Probing non perturbative QCD with the cusp effect in kaon decays by NA48

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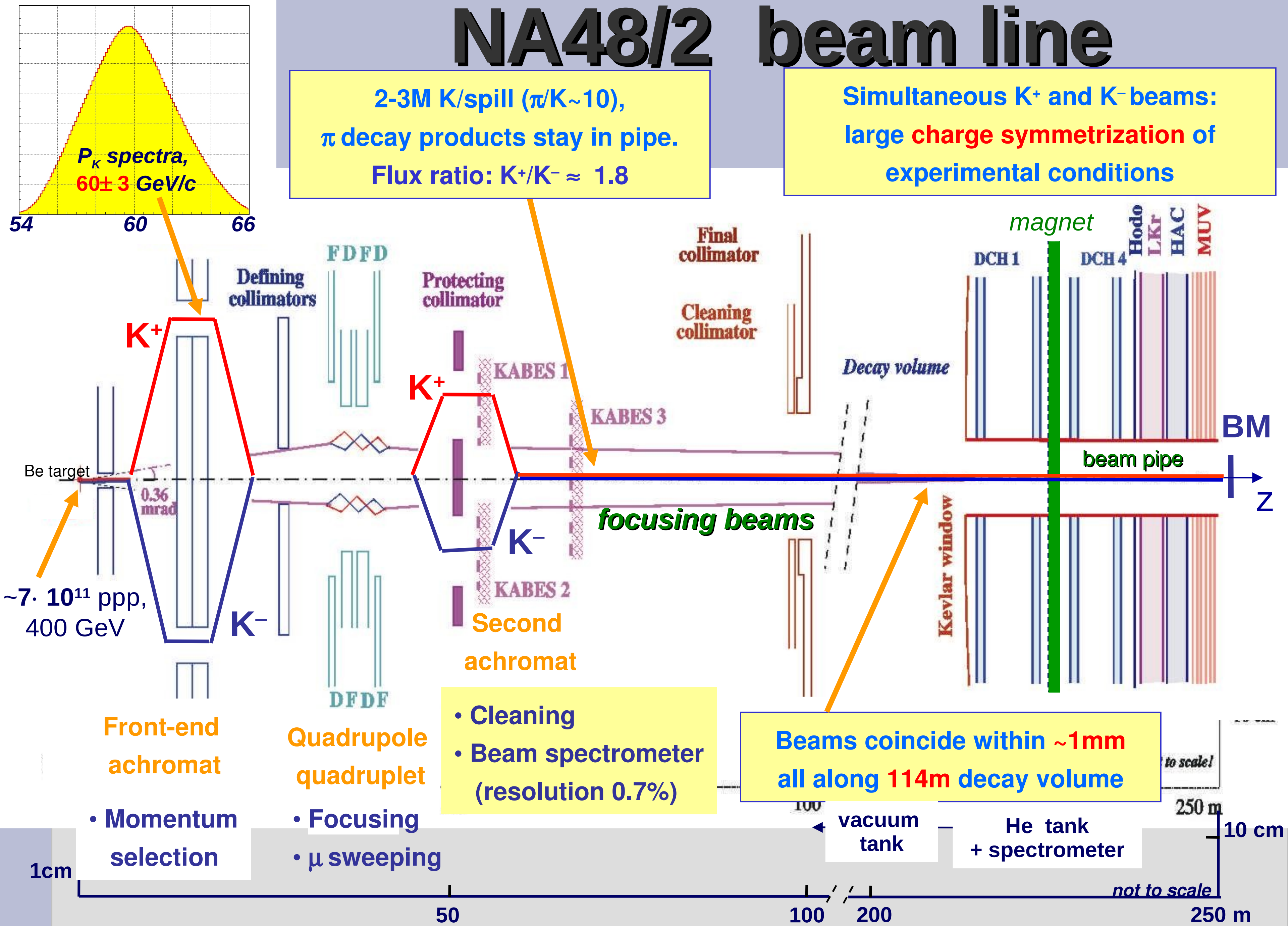
on behalf of the NA48/2 Collaboration:

Cambridge, CERN, Chicago, Dubna, Edinburgh, Ferrara, Firenze, Mainz, Northwestern, Perugia, Pisa, Saclay, Siegen, Torino, Vienna

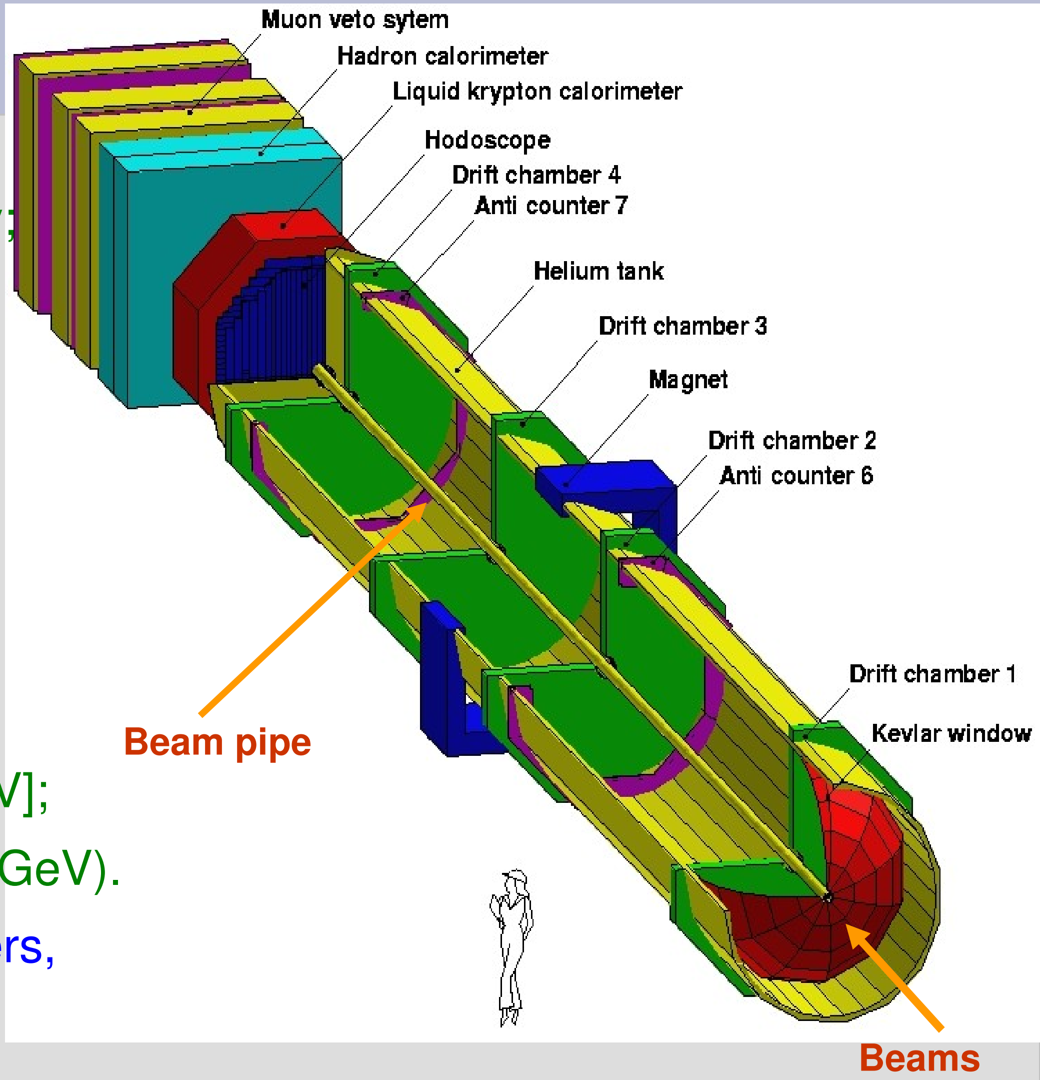
NA48/2 beam line



The NA48 detector

Main detector components:

- Magnetic spectrometer (4 DCHs):
  4 views/DCH: redundancy ⇒efficiency;
  used in trigger logic;
  $\Delta p/p = 1.0\% + 0.044\% * p$ [GeV/c].
- Hodoscope
  fast trigger;
  precise time measurement (150ps).
- Liquid Krypton EM calorimeter (LKr)
  High granularity, quasi-homogenious;
  $\sigma_E/E = 3.2\%/E^{1/2} + 9\%/E + 0.42\%$ [GeV];
  $\sigma_x=\sigma_y=0.42/E^{1/2} + 0.6$mm (1.5mm@10GeV).
- Hadron calorimeter, muon veto counters,
  photon vetoes.



