\rm PS}(\mu_l,\mu_l,\mu_s) = \left(f_0^{(K)} + f_m^{(K)}\,\xi_{ss}\right)\cdot\left[1 - \frac{3}{4}\xi_{ll}\ln\xi_{ll} + \left(b_0^{(K)} + b_m^{(K)}\,\xi_{ss}\right)\xi_{ll} + \left(A_a + A_{as}\,\xi_{ss}\right)\frac{a^2}{r_0^2}\right]\cdot K_{\rm FSE}(L)$$

where $\xi_{ij} = m_{\rm PS}^2(\mu_{\rm sea},\mu_i,\mu_j)/(4\pi f_0)^2$ in terms of meson masses



A fit based on NLO SU(3)-ChPT finds good compatibility

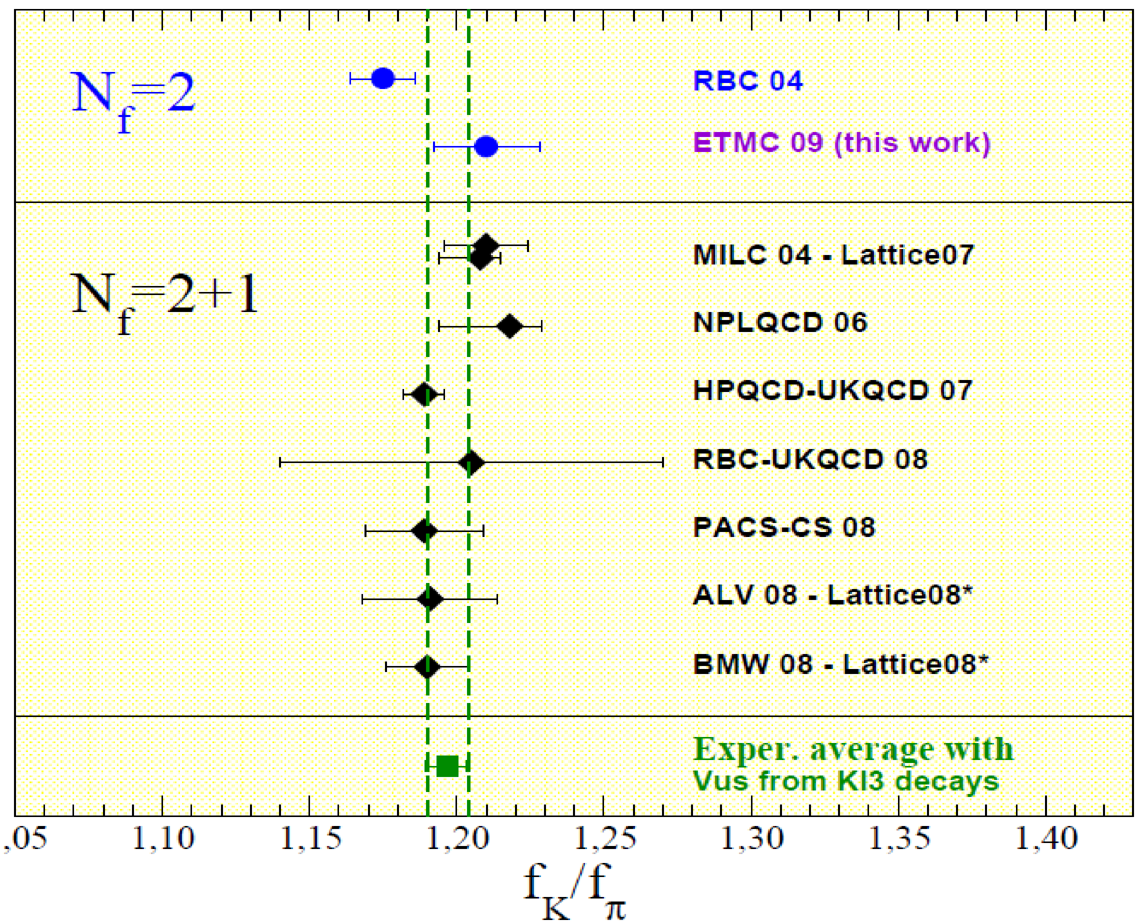
$$f_K/f_\pi = 1.210(5)_{\text{SU(3)}} \quad 1.210(5)_{\text{SU(2)}}$$

However SU(3)-ChPT looks less robust that SU(2)

Stat/chiral $\mathcal{O}(a^2)$ FSE

ETMC arXiV:0904.0954

$$f_K = 158.1(\boxed{0.8})(\boxed{2.0})(\boxed{1.1})\ \mathrm{MeV} \quad , \quad f_K/f_\pi = 1.210(\boxed{6})(\boxed{15})(\boxed{9}) \,,$$



<u>Warning from ETMC: Continuum Limit important!</u>

2007 → 2009

f_K/f_π: **1.227**(9)(24) → **1.210**(18)

Determination of $f_+(0)$

$$\Gamma(K_{\ell 3(\gamma)}) \propto |V_{us}|^2 \; f_+(0)^2$$

❖ **ETMC determination of $f_+(q^2)$ *at* $q^2=0$**

ETMC in preparation

☹ N_F=2 → the strange quark is quenched

😐 **one lattice spacing: a≈0.09fm, *other values in progress:***

➲ ***to estimate systematics we exploit* a≈0.07fm for M_π≈ 300 & 470 MeV**

😐 **$LM_\pi \geq 4$ to keep finite size effects negligible**

➘ **M_π≈ 260, 300, 375, 435, 470, 575 MeV**

L=2.9fm V=32³x64	L=2.2fm V=24³x48

(unitarity pions $\mu_l=\mu_{sea}$)

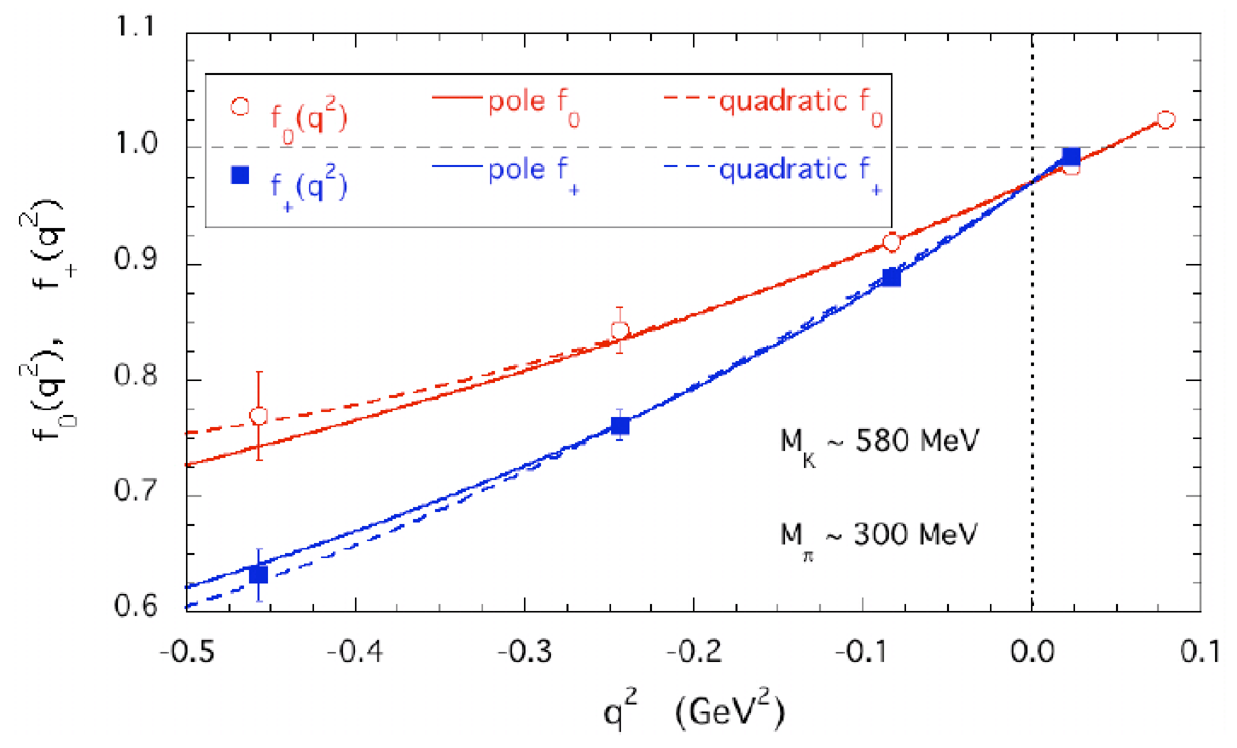
😐 *several q^2 in order to extract $f_{+,0}(q^2)$ at q^2=0*

$$\langle K|\bar{s}\gamma^\mu u|\pi\rangle = f_+(q^2)\left((p_K+p_\pi)^\mu - \frac{M_K^2-M_\pi^2}{q^2}q^\mu\right) + f_0(q^2)\frac{M_K^2-M_\pi^2}{q^2}q^\mu$$

$$f_+(0)=f_0(0)$$

1)Extracting $f_+(0)$ from $f_{0+}(q^2)$

($f_{0+}(q^2)$ by appropriate double-ratios, FNAL'02, SPQCD'04)



$f_+(0)=f_0(0)$

pole fit

$$f_{0,+}\left(q^2\right) = f_+(0)/\left(1 - s_{0,+}\, q^2\right)$$

quadratic fit

$$f_{0,+}\left(q^2\right) = f_+(0)\left(1 + s_{0,+}\, q^2 + c_{0,+} q^4\right)$$

Systematic uncertainty in the determination of $f_+(0)$

Chiral Extrapolation for $f_+(0)$

M_π (MeV)	M_K^{ref} (MeV)	$f_+(0)$ (pole)	$f_+(0)$ (quadratic)
260	520	0.97519 (499)	0.97374 (505)
300	530	0.98052 (440)	0.97950 (390)
375	555	0.98916 (264)	0.98813 (248)
435	575	0.99343 (130)	0.99273 (131)
470	590	0.99421 (79)	0.99413 (85)
575	635	0.99823 (15)	0.99827 (19)

SU(3)-CHPt

$$f_+(0) = 1 + (M_K^2 - M_\pi^2)^2$$

$$f_+(0) = 1 + f_2 + \Delta f$$

f_2=-0.023

(NO UNCERTAINTY!)

