Studies of the QCD and QED effects on Isospin breaking

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Electromagnetic Splittings

QED + QCD simulations

[T.Blum, T.Doi, M.Hayakawa, T.Izubuchi, S.Uno, N.Yamada and R.Zhou] , in preparation
[R. Zhou, T.Blum, T.Doi, M.Hayakawa, T.Izubuchi, and N.Yamada] ,
"Isospin symmetry breaking effects in the pion and nucleon masses" PoS(LATTICE 2008) 131.
[T. Blum, T. Doi, M. Hayakawa, TI, N. Yamada] ,
"Determination of light quark masses from the electromagnetic splitting of psedoscalar meson masses computed with two flavors of domain wall fermions"
Phys. Rev.D76 (2007) 114508 (38 pages)
"The isospin breaking effect on baryons with Nf=2 domain wall fermions"
PoS(LAT2006) 174 (7 pages)
"Electromagnetic properties of hadrons with two flavors of dynamical domain wall fermions"
PoS(LAT2005) 092 (6 pages)
"Hadronic light-by light scattering contribution to the muon g-2 from lattice QCD: Methodology"
PoS(LAT2005) 353(6 pages)

Isospin Breaking Effects

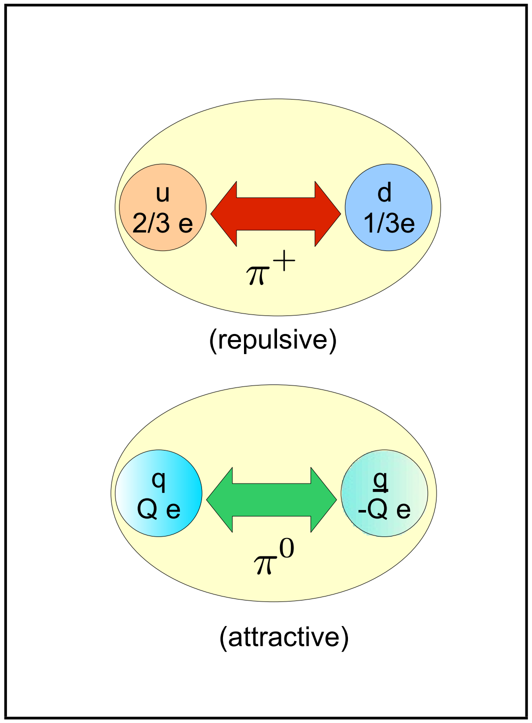
- The first principle calculations of isospin breaking effects due to electromagnetic (EM) and the up, down quark mass difference are necessary for accurate hadron spectrum, quark mass determination.

- Isospin breaking's are measured very accurately :

$$m_{\pi^\pm} - m_{\pi^0} = 4.5936(5)\text{MeV},$$

$$m_N - m_P = 1.2933317(5)\text{MeV}$$

- From $\Gamma(\pi^+ \to \mu^+\nu_\mu, \mu^+\nu_\mu\gamma)$ + V_{ud}(exp)
$f_{\pi^+} = 130.7 \pm 0.1 \pm 0.36$MeV   PDG 2004



- the last error is due to the uncertainty in the part of $\mathcal{O}(\alpha)$ radiative corrections that depends on the hadronic structure of the π meson.

$$\Gamma(PS^+ \to \mu^+\nu_\mu, \mu^+\nu_\mu\gamma) \propto [1 + C_{\text{PS}}\,\alpha]_{\text{had. struc.}}$$

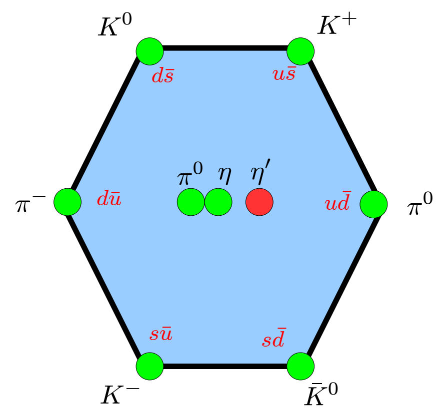
$$C_\pi \sim 0 \pm 0.24, \quad C_\pi - C_K = 3.0 \pm 1.5$$

c.f. Marciano 2004, MILC, RBC/UKQCD : V_{us} from f_π/f_K (Lattice) + $\Gamma(\pi_{l2})/\Gamma(K_{l2})$.

Isospin Breaking Effects (contd.)

- PS meson spectrum and quark masses.

  - Asymmetry due to Quark mass differences : $m_u \neq m_d \neq m_s$
  - Asymmetry due to QED interactions : $Q_u = 2/3e, Q_d = Q_s = -1/3e$
  - QCD axial anomaly makes m'_η heavy.



- Could $m_u \simeq 0$, which would explain the very small Neutron EDM ? (Strong CP problem) [D.Nelson,G.Fleming, G.Kilcup,PRL90:021601,2003.]