NA48/2 data:

2003 run: ~ 50 days

2004 run: ~ 60 days

A view of the NA48/2 beam line

Total statistics in 2 years:

$K^{\pm} \rightarrow \pi^{-}\pi^{+}\pi^{\pm}$: ~$4 \cdot 10^{9}$

$K^{\pm} \rightarrow \pi^{0}\pi^{0}\pi^{\pm}$: ~$1 \cdot 10^{8}$

Rare $K^{\pm}$ decays:

BR's down to 10^{-9}

can be measured

>200 TB of data recorded

Pion scattering lengths

The important free parameter of ChPT is the quark condensate <qq>, it determines the relative size of mass and momentum terms in the power expansion.

a_0 and a_2 are S-wave $\pi\pi$ scattering lengths in isospin states I=0 and I=2, correspondingly. They enter into all $\pi\pi$ scattering amplitudes.

The relation between <qq> and the scattering lengths a_0 and a_2 is known from this theory with a high precision, so the experimental measurement of a_0 and a_2 provides an important constraints for ChPT Lagrangian parameters.

Pion scattering lengths can be measured in the

study of the cusp-effect in $K^{\pm}\to\pi^{\pm}\pi^{0}\pi^{0}$ decays

[2003 result: PLB 633 (2006) 173, Cabibbo-Isidori theoretical framework]

$K^{\pm}\to\pi^{\pm}\pi^0\pi^0$ selection

Assume $\pi^0\to\gamma\gamma$ for each $\gamma\gamma$:

$$m_0^2 = 2E_iE_k(1-\cos\alpha) \approx E_iE_k\alpha^2 = E_iE_k \frac{(D_{ik})^2}{(z_{ik})^2}$$

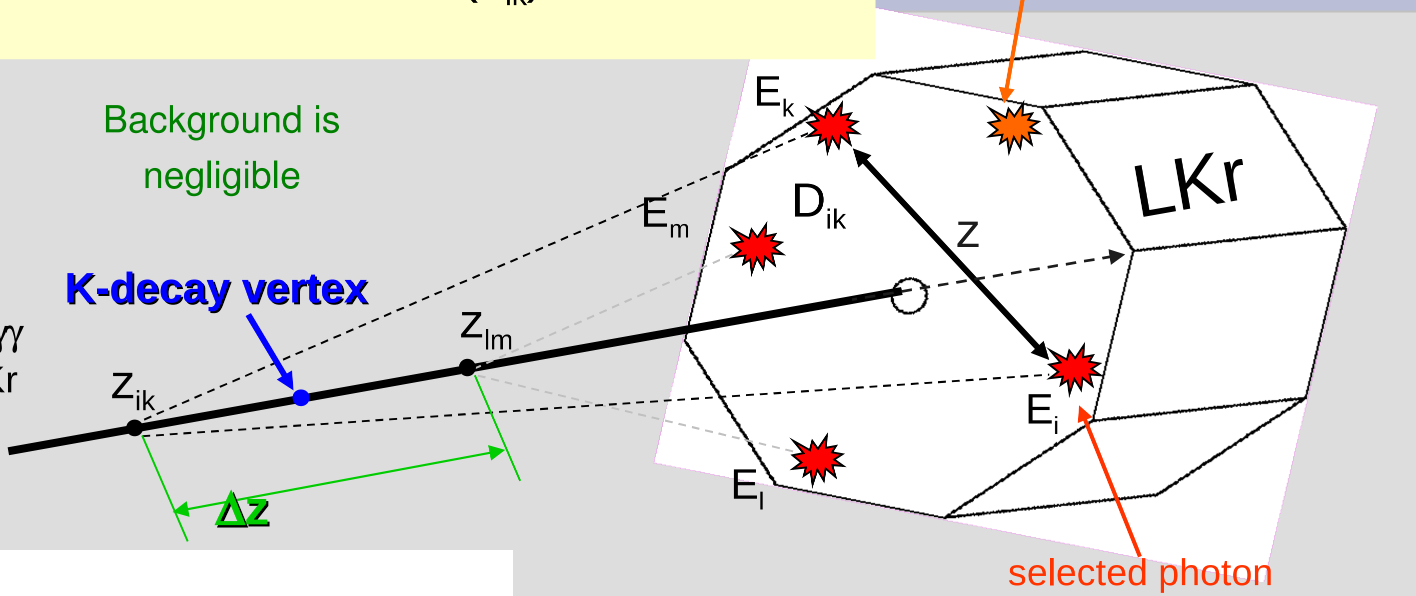
m_0^2 – mass of π^0

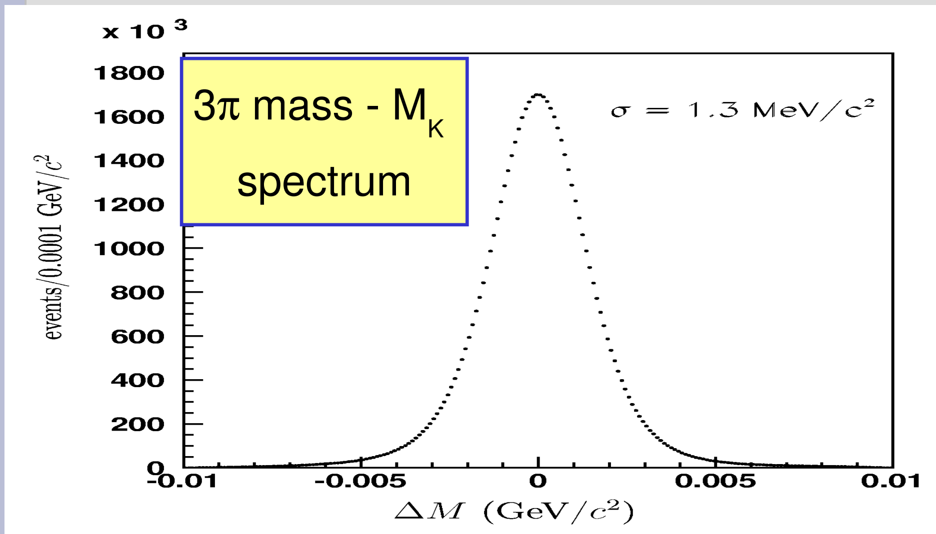
E_i, E_k – energy of γ_i, γ_k

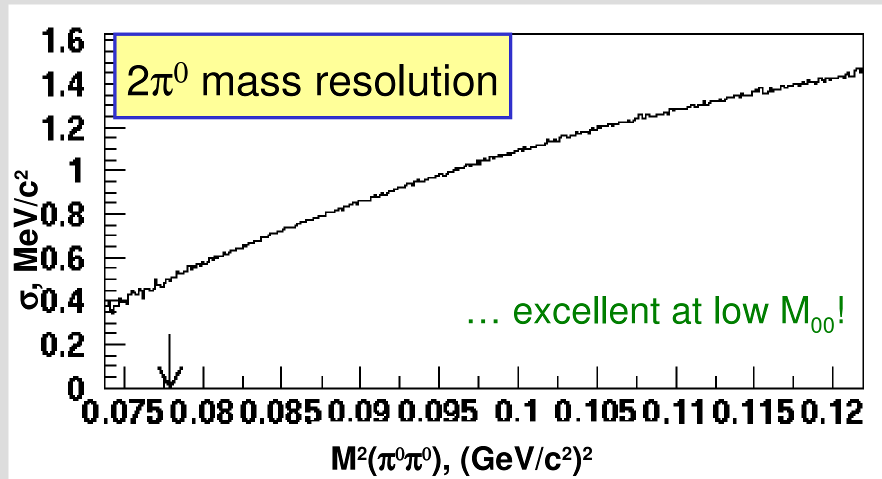
D_{ik} – distance between γ_i and γ_k on LKr