NO LECs

Δf on the lattice?

=>$f_+(0)$

SU(2) CHPT

Expansion for $M_\pi << M_K$

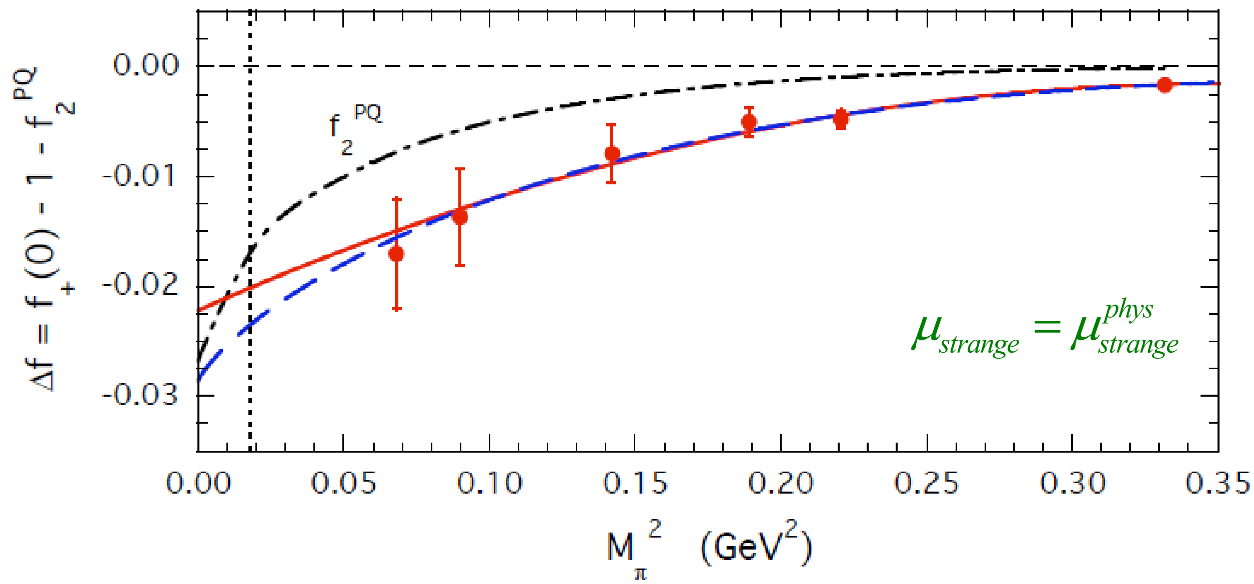
$$f_+(0) = F_+ - \frac{3}{4}\frac{M_\pi^2}{16\pi^2 f_\pi^2}\log\left(\frac{M_\pi^2}{\mu^2}\right) + c_+(\mu) M_\pi^2$$

F_+ and c_+ are LEC's depending on m_s

F_+ and C_+ on the lattice?

=> $f_+(0)$

Chiral extrapolation I: SU(3)-ChPT



$A_0 + A_1 M_\pi^2 + A_2 M_\pi^4$

$A_0 + A_1 M_\pi^2 + A_3 M_\pi^2 \log\left(M_\pi^2\right)$

* dominant chiral logs from f_2^{PQ}
* smooth extrapolation for Δf

$$\Delta f = -0.0233(61)(32) \Longleftrightarrow f_+^{SU(3)}(0) = 0.9599(61)(32)$$

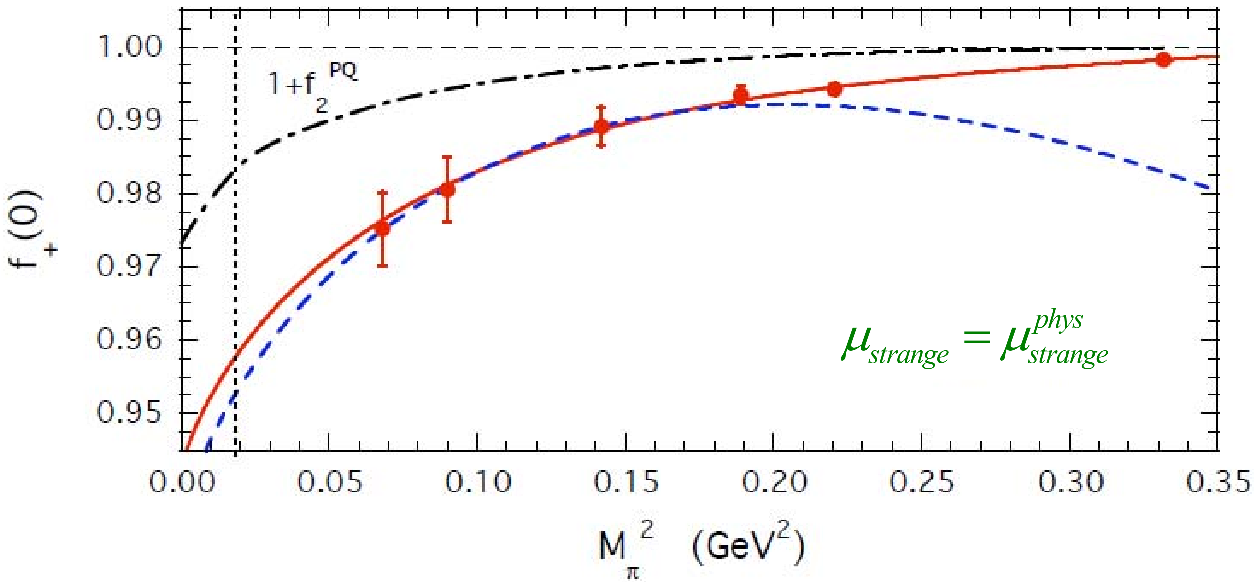
Stat. error

chiral & q^2 fit uncertainties

Chiral extrapolation II: SU(2)-ChPT

$$f_+(0) = F_+ - \frac{3}{4}\frac{M_\pi^2}{16\pi^2 f_\pi^2}\log\left(\frac{M_\pi^2}{\mu^2}\right) + c_+(\mu)M_\pi^2 + O\left(M_\pi^4\right)$$

F_+ and c_+ are LEC's depending on m_s



SU(2) NLO (lowest three points: M_π < 400 MeV

SU(2) NLO + quadratic term $\propto M_\pi^4$

$$f_+^{SU(2)}(0) = 0.9563(53)(13)$$

Stat. error

chiral & q^2 fit uncertainties

(Preliminary)

$$f_+^{SU(3)}(0) = 0.9599(61)(32)$$
$$f_+^{SU(2)}(0) = 0.9563(53)(13)$$

$$f_+(0) = 0.9581(57)(35)$$

❖Estimate of remnant systematics: L, a & m_s-quenched
(*awaiting for a complete systematics study*)

- **Finite size effects:** add (in quadrature) 0.0019 to sys..