- Positive mass difference between Neutron (udd) and Proton (uud) stabilizes proton thus make our world as it is. $m_N - m_P = 1.2933317(5)$MeV

- $m_\rho^+ - m_\rho^0$, $\Gamma_{\rho^+}, \Gamma_{\rho^0}$ are related to the conversion of $\Gamma(\tau \to$ Hadrons) to $\Gamma(e^+e^- \to$ Hadrons) to determine leading QCD correction to muon $g-2$.

EM splittings

- Axial WT identity with EM for massless quarks $(N_F = 3)$,

$$\mathcal{L}_{\rm em} = eA_{{\rm em}\,\mu}(x)\bar{q}Q_{\rm em}\gamma_\mu q(x),\ \ Q_{\rm em} = {\rm diag}(2/3, -1/3, -1/3)$$

$$\partial^\mu \mathcal{A}^a_\mu = ieA_{{\rm em}\,\mu}\,\overline{q}\,[T^a,\, Q_{\rm em}]\,\gamma^\mu\gamma_5 q - \frac{\alpha}{2\pi}\,tr\left(Q^2_{\rm em}T^a\right)F^{\mu\nu}_{\rm em}\widetilde{F}_{{\rm em}\,\mu\nu}\,,$$

neutral currents, four $\mathcal{A}^a_\mu(x)$, are conserved (ignoring $\mathcal{O}(\alpha^2)$ effects): π^0, K^0, $\overline{K^0}$, η_8 are still a NG bosons.

- ChPT with EM at $\mathcal{O}(p^4, p^2e^2)$:

$$M^2_{\pi^\pm} = 2mB_0 + 2e^2\frac{C}{f_0^2} \qquad +\mathcal{O}(m^2\log m, m^2) + I_0e^2m\log m + K_0e^2m$$

$$M^2_{\pi^0} = 2mB_0 \qquad +\mathcal{O}(m^2\log m, m^2) + I_\pm e^2m\log m + K_\pm e^2m$$

Dashen's theorem :
The difference of squared pion mass is independent of quark mass up to $\mathcal{O}(e^2m)$,

$$\Delta M^2_\pi \equiv M^2_{\pi^\pm} - M^2_{\pi^0} = 2e^2\frac{C}{f_0^2} + (I_\pm - I_0)e^2m\log m + (K_\pm - K_0)e^2m$$

C, $K_\pm$, K_0 is a new low energy constant. $I_\pm$, I_0 is known in terms of them.

QCD+QED lattice simulation

- In 1996, Duncan, Eichten, Thacker carried out SU(3)×U(1) simulation to do the EM splittings for the hadron spectroscopy using quenched Wilson fermion on $a^{-1} \sim 1.15$ GeV, $12^3 \times 24$ lattice. [Duncan, Eichten, Thacker PRL76(96) 3894, PLB409(97) 387]

- Using $N_F = 2 + 1$ Dynamical DWF ensemble (RBC/UKQCD) would have benefits of chiral symmetry, such as better scaling and smaller quenching errors.

- Especially smaller systematic errors due to the the quark massless limits, $m_f \to -m_{res}(Q_i)$, has smaller Q_i dependence than that of Wilson fermion, $\kappa \to \kappa_c(Q_i)$ (PCAC).

- Generate Coulomb gauge fixed (quenched) non-compact U(1) gauge action with $\beta_{QED} = 1$. $U_\mu^{EM} = \exp[-iA_{\text{em}\,\mu}(x)]$.

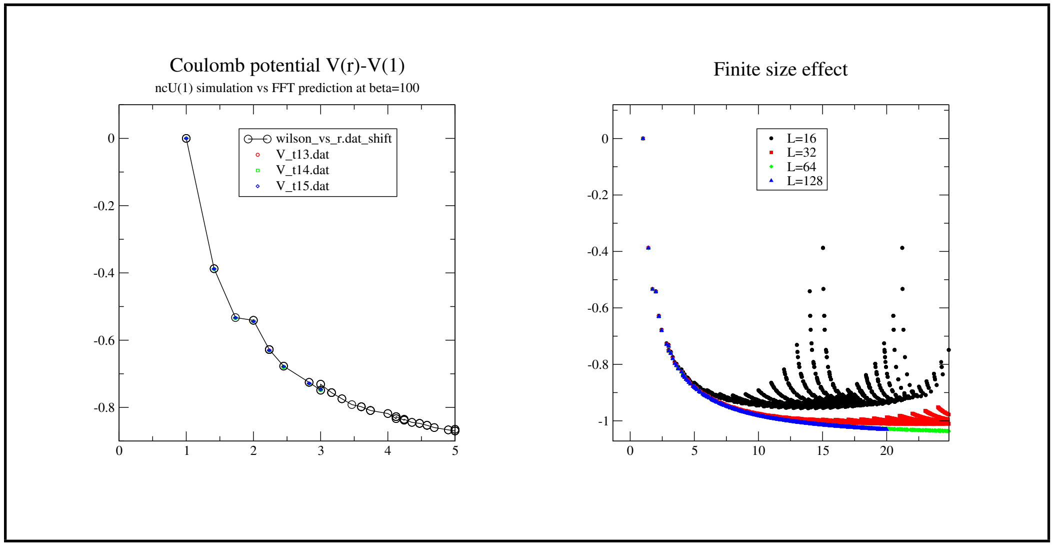
- Quark propagator, $S_{q_i}(x)$ with EM charge $Q_i = q_i e$ with Coulomb gauge fixed wall source