z_{ik} – distance from $\pi^0\to\gamma\gamma$ decay vertex to LKr

Background is negligible



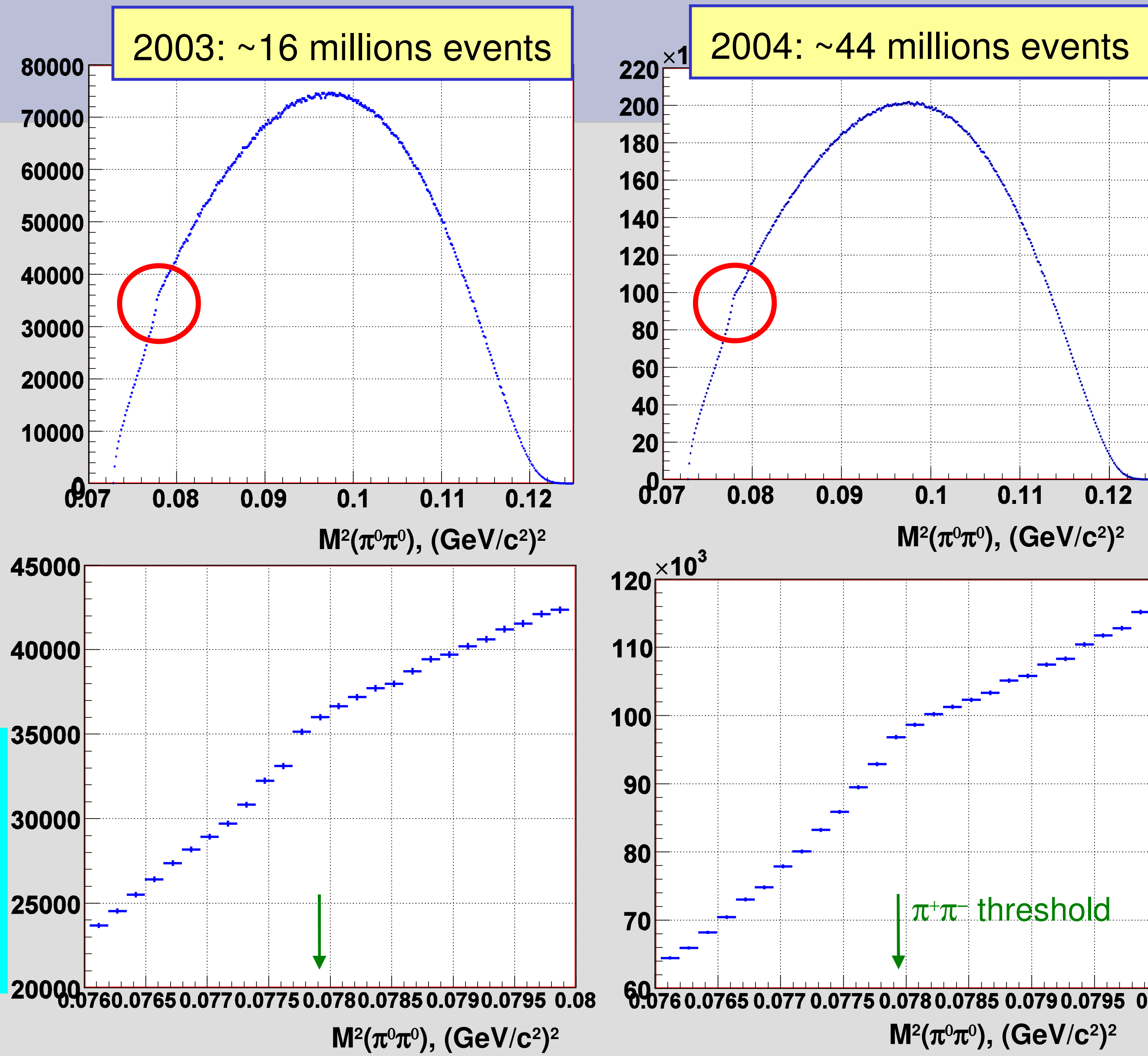




Observation of the cusp

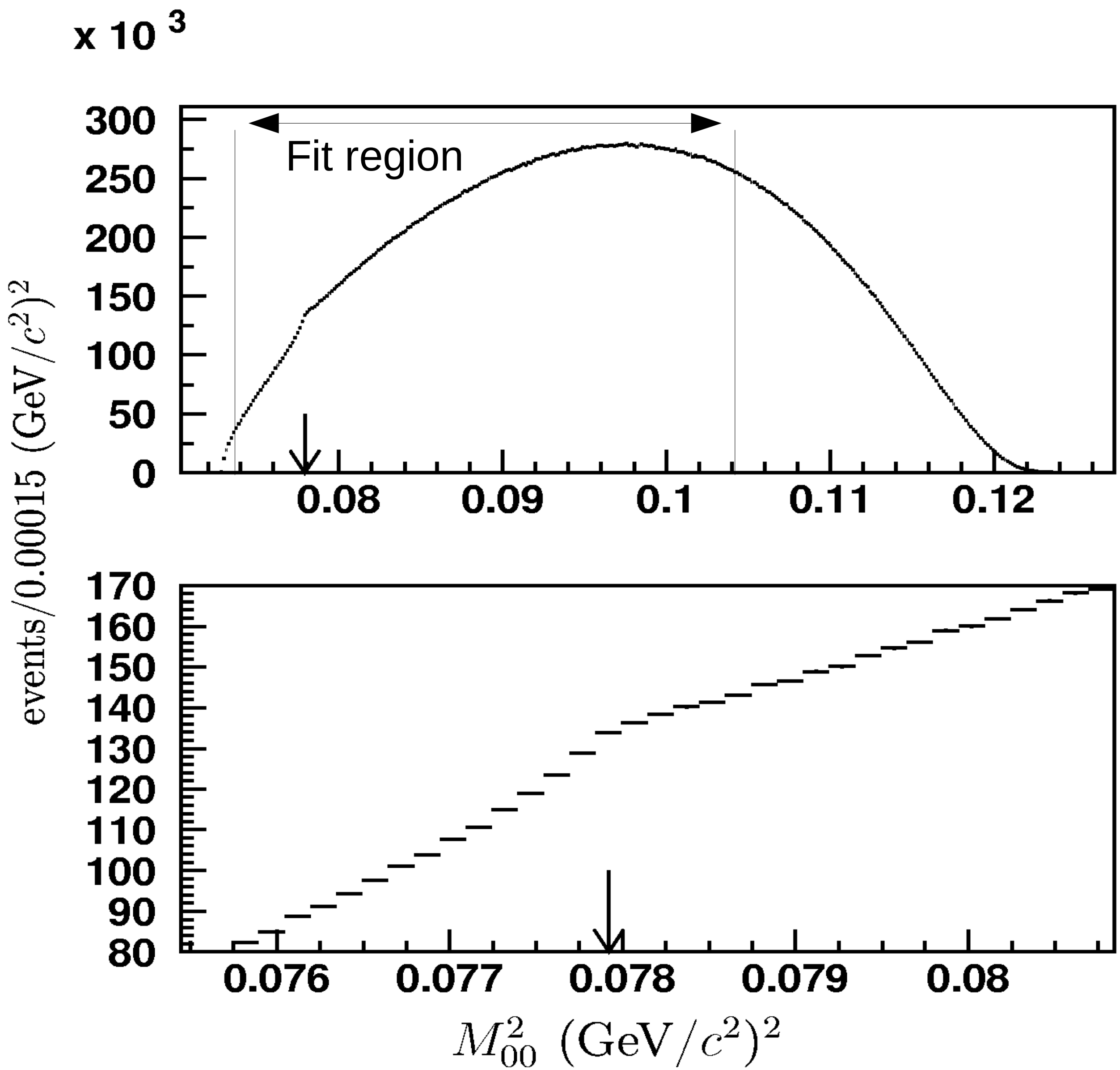
First observation of the cusp was made with the 2003 data

Inclusion of 2004 data: statistics increased by a factor of 3.7



Uniform selection conditions and MC/Data statistics, so we sum the 2003 and 2004 data for the joint analysis

Full data statistics



2003 + 2004

60.31 millions events

Fit region is choosen to reach a minimum total error (systematical contribution grows with upper M_{00})

226-th bin instead of 176-th (as was in our first cusp paper (2006))

Statistical uncertainties are shown !