  by comparing $M_\pi \approx$ 300 between L=24 & L=32

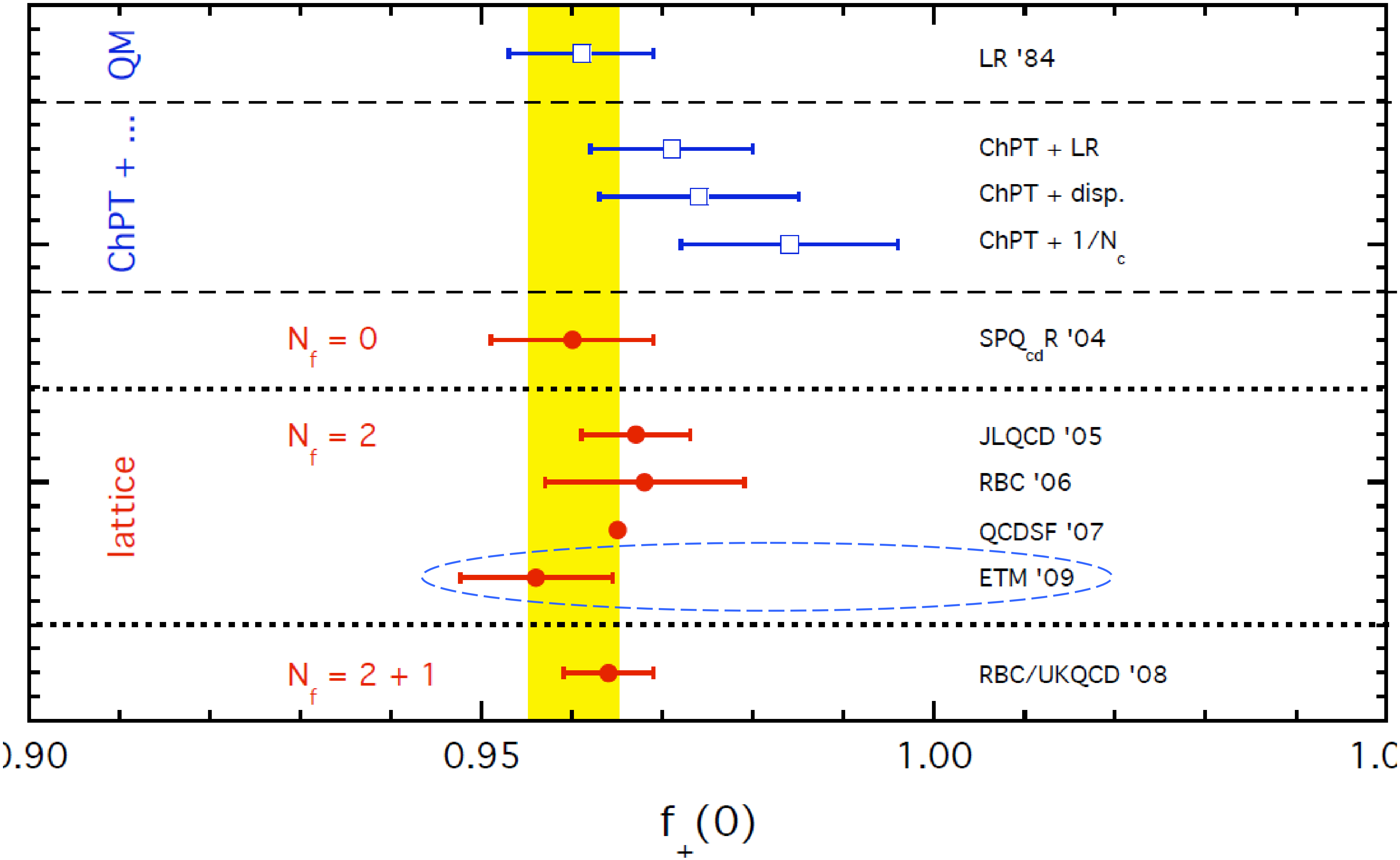
- **Finite lattice spacing:** add +0.0037 to c. value & sys.

  by comparing $M_\pi \approx$ 300 & 470 MeV between a=0.09 & 0.07 fm

- **Quenching m_s (NF=2 vs NF=2+1):** -0.0058 to c. value, 0.0029 to sys.

  $f_2^{NF=2+1}$-$f_2^{NF=2}$ known in SU(3)-CHPT

$$f_+(0) = 0.9560(57)_{stat}(62)_{syst} = 0.9560 \pm 0.0084$$



$$f_+(0) = 0.9560(57)_{stat}(62)_{syst} = 0.9560 \pm 0.0084$$

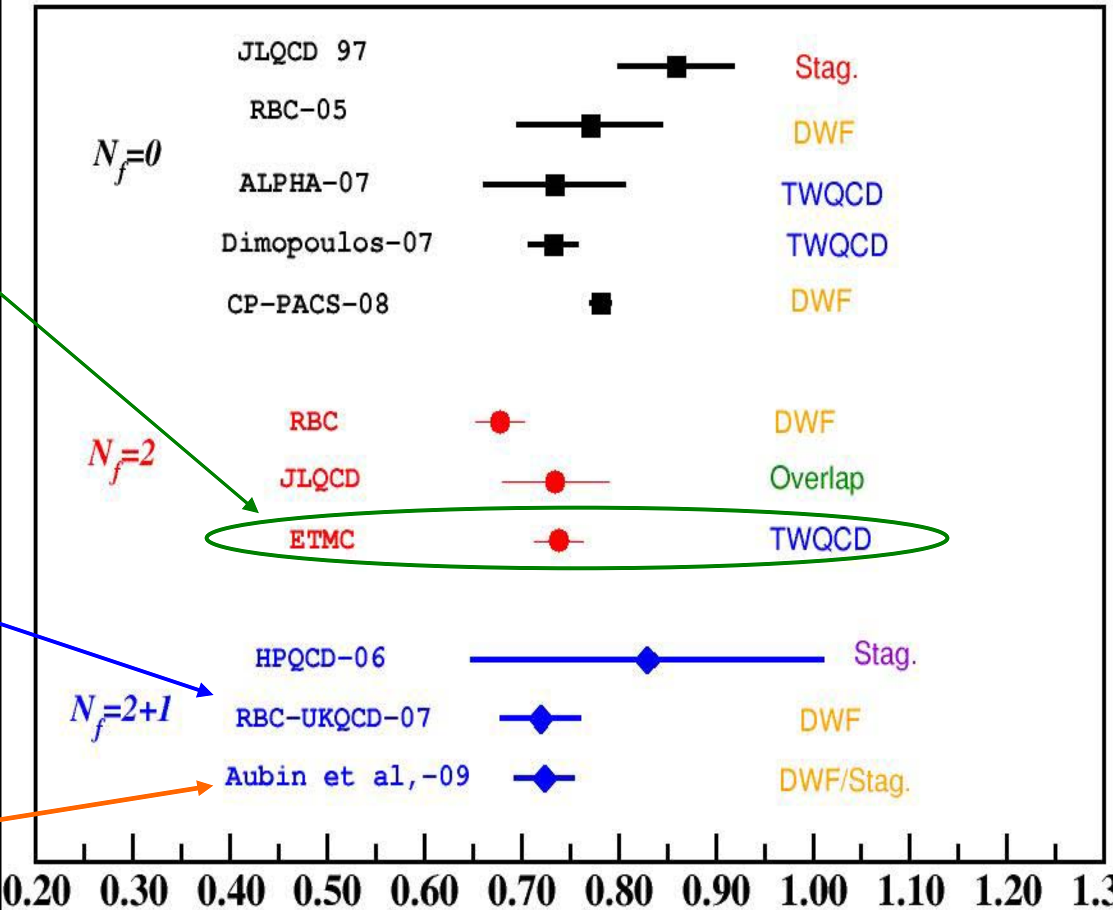
(Preliminary)

B_K from N_F=2 tmQCD: (Preliminary)

ETMC: N_F = 2: ☹ **TmQCD**
☺ continuum limit
=> a^{-1}=>1.97, 2.19,2.81 Gev,
L=2.2/2.9 fm, $m_\pi \geq$260 MeV
☺ sea q. masses dep.
☹ Quenching the strange

RBC: N_F=2+1: ☺ DWF
☹ no continuum limit
⇒ a^{-1}=1.73 Gev,
L=2.74 fm, m_π>=330MeV
☹ 2 sea q. masses!!!

Aubin et al. 0905.3947
☹ **Staggered**/DWF
☺ continuum limit



JLQCD: N_F = 2: **Overlap** ☺
☹ no continuum limit
=> a^{-1}=1.67 Gev,
L=1.9 fm, m_π>=290

$$B_K^{RGI} = 0.739(25)(?)$$

(Very Preliminary)

European Twisted Mass Collaboration

Determination of B_K

$$\langle \overline{K} | \bar{s}\gamma_L^\mu d \bar{s}\gamma_L^\mu d | K \rangle = \frac{8}{3}\left(f_K m_K\right)^2 B_K$$

- **ETMC determination of B_K**

☹ ***Notoriously, Wilson fermions are problematic for B_K ;***

➘**Wilson Fermions**→***no chiral symmetry→ mixing with dim-6 operators***

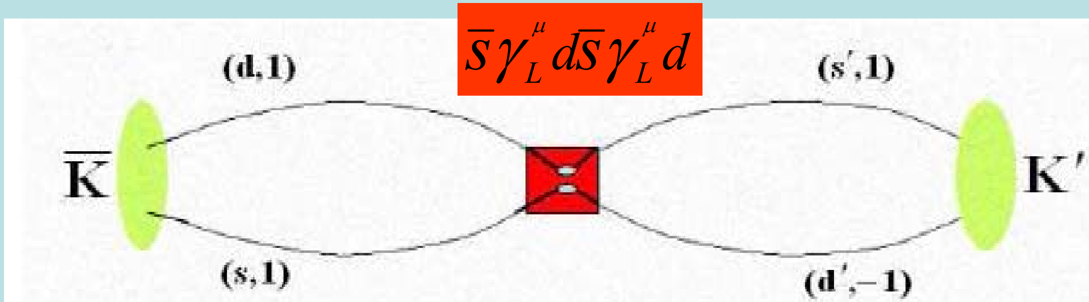
***Way Out:* combining Wilson fermions with different r-parameters**