Theory: final state rescattering

N. Cabibbo, PRL 93 (2004) 121801

$$M(K^{\pm}\to\pi^{\pm}\pi^{0}\pi^{0}) = M_0 + M_1$$

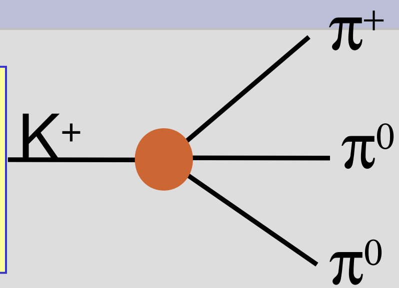
Direct emission ($k_0, h_0 << g$):

$$M_0 = A_0(1+g_0u/2+h_0u^2/2+k_0v^2/2)$$

Kaon rest frame:

$u = 2m_K\cdot(m_K/3-E_{odd})/m_\pi^2$

$v = 2m_K\cdot(E_1-E_2)/m_\pi^2$



$$M_+ = A_+(1+g_+u/2+h_+u^2/2+k_+v^2/2)$$

Negative interference under threshold

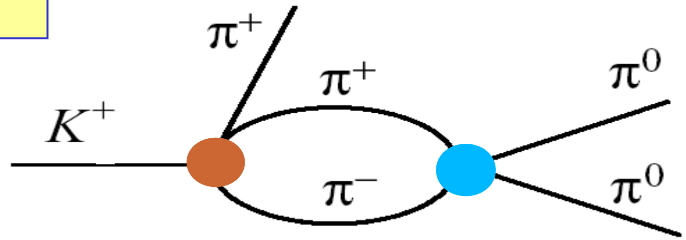
Rescattering amplitude:

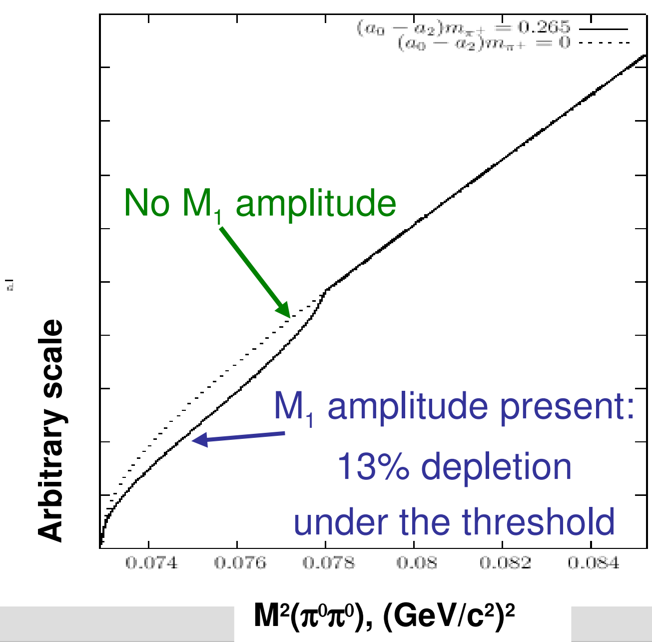
$$M_1 = -2/3(a_0-a_2)m_+M_+\sqrt{1-\left(\frac{M_{00}}{2m_+}\right)^2}$$

Combination of S-wave ππ scattering lenghts

$K^{\pm}\to3\pi^{\pm}$ amplitude at threshold

(isospin symmetry assumed here)



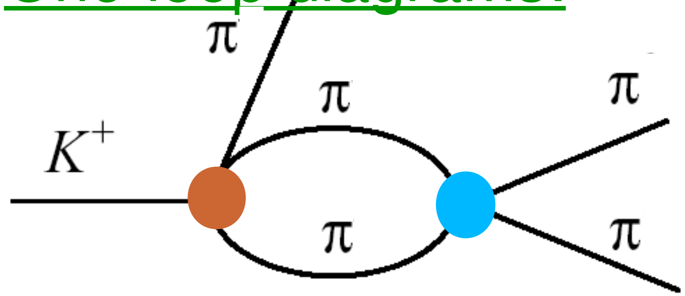


Theory: two-loop diagrams

N. Cabibbo and G. Isidori (CI), JHEP 503 (2005) 21

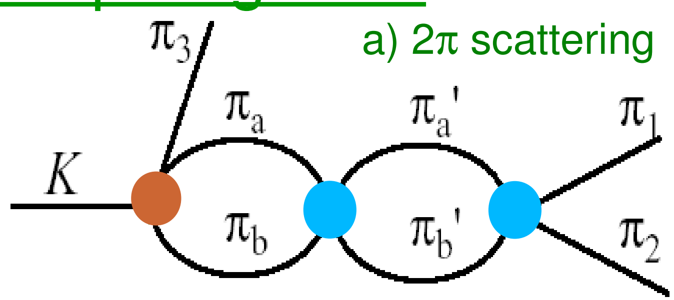
- Five S-wave scattering lengths (a_x, a_{++}, a_{+-}, a_{+0}, a_{00}) expressed as linear combinations of a_0 and a_2
- Isospin symmetry breaking accounted for following J. Gasser. For example, $a_x = (1+\varepsilon/3)(a_0-a_2)/3$, where $\varepsilon=(m_+^2-m_0^2)/m_+^2=0.065$ is isospin breaking parameter
- Radiative corrections missing; (a_0-a_2) precision ~5%
- V-dependent terms ~ $(k`/2)V^2$ introduced both into "unperturbed" $K^\pm\to\pi^\pm\pi^0\pi^0$ and $K^\pm\to\pi^\pm\pi^+\pi^-$ amplitudes.

One-loop diagrams:

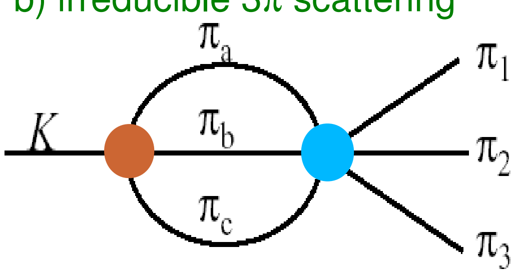


Two-loop diagrams:

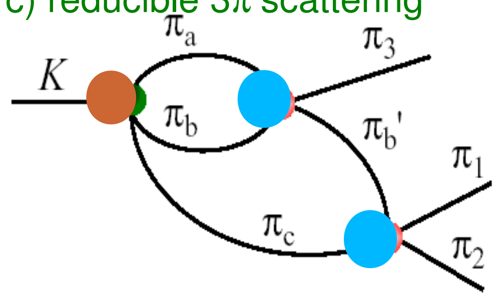
a) 2π scattering

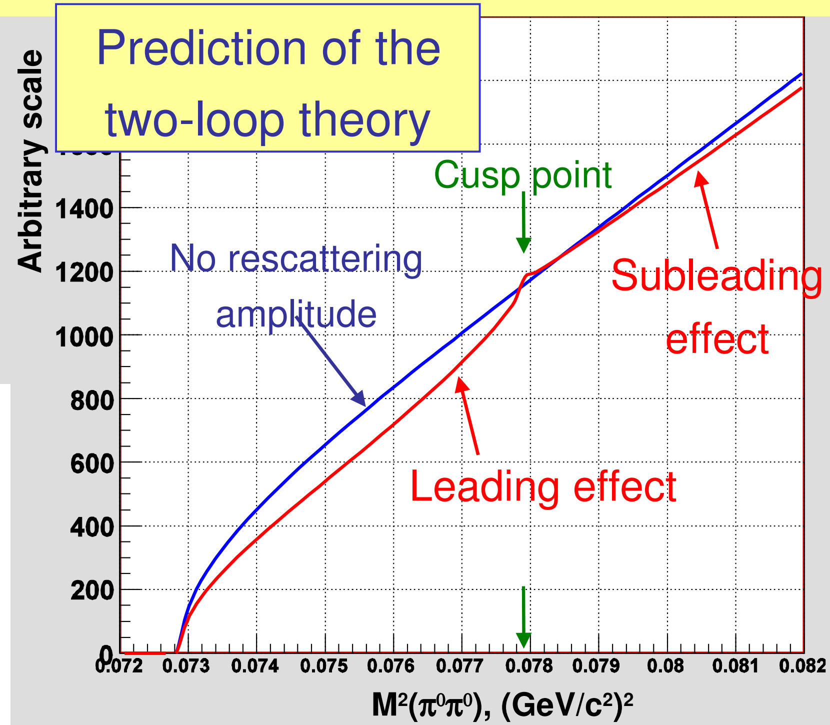


b) irreducible 3π scattering



c) reducible 3π scattering





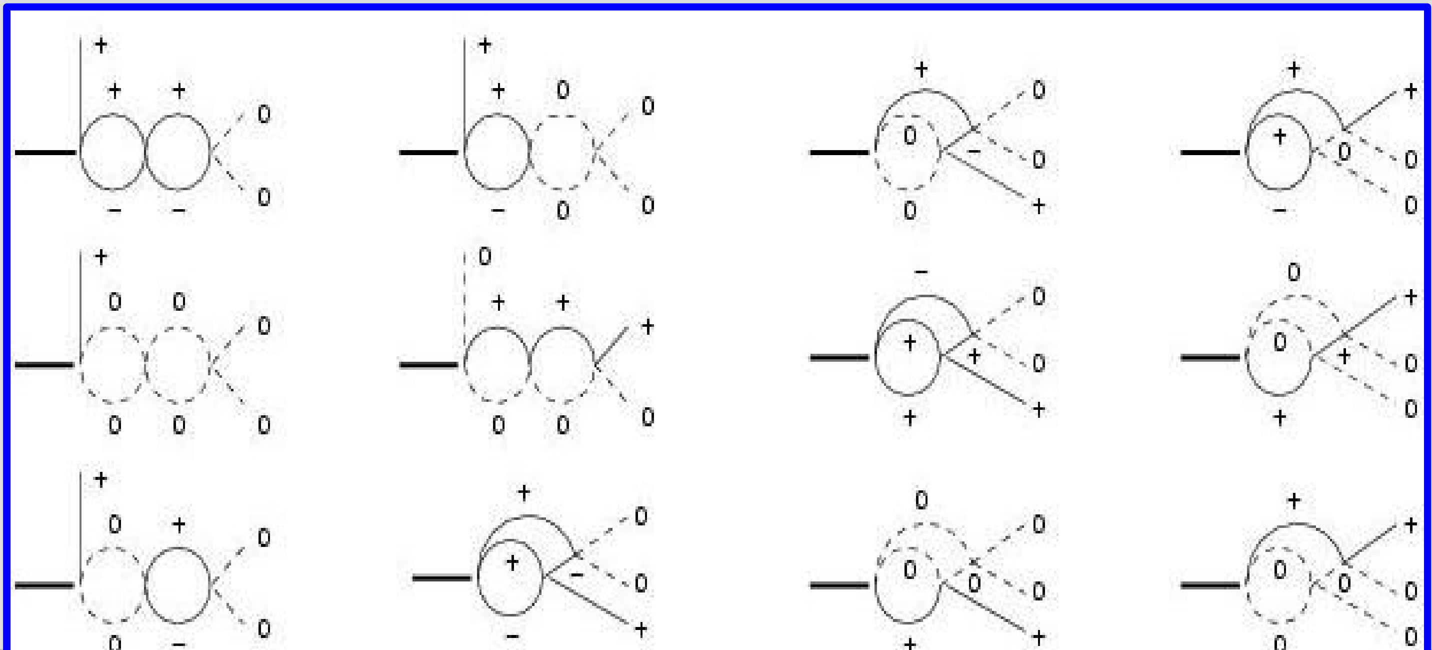
Theory: effective fields

G. Colangelo, J. Gasser, B. Kubis, A. Rusetsky (Bern-Bonn group, BB)

Phys.Lett. B638 (2006) 187-194

- Non-relativistic Lagrangian for effective fields; expanding in another small parameters.
- Valid in the whole decay region.
- Another (in comparison with CI) part of amplitude is absorbed in the polinomial terms (so another correlations).
- At two loops, algebraically different formulae for amplitude
- FORTRAN code provided by authors

NA48 technical intervention: polinomial parts of amlitudes are expressed in terms of (U,V)-slopes g,h,k (numerically different from CI case ones).



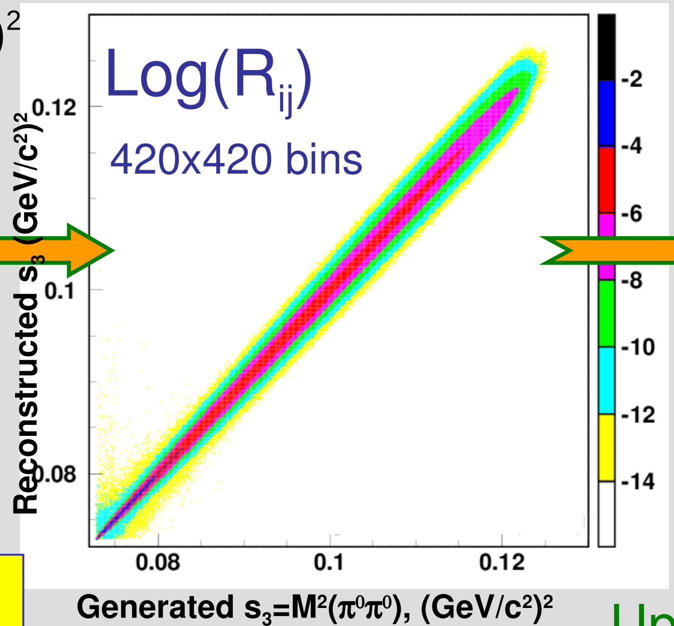
Fitting procedure

1-dimensional fit of the M_{00} projection

Detector response matrix R_{ij} obtained with a GEANT-based Monte-Carlo simulation

Bin width 0.00015 $(GeV/c^2)^2$

Generated distribution $G(g_0,h_0,a_0,a_2,M_{00})$



Reconstructed distribution:

$$F_j^{MC} = \Sigma R_{ij} G_i$$

MINUIT minimization of χ^2 of data/MC spectra shapes

Up to 5 free parameters

$$\chi^2(g,h',m_+(a_0-a_2),m_+a_2,N)=\sum_{s_3\text{ bins}} \frac{(F_{DATA}-NF_{MC})^2}{\delta F_{DATA}^2+N^2\delta F_{MC}^2}$$

If one fix a_2 or link it to a_0 by some relation, result for (a_0-a_2) is more precise

Charged decay ($K^{\pm}\to\pi^{\pm}\pi^{+}\pi^{-}$) amplitude parameters

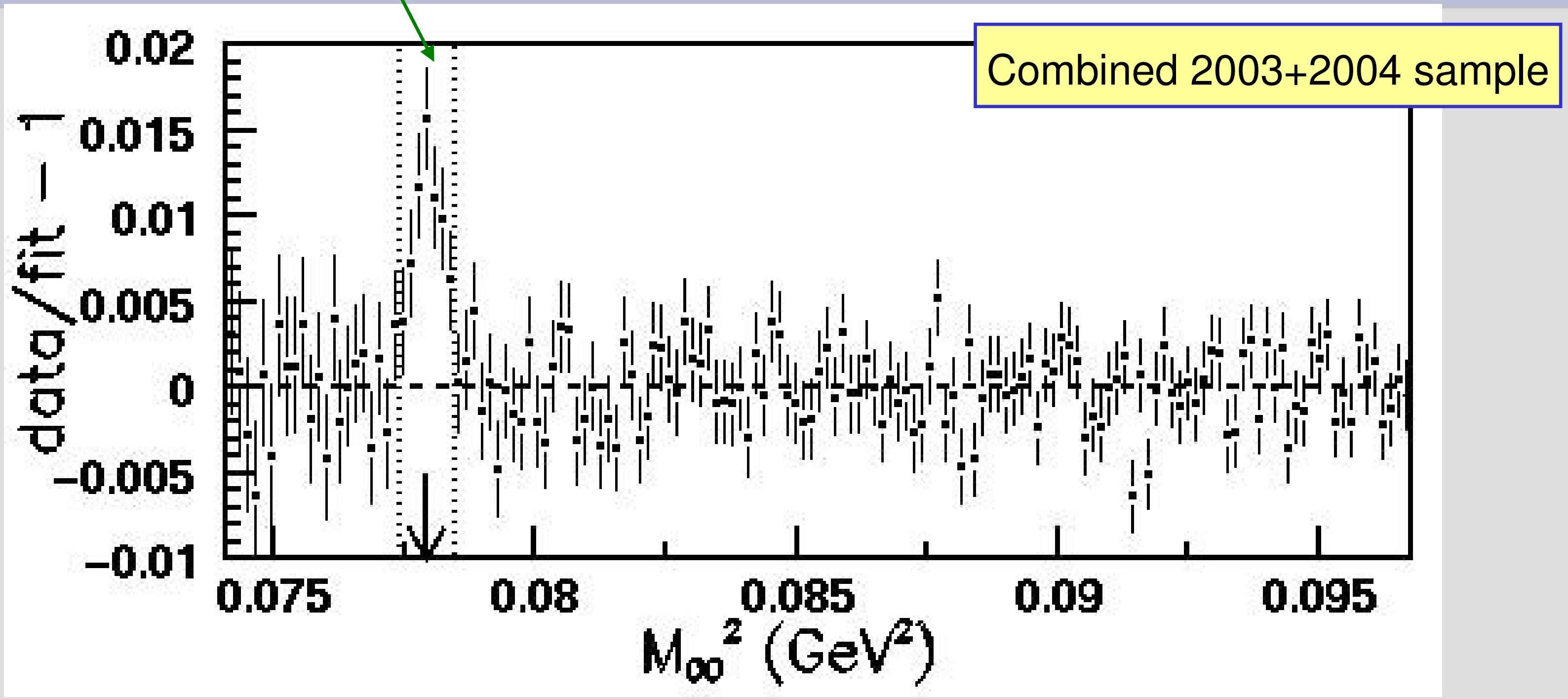
- For the CI fit we take the measured by NA48/2 parameters of the $K^{\pm}\to 3\pi^{\pm}$ amplitude $M_{+} = A_{+}(1+g_{+}u/2+h_{+}u^{2}/2+k_{+}v^{2}/2)$:
- $g_{+} = -0.21117(15)$; $h_{+} = 0.00671(26)$; $k_{+} = -0.00477(8)$