Frezzotti & Rossi hep-lat/0407002

$$\bar{s}\gamma_L^\mu d \bar{s}\gamma_L^\mu d$$

$\phi_K = \bar{d}\gamma_5 s$

$r_d = r_s = 1$



$\phi_{K'} = \bar{d}'\gamma_5 s'$

$-r_{d'} = r_{s'} = 1$

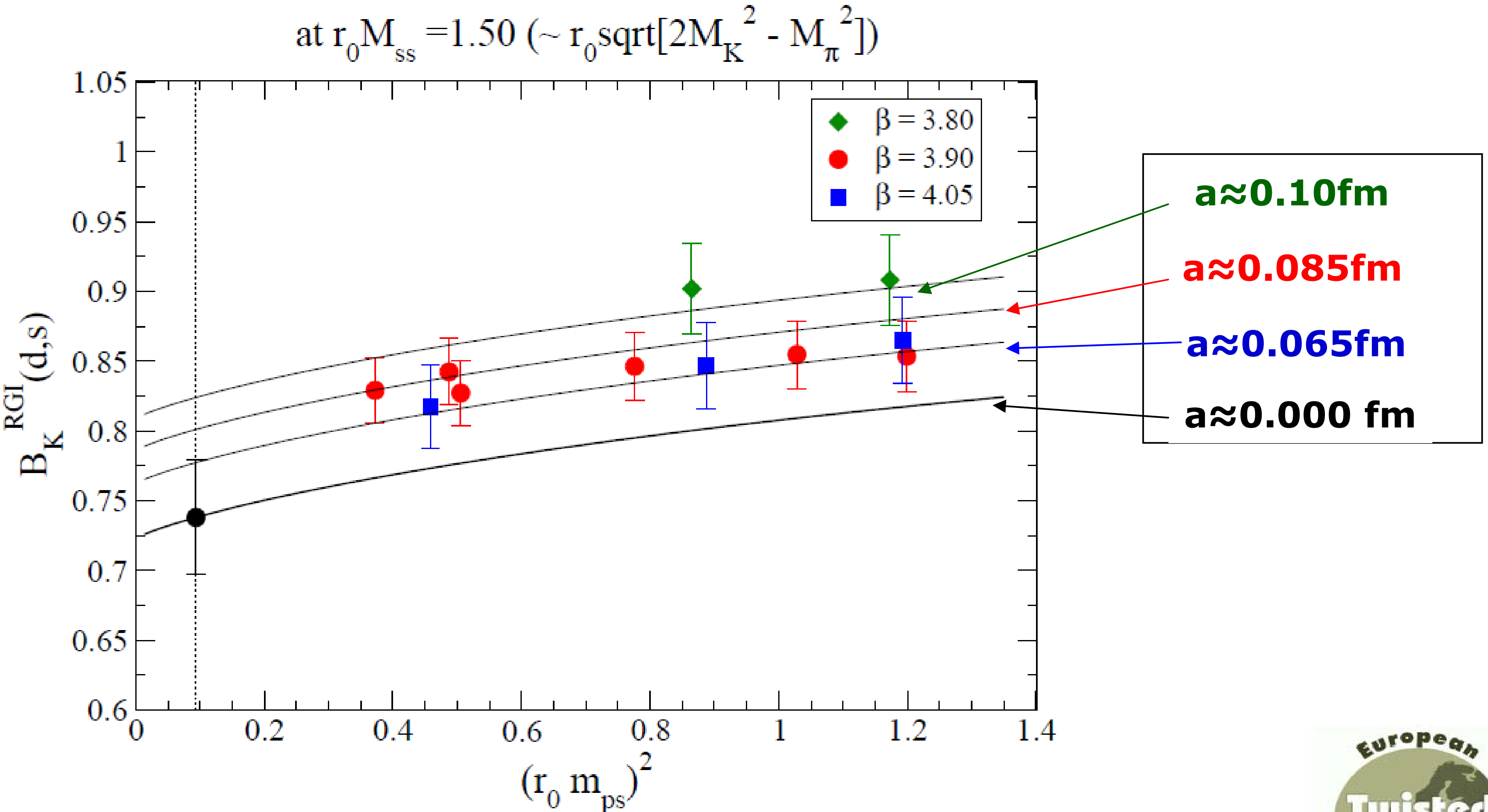
➲ ***Multiplicative renormalization as in the continuum***

➲ ***$\mathcal{O}(a^2)$ Cut-off effects***

► *Chiral-fit Setup:* **$SU(2)_L$ x $SU(2)_R$ fit (μ_l) + static strange quarks (μ_h)**

Unitarity case

$$B_K^{bare}(\mu_h, \mu_v = \mu_{sea}) = B_\chi^{bare}(\mu_h)\left[1 + \left(P_v(\mu_h) + P_{sea}(\mu_h)\right)\mu_{sea} - \frac{2B_0}{32\pi^2 f_0^2}\mu_{sea}\log\mu_{sea}\right]$$

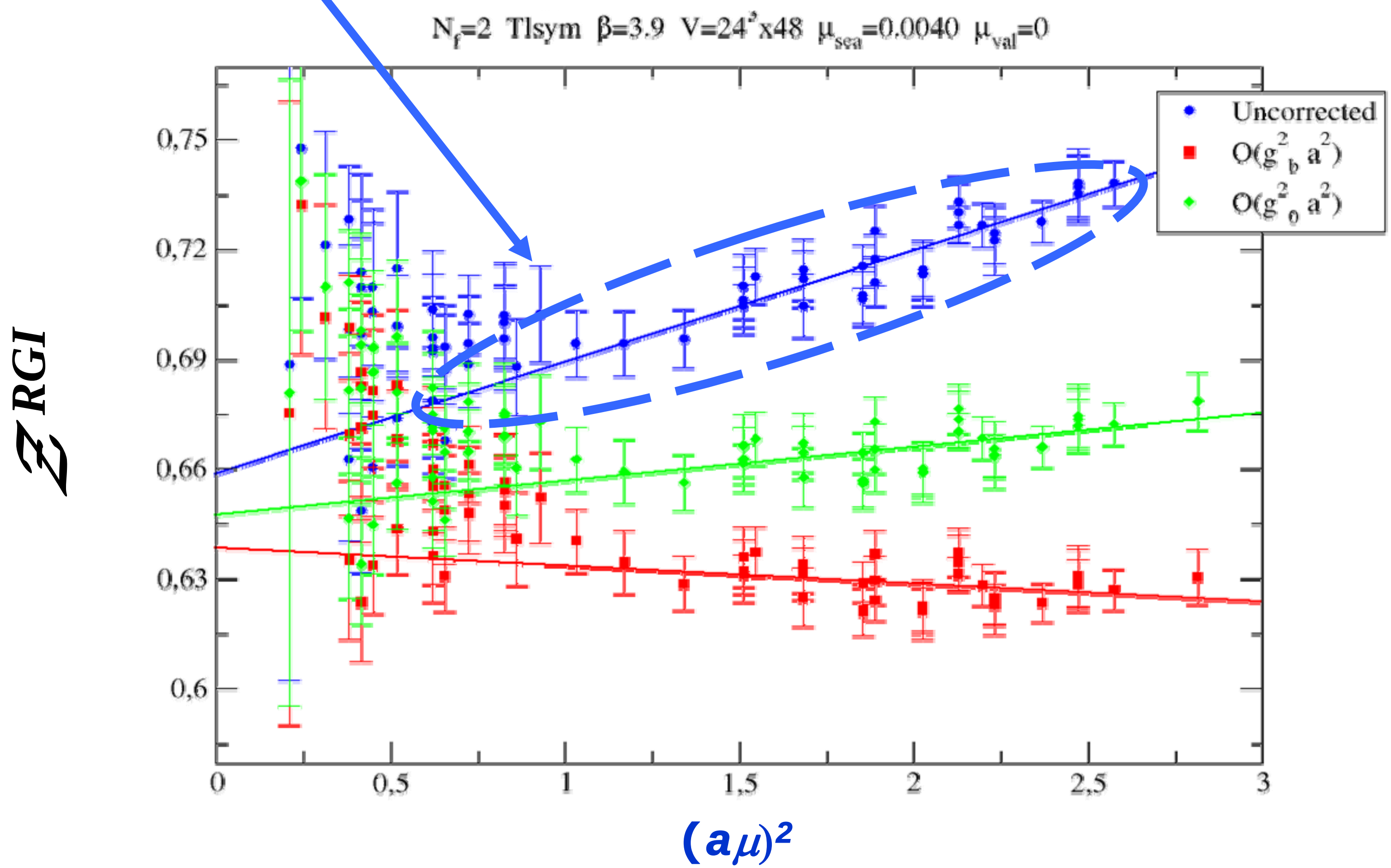


$B_K^{RGI} = 0.739(25)$ (Very Preliminary)