- For BB case the similar parameters are not numerically the same due to the presence of additional rescattering terms in the in M_{+} amplitude. To define them: simultaneous fit of the NA48/2 $K^{\pm}\to 3\pi^{\pm}$ and $K^{\pm}\to\pi^{\pm}\pi^{0}\pi^{0}$ Dalitz plots by the CGKR code; central result (slightly depends on other fit conditions) is:
- : $g_{+} = -0.1894(4)$; $h_{+} = 0.0003(20)$; $k_{+} = -0.0077(19)$

Pionium signature

Points excluded from the fit due to absence of EM corrections in the model

7 data bins skipped around the $M(\pi^+\pi^-)$ threshold



Excess of events in the excluded interval (CI fit),

if interpreted as due to pionium decaying as $A_{2\pi} \rightarrow \pi^0\pi^0$,

gives $R = \Gamma(K^\pm \rightarrow \pi^+ A_{2\pi}) / \Gamma(K^\pm \rightarrow \pi^\pm\pi^+\pi^-) = (1.8 \pm 0.3) \times 10^{-5}$.

⇒ Prediction [Z.K. Silagadze, JETP Lett. 60 (1994) 689]: $R = 0.8 \times 10^{-5}$.

Electromagnetic corrections to final state interactions in $K \to 3\pi$ decays

(Gevorkian, Tarasov, Voskresenskaya, Phys.Lett. **B649** 159 (2007))

Two contributions from $K^\pm \to \pi^\pm\pi^+\pi^-$ decay to the $K^\pm \to \pi^\pm\pi^\circ\pi^\circ$ cusp region:

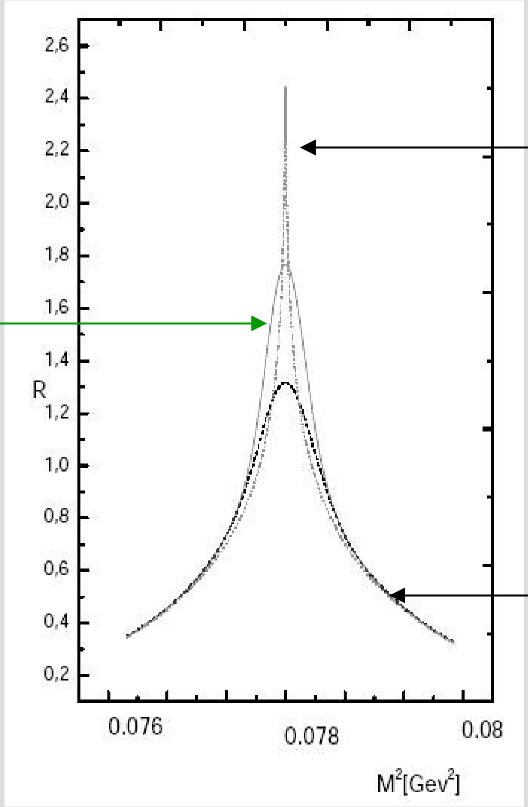
- **Pionium formation : $\pi^+\pi^-$ atom $\to \pi^\circ\pi^\circ$ (negligible width)**
- **Additional $\pi^+\pi^-$ unbound states with resonance structure $\to \pi^\circ\pi^\circ$**

Our observable is an additional joint narrow (with respect to our resolution) contribution into the decay width at the very cusp point.

This measured narrow contribution N with respect to full width :

$N / \Gamma(K^\pm \to \pi^\pm\pi^\circ\pi^\circ) = (5.6 \pm 1.0) \times 10^{-5}$

$\pi^+\pi^-$ atoms plus
$\pi^+\pi^-$ resonant structure with experimental resolution



$\pi^+\pi^-$ resonant structure (no experimental resolution)

$\pi^+\pi^-$ resonant structure with experimental resolution

In our fits we use a free parameter f_{atom} (relative excess in threshold bin of width 0.00015 GeV^2) to represent Pionium + resonant structure. Its predicted value 5.8% is in a good agreement with the measurement (6 +/- 1 %).

Bern — Bonn radiative correction to pion rescattering amplitude for 3pi decays.

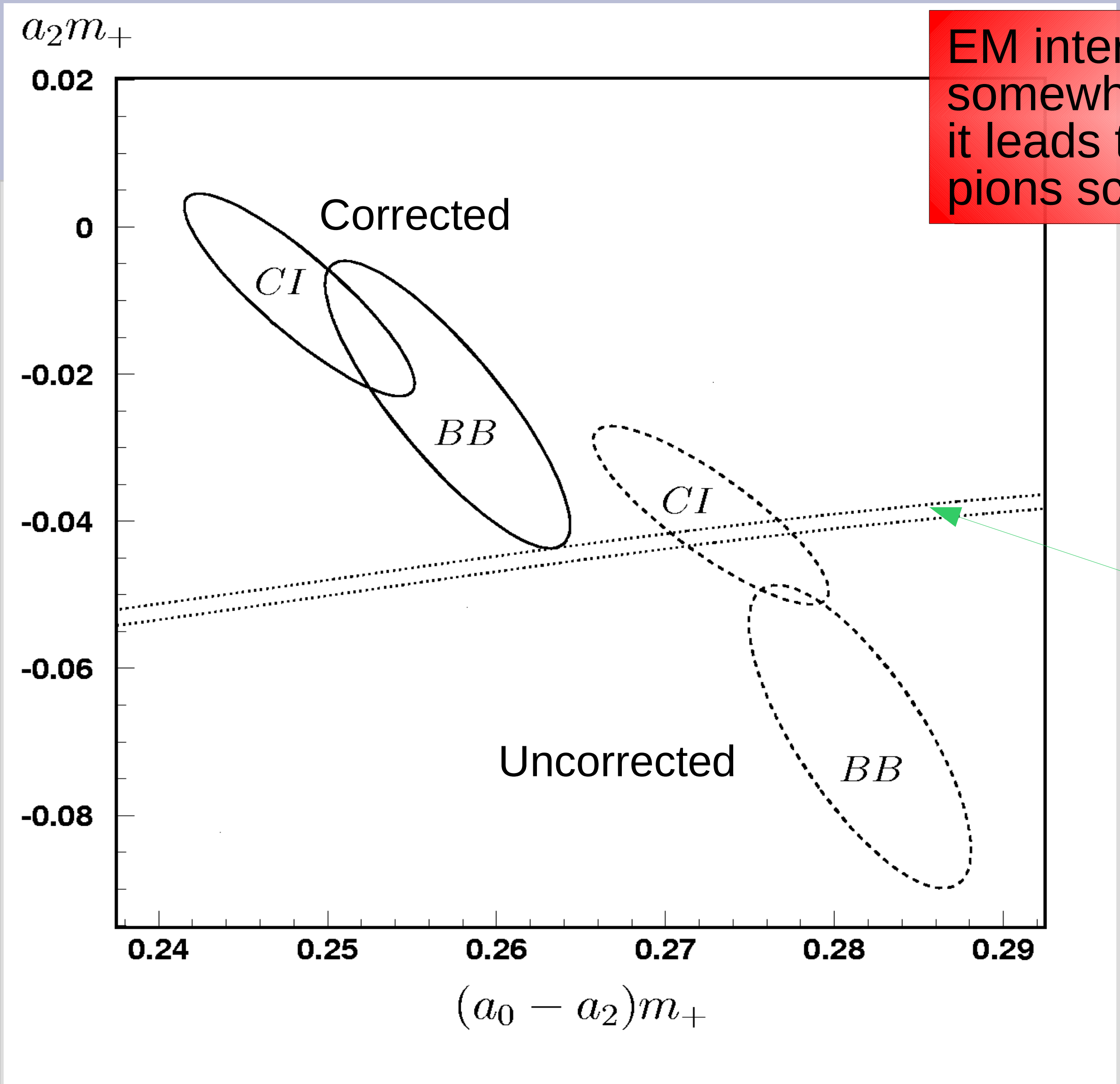
Bissegger M., Fuhrer A., Gasser J., Kubis B., Rusetsky A.
Nucl. Phys., B806, 2009, 178-223.

Electromagnetic part is included into Lagrangian and all the lowest-order diagrams are taken into account — more consequent calculation for the wide U region.

But the bound states and resonant peak at the very threshold can not be calculated perturbatively. That is why we use free parameter f_atom to represent the peak-like contribution in our fits of experimental data.

The correction could be extracted as a relative effect for BB amplitude and implemented also in Cabibbo-Isidori case as a multiplicative factor.

Effect of radiative corrections



EM interaction makes cusp somewhat stronger, so ignoring it leads to overestimation of the pions scattering constants

68% - probability ellipses for statistical errors only to illustrate the shift.

Chiral symmetry constraint [Colangelo et al., PRL 86 (2001) 5008]:

$a_2 = (–0.0444 \pm 0.0008)$
$+ 0.236(a_0–0.22)$
$– 0.61(a_0–0.22)^2$
$– 9.9(a_0–0.22)^3$

Example of parameters correlation coefficients (statistical only), BB fit case

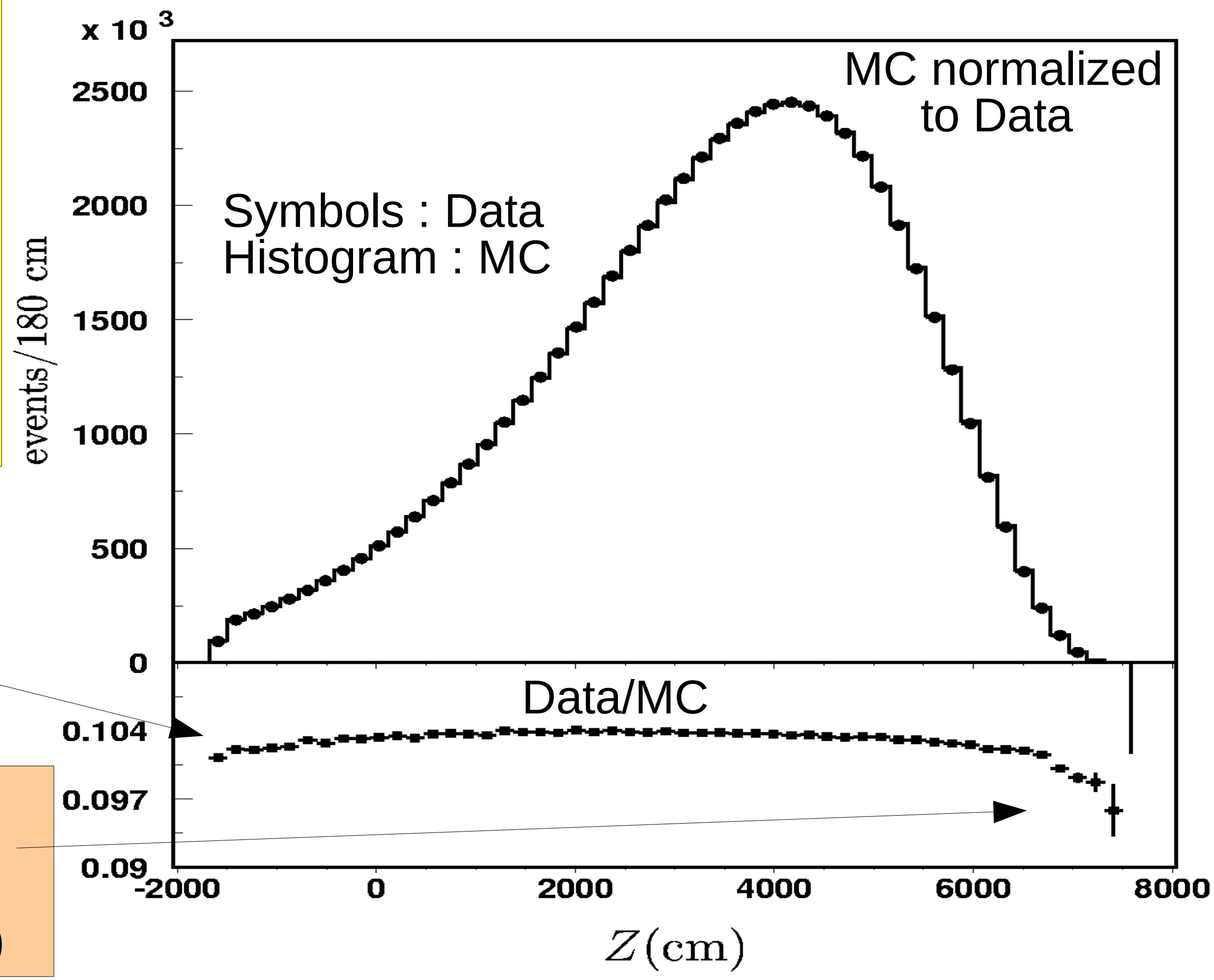
	g_0	h_0	g	h	k	f_{atom}	$a_0 - a_2$
g_0	1.000						
h_0	0.997	1.000					
g	−0.972	−0.965	1.000				
h	0.234	0.220	−0.255	1.000			
k	−0.211	−0.225	0.194	0.889	1.000		
f_{atom}	0.597	0.570	−0.652	0.172	−0.111	1.000	
$a_0 - a_2$	−0.870	−0.843	0.934	−0.404	−0.001	−0.682	1.000
a_2	0.977	0.982	−0.976	0.141	−0.310	0.597	−0.839

Systematics example: decays Z distribution

• Good agreement Between Data and MC.

• But: huge statistics, difference is visible.

• Define the sources, measure the result sensitivity -> syst.err.



Trigger effect

Difference in forward decays acceptance (track radius cut + beam shape)

Systematic uncertainties for independent a_0 and a_2

In units of 10^{-4}

Source	g_0	h_0	a_0	a_2	$a_0 - a_2$	f_{atom}
Acceptance(Z)	31	21	16	20	4	0
Acceptance(V)	6	1	7	8	1	4
Trigger efficiency	26	22	29	39	10	13
LKr resolution	10	9	21	29	9	60
LKr nonlinearity	34	36	56	67	12	1
P_K spectrum	12	11	18	32	13	10
MC(T)	2	1	4	1	5	25
k_0 error	5	5	4	6	2	1
Hadronic showers	2	4	8	18	10	20
Total systematic	56	50	72	94	25	70
Statistical	47	46	92	129	48	97

Correlation between a_0 and a_2 are taken into account

Final result for independent a_0 and a_2

$$(a_0 - a_2)m_+ = \;\; 0.2571 \pm 0.0048_{stat.} \pm 0.0025_{syst.} \pm 0.0014_{ext.}$$

$$a_2 m_+ = -0.024 \pm 0.013_{stat.} \pm 0.009_{syst.} \pm 0.002_{ext.}$$

- External uncertainty: due to $(A_{++-}/A_{+00})|_{threshold} = (1.93\text{-}1.94) \pm 0.015$;

  (central value depends on fitted matrix element, error comes from

  branching rates PDG data)

- Additional theoretical uncertainty (higher order terms neglected) is estimated from the difference between results, obtained using Bern-Bonn and Cabibbo-Isidori formulae, that assume different next-order contributions:

  $\delta \; (a_0 - a_2)m_+ = 0.0088$; $\quad \delta \; a_2 m_+ = 0.015$.