▶ *Renormalization Constants in* **RI-MOM scheme**: **PRELIMINARY**

$$B_K(\text{ren}) = \mathcal{Z}\, B_K(\text{bare})$$

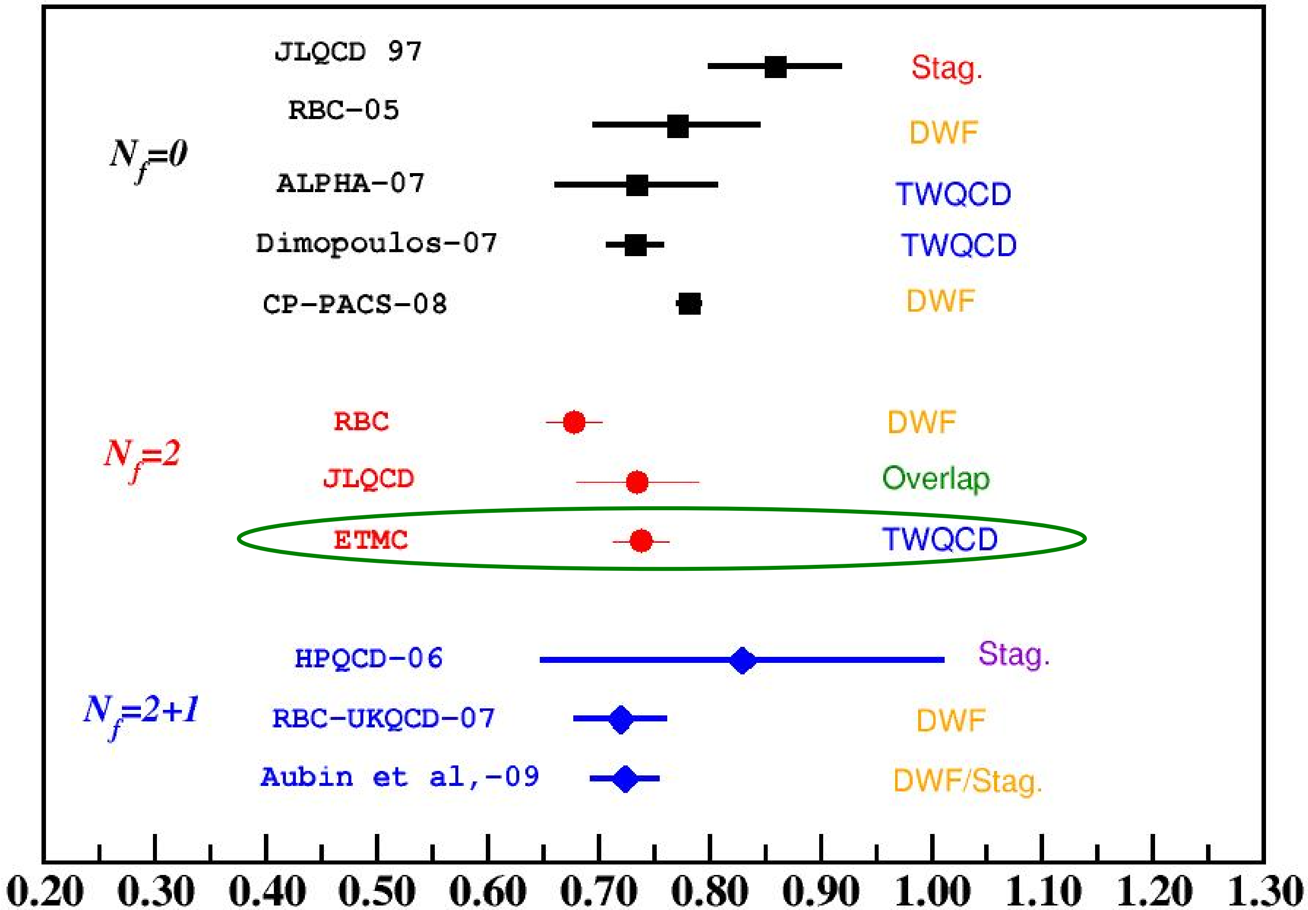
❖ ***However, $\mathcal{O}(a^2g^2)$ looks sizable***



Substraction of $\mathcal{O}(a^2g^2)$ by explicit calculation

H. Papadopoulos et al. '09

B_K^{RGI}



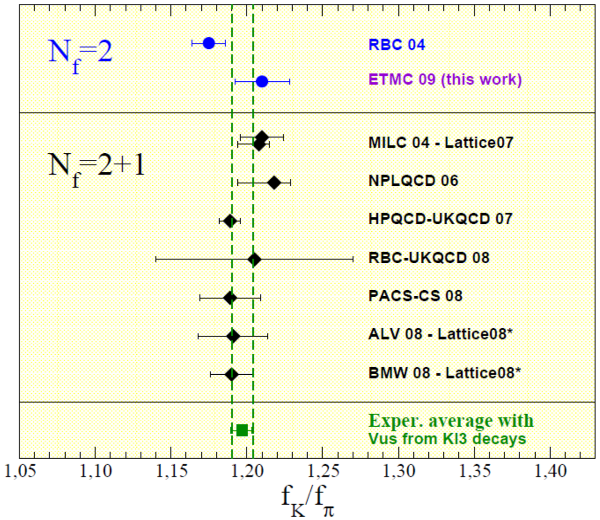
☹ ***N_F = 2:*** ☺ continuum limit
☺ sea q. masses dep.
☺*NP*-renormalisation
☹ **Quenching the strange**

$$B_K^{RGI} = 0.739(25)$$

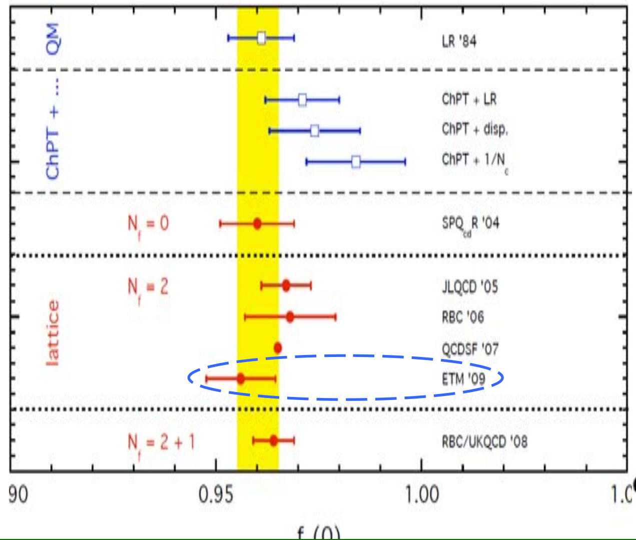
(Very Preliminary)

Summary of Kaon Physics from ETMC

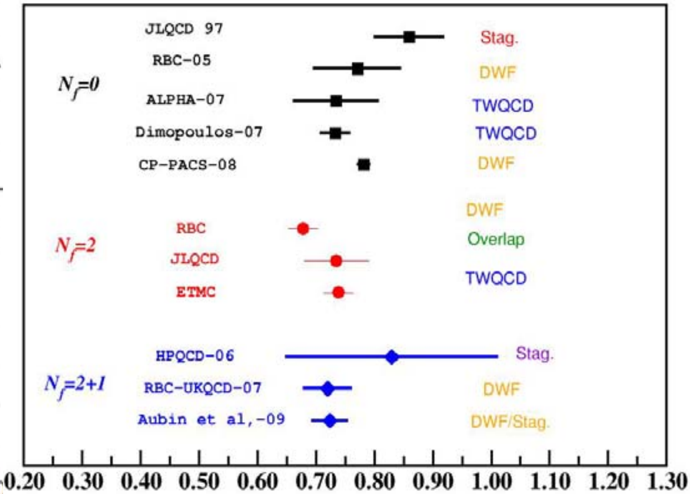
$f_K(0)/f_\pi(0)$



$f_+(0)$

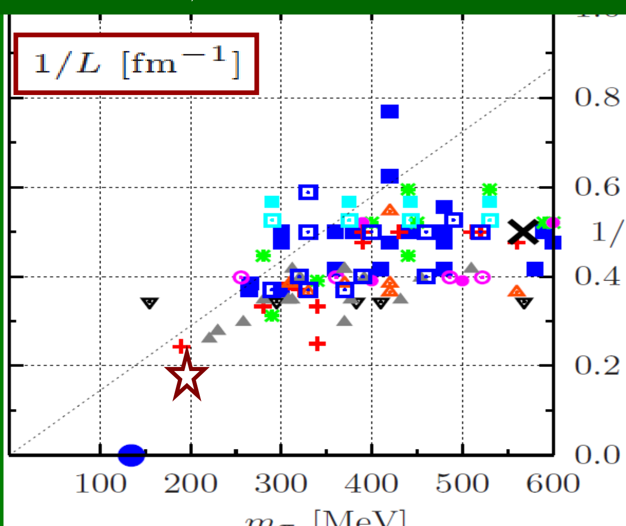


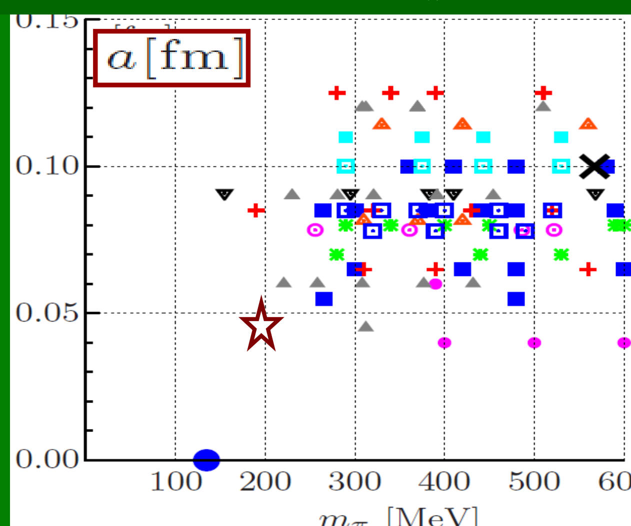
B_K^{RGI}

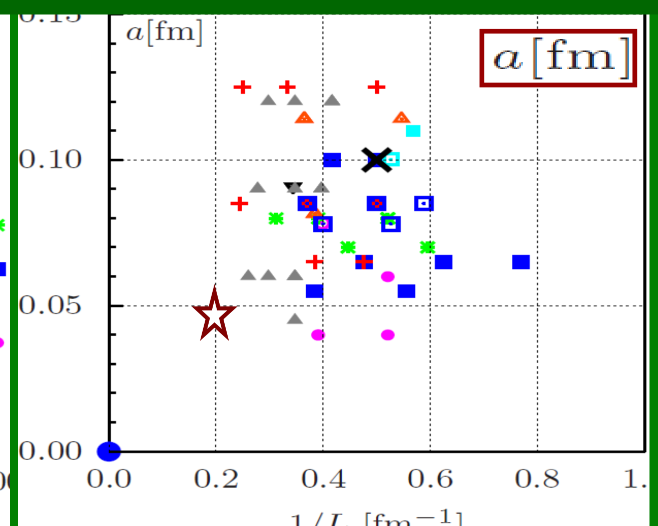


Recent developments will allow soon high precision Kaon physics

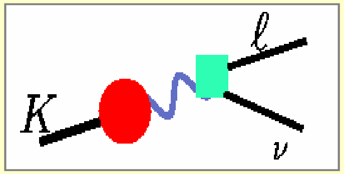
$a \sim 0.05$ *fm*, $L \geq 4.5$ *fm* and $200 \leq m_\pi \leq 300$ MeV is not far away







Determination of f_K/f_π



❖ **ETMC determination of f_K/f_π**

ETMC arXiV:0904.0954

☹ N_F=2 → the strange quark is quenched,

☺ **3 values of the lattice spacing: a≈0.10fm, a≈0.085fm, a≈0.065fm** ⇒ **Continuum Limit**

Moreover for Twisted Mass fermions (at maximal twist)

Advantages:

- ***Pseudoscalar meson masses and decay constants are automatically $\mathcal{O}$(a)-improved***
- **Axial currents do not require renormalization constant:**

Chiral Regime of QCD - summary

- SU(2) chiral perturbation theory

$$m_\pi^2 = M^2 \left[1 + \frac{M^2}{32\pi^2 F^2} \log\left(M^2/\Lambda_3^2\right) + \dots\right],$$

$$f_\pi = F - \frac{M^2}{16\pi^2 F^2} \log\left(M^2/\Lambda_4^2\right) + \dots$$

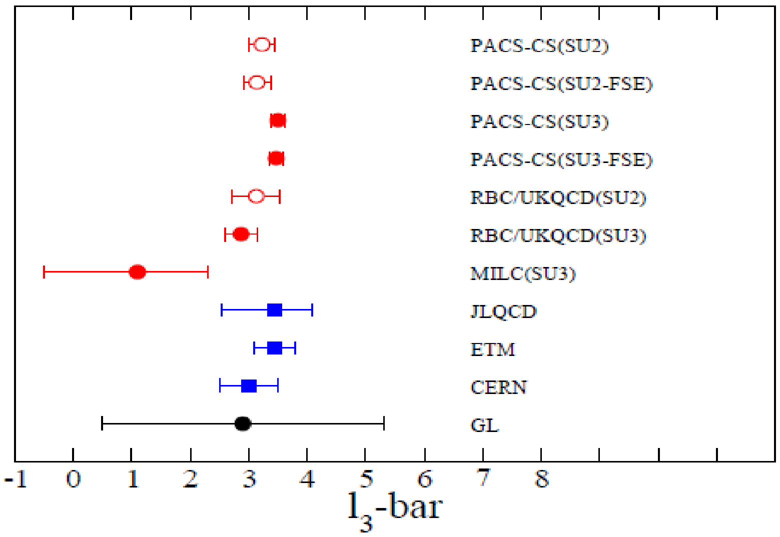
$$\bar{l}_3 = \log\frac{\Lambda_3^2}{m_\pi^2} = 2.9 \pm 2.4, \qquad \bar{l}_4 = \log\frac{\Lambda_4^2}{m_\pi^2} = 4.4 \pm 0.2.$$

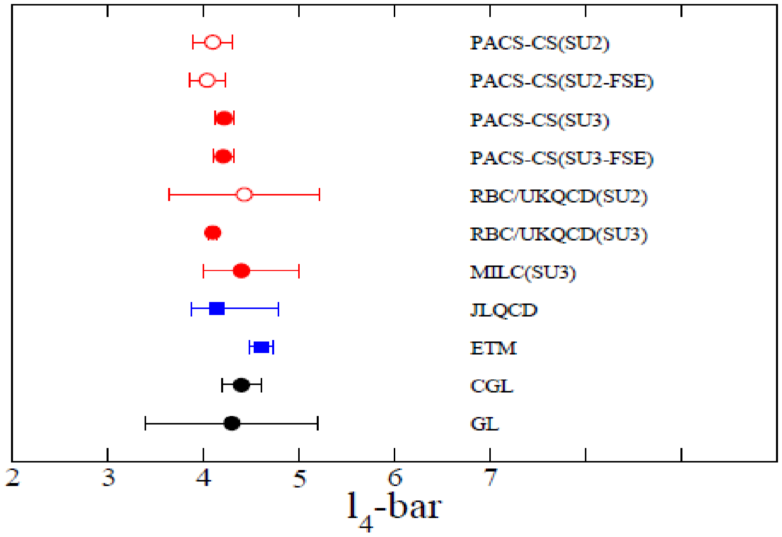
Colangelo, Gasser & Leutwyler '01

- First hints of chiral Logs by different approaches and groups:

  ➲ great success from Lattice approaches based on first principle;
  Wilson-like, Overlap and Domain-Wall fermions

  ➲ information qualitatively deeper than using Staggered fermions;
  no clear clue of chiral logs because of fits with ~40 parameters



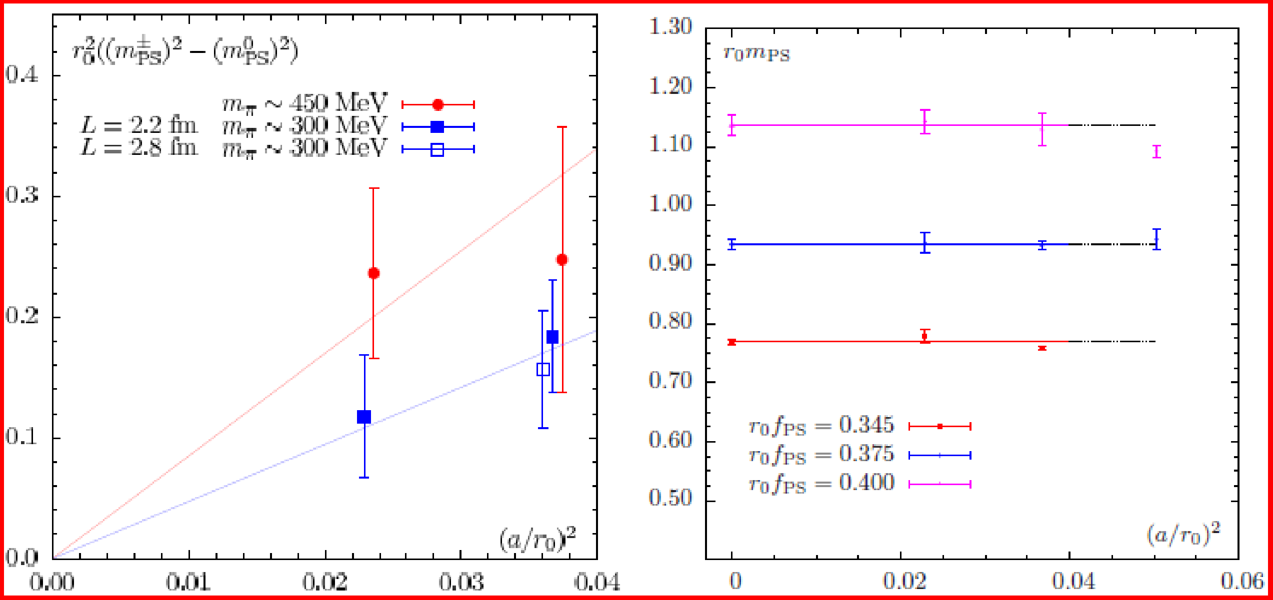


➲ a slight lower value of l_3-bar with staggered fermions (MILC)!!!