  The relative errors of all kinds for a_2 are essentially larger than $(a_0\text{-}a_2)$ ones, as a_2 corresponds to second-order contribution to cusp shape.

Systematics with ChPT constraint (a_2 is a function of a_0).

In units of 10^{-4}

Source	g_0	h_0	a_0	a_2	$a_0 - a_2$	f_{atom}
Acceptance(Z)	24	14	4	1	3	9
Acceptance(V)	8	4	2	1	2	0
Trigger efficiency	14	16	9	2	7	8
LKr resolution	0	1	2	1	2	46
LKr nonlinearity	12	13	13	3	10	31
P_K spectrum	0	0	2	1	2	5
MC(T)	2	2	6	1	4	24
k_0 error	7	7	0	0	0	2
Hadronic showers	5	3	4	1	3	17
Total systematic	33	26	18	4	14	64
Statistical	9	9	32	8	24	77

Final result with ChPT constraint

[Colangelo et al., PRL 86 (2001) 5008]

$$(a_0 - a_2)m_+ = 0.2633 \pm 0.0024_{stat.} \pm 0.0014_{syst.} \pm 0.0019_{ext.}$$

Theory precision uncertainty for this case is estimated in theory:

⇨ **$\delta(a_0 - a_2)m_+ = 0.0053$ (2%).** S.Gallorini 2008

Theoretical ChPT prediction : 0.265 +/- 0.004
(Colangelo, Gasser, Leutwyler 2000)

Experimental precision is compatible with the theoretical one, the largest is the theoretical uncertainty of $(a_0 - a_2)$ extraction !

Results: other Dalitz plot parameters

$$M(K^{\pm}\to\pi^{\pm}\pi^{0}\pi^{0}) = M_0 + M_1$$

Unperturbed amplitude is $M_0 \sim (1+g_0 u/2+h_0 u^2/2+k_0 v^2/2)$

<u>NB:</u> even without M_1 not the same parameters as the PDG ones:

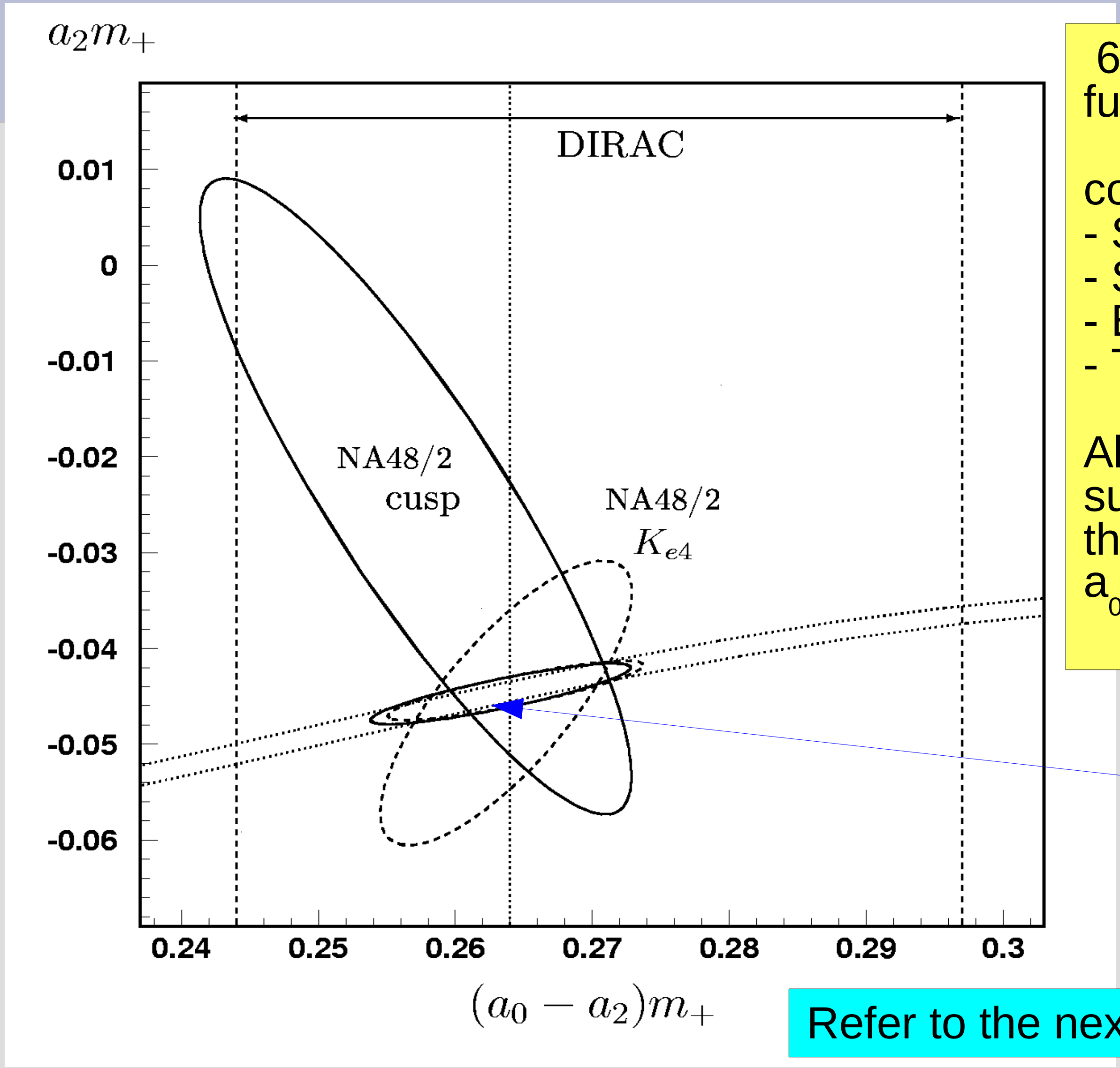
$|M_0|^2_{(PDG)} \sim (1+gu+hu^2+kv^2)$ $[g_0\approx g,\ h_0\approx h-g^2/4,\ k_0\approx k]$

- <u>Technique:</u>
  1. k_0 is extracted from 2-dimensional CI and BB fits
  2. (a_0-a_2, g_0, h_0); ChPT $a_2(a_0)$; fixed k_0 (its uncertainty -> systematics)

CI parameters:	BB parameters:	uncertainties:
$k_0^{CI} = -0.0095$	$k_0^{BB} = -0.0081$	$\pm 0.0002_{stat.} \pm 0.0005_{syst.}$
$g_0^{CI} = 0.653$	$g_0^{BB} = 0.622$	$\pm 0.001_{stat.} \pm 0.003_{syst.}$
$h_0^{CI} = -0.043$.	$h_0^{BB} = -0.052$.	$\pm 0.001_{stat.} \pm 0.003_{syst.}$

For the free a_2 the errors are larger.

Final results of pion scattering lengths measurement from the cusp and comparison to Ke4 results



68% - probability ellipses, full uncertainties:

combined
- Statistical,
- Systematical,
- External,
- Theoretical.

All covariances are summed together to obtain the final correlation between a_0-a_2 and a_2 (-0.879).

Cusp and Ke4 results using Chiral constraint (shown with a Gaussian error)

Refer to the next talk for Ke4 analysis details

Conclusions

- 60.31 millons of fully reconstructed $K^{\pm}\to\pi^{\pm}\pi^{0}\pi^{0}$ decays, cusp anomaly in $\pi^{0}\pi^{0}$ mass distribution at $2m_{\pi\pm}$ due to pions rescattering.
- Complete analyses are done both for Cabibbo-Isidori and Bern-Bonn theoretical calculations, taking into account electromagnetic correction.
- Final result with ChPT link between a_0 and a_2 :

$$(a_0-a_2) = 0.263 \pm 0.003(\text{experiment}) \pm 0.005(\text{theory})$$

[CERN-PH-EP/2009-010, submitted to EPJC],

is in good agreement with Ke4 and with the prediction of ChPT.

- Experimental uncertainty is smaller, than the error of theoretical ChPT prediction 0.265 ± 0.004 (Colangelo, Gasser, Leutwyler 2000).