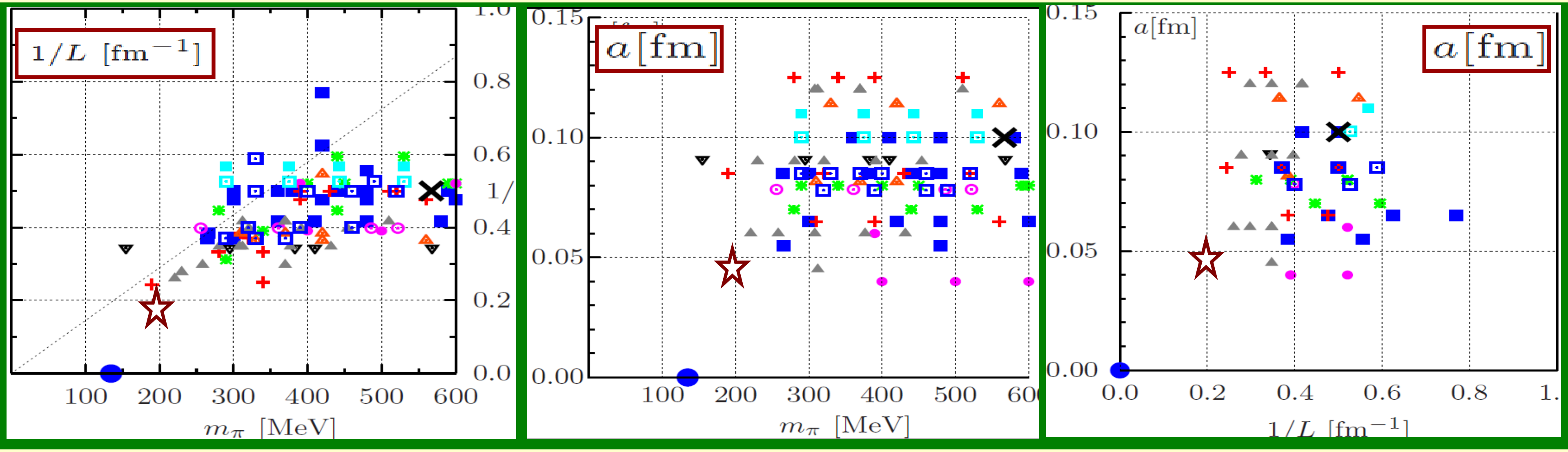
Twisted Mass QCD Fermions: isospin breaking

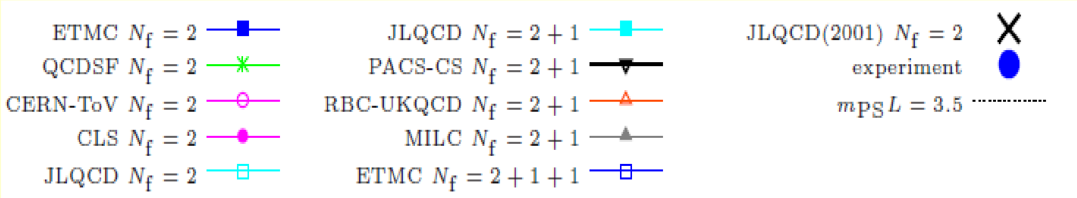
	β	$a\mu_q$	R_O
$af_{\rm PS}$	3.90	0.004	0.04(06)
	4.05	0.003	−0.03(06)
$am_{\rm V}$	3.90	0.004	0.02(07)
	4.05	0.003	−0.10(11)
$af_{\rm V}$	3.90	0.004	−0.07(18)
	4.05	0.003	−0.31(29)
am_{Δ}	3.90	0.004	0.022(29)
	4.05	0.003	−0.004(45)

$R_O = (O^{\pm} - O^0)/O^{\pm}$, measuring the isospin breaking effects in twisted mass lattice QCD.



- a, L & m_π Systematics at 1%. - How far are we?





Plots by G. Herdoiza

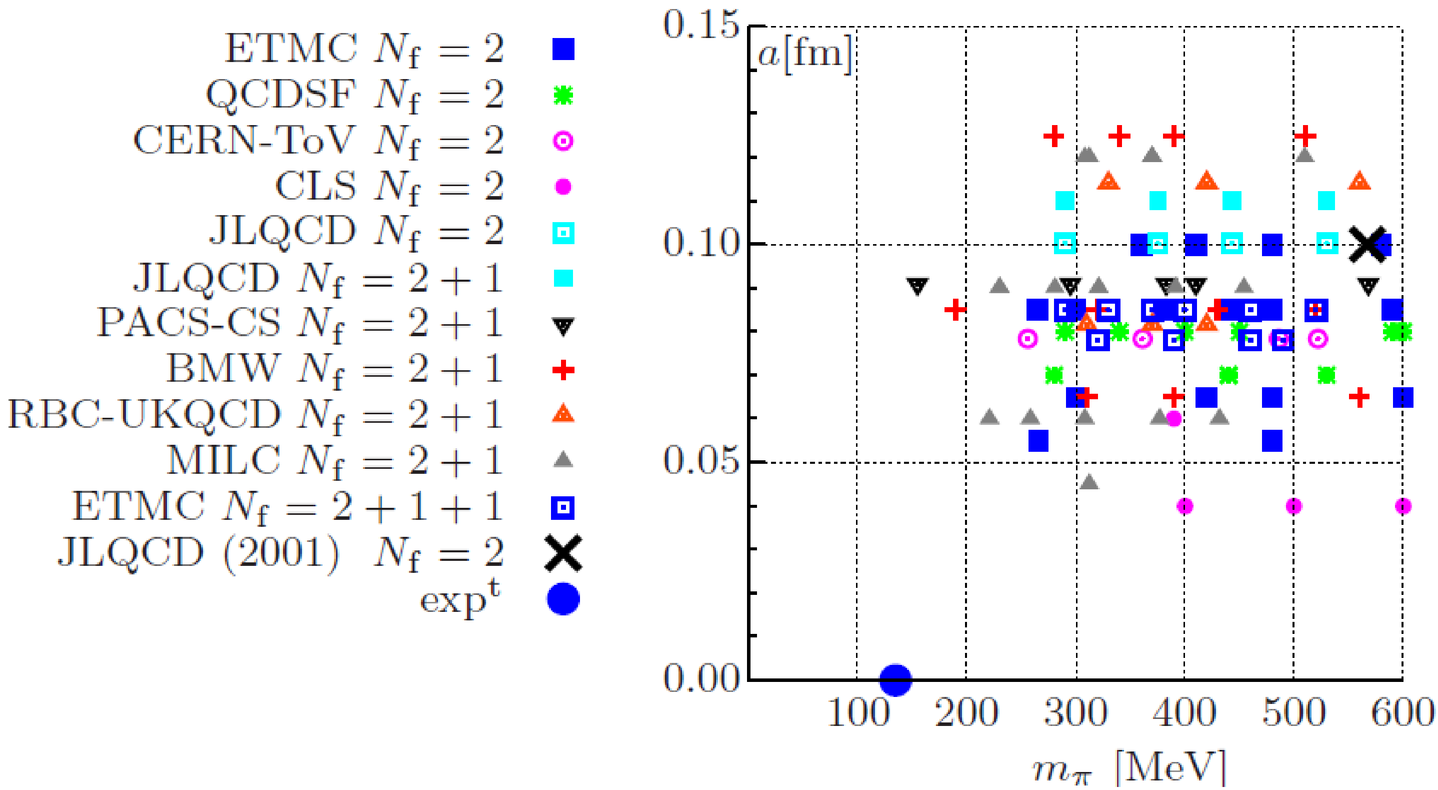
$$\star\ Q_{lat} = Q_{phys} \times (1+\delta_a)\,(1+\delta_L)\,(1+\delta_m)$$

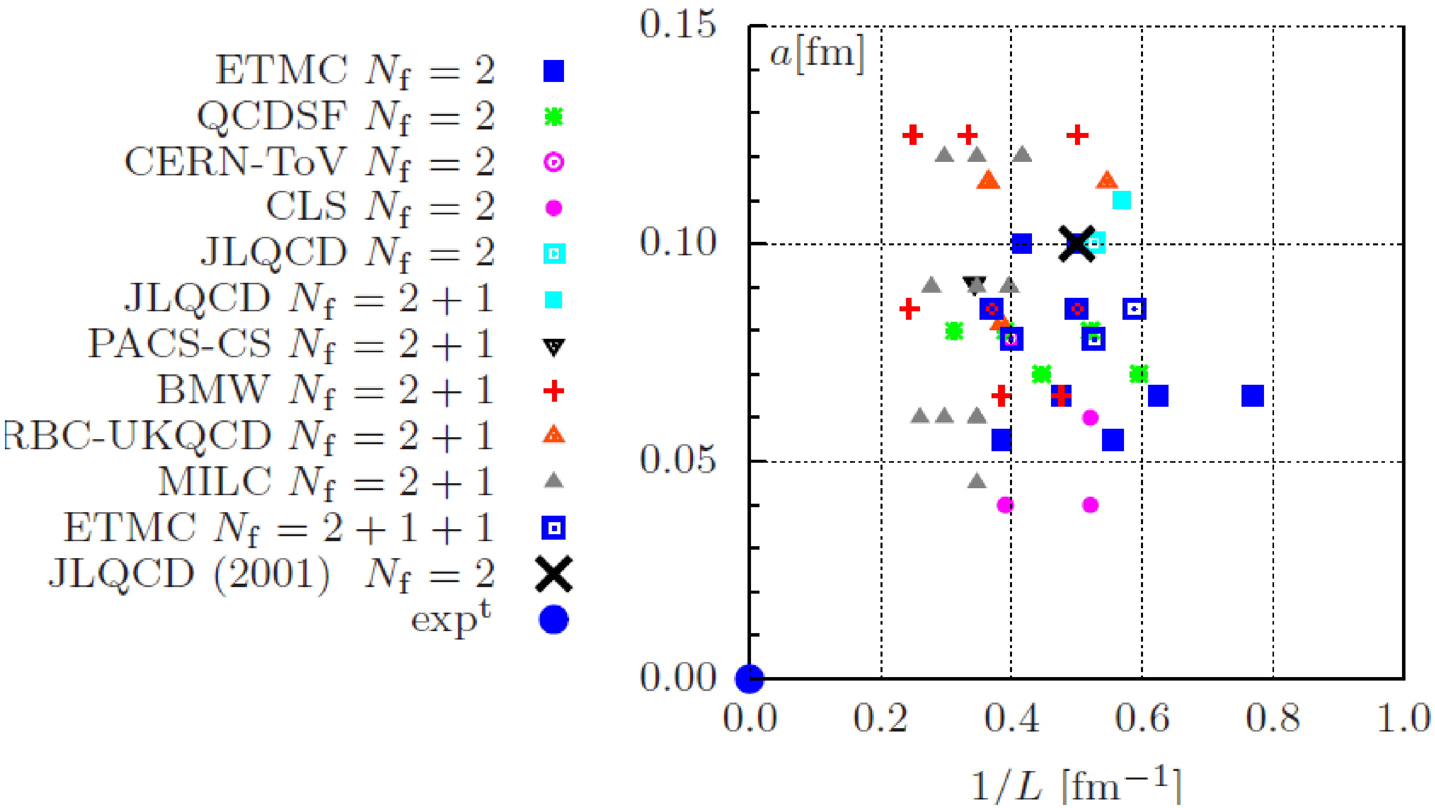
$f_K/f_\pi - 1$

$f_+(0)-1$, B_K

sys. at 1 % ? How far are we?

1. ***discretization errors:*** => $a \leq$ **0.05 fm** (4 GeV), ($a \geq 0.07$ fm)
2. ***finite size effects:*** $L * m_\pi \geq 4.0$ => $L \geq$ **4.5 fm** ($L \leq 3$ fm)
3. ***chiral regime:*** => **200 ≤ m_π ≤ 300 MeV**


ETMC $N_f = 2$
QCDSF $N_f = 2$
CERN-ToV $N_f = 2$
CLS $N_f = 2$
JLQCD $N_f = 2$
JLQCD $N_f = 2+1$
PACS-CS $N_f = 2+1$
BMW $N_f = 2+1$
RBC-UKQCD $N_f = 2+1$
MILC $N_f = 2+1$
ETMC $N_f = 2+1+1$
JLQCD (2001) $N_f = 2$
exp^t
a[fm]
0.15
0.10
0.05
0.00
100
200
300
400
500
600
m_π [MeV]



ETMC $N_f = 2$
QCDSF $N_f = 2$
CERN-ToV $N_f = 2$
CLS $N_f = 2$
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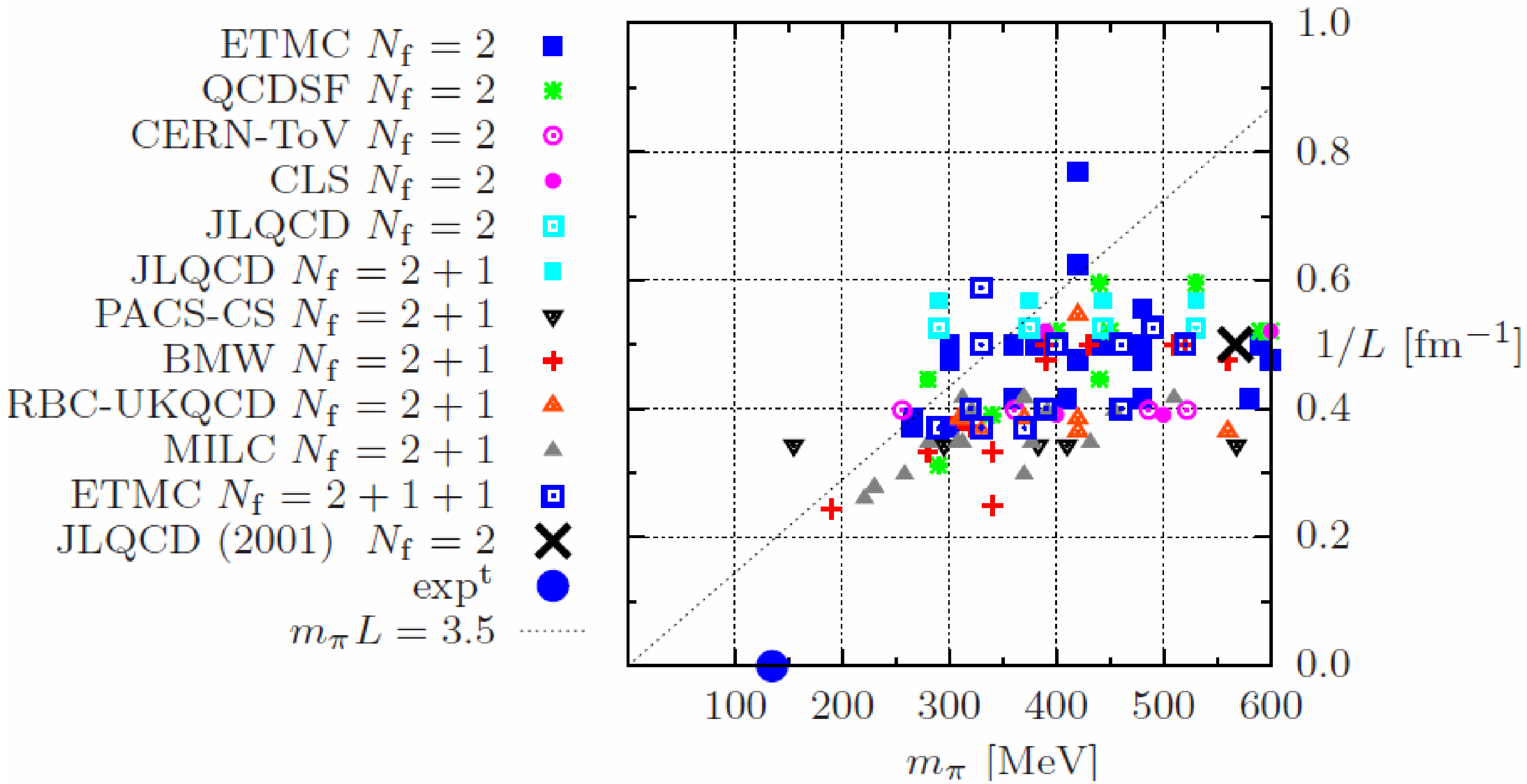
BMW $N_f = 2+1$
RBC-UKQCD $N_f = 2+1$
MILC $N_f = 2+1$
ETMC $N_f = 2+1+1$
JLQCD (2001) $N_f = 2$
expt
a[fm]
0.15
0.10
0.05
0.00
0.0
0.2
0.4
0.6
0.8
1.0
$1/L$ [fm^{-1}]


ETMC $N_f = 2$
QCDSF $N_f = 2$
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CLS $N_f = 2$
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BMW $N_f = 2+1$
RBC-UKQCD $N_f = 2+1$
MILC $N_f = 2+1$
ETMC $N_f = 2+1+1$
JLQCD (2001) $N_f = 2$
exp[t]
$m_\pi L = 3.5$
1.0
0.8
0.6
0.4
0.2
0.0
$1/L$ [fm^{-1}]
100
200
300
400
500
600
m_π [MeV]