$$D\left[(U_\mu^{EM})^{Q_i} \times U_\mu^{SU(3)}\right] S_{q_i}(x) = b_{src}, \quad (i = \text{up,down})$$

$$q_{\text{up}} = 2/3, \quad q_{\text{down}} = -1/3$$

photon field on lattice

- non-compact $U(1)$ gauge is generated by using Fast Fourier Transformation (FFT). Coulomb gauge $\partial_j A_{\text{em}\,j}(x) = 0, \tilde{A}_{\text{em}\,\mu=0}(p_0, 0) = 0$ with eliminating zero modes. ($N_F = 2+1$: Feynman gauge)
- static lepton potential on $16^3 \times 32$ lattice ($\beta_{QED} = 100$, 4,000 confs) vs lattice Coulomb potential.
- L=16 has significant finite volume effect for $ra > 6 \sim 1.5 r_0 \sim 0.75$ fm. It would be worth considering for generation of U(1) on a larger lattice and cutting it off.



ChPT+EM at NLO

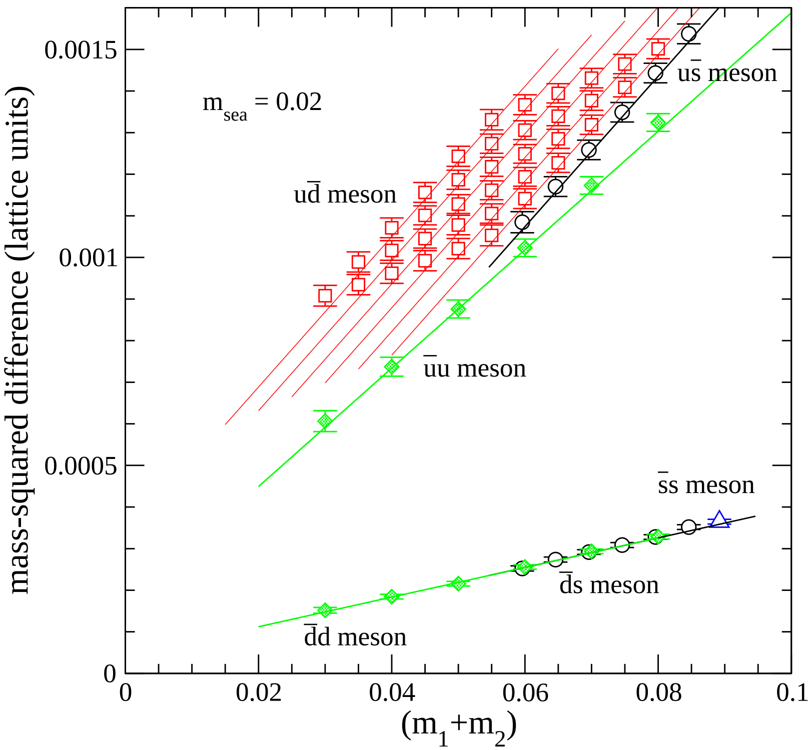
- Double expansion of $M^2_{\rm PS}(m_1, q_1; m_3, q_3)$ in $\mathcal{O}(\alpha), \mathcal{O}(m_q)$.
  QCD LO:
  $$M^2_{\rm PS} = \chi_{13} = B_0(m_1 + m_3)$$
  QCD NLO: $(1/F_0^2 \times)$
  $$(2L_6 - L_4)\chi^2_{13} + (2L_5 - L_8)\chi_{13}\bar{\chi}_1 + \chi_{13} \sum_{I=1,3,\pi,\eta} R_I \chi_I \log(\chi_I/\Lambda^2_\chi),$$
  QED LO: (Dashen's term)
  $$\frac{2C}{F_0^2}(q_1 - q_3)^2$$
  QED NLO: ($\bar{Q}_2 = \sum q^2_{{\rm sea}-i}$, no $\bar{Q}_1$ in SU(3)$_{N_F}$)
  $$-Y_1\bar{Q}_2\chi_{13} + Y_2(q_1^2\chi_1 + q_3^2\chi_3) + Y_3 q^2_{13}\chi_{13} - Y_4 q_1 q_3 \chi_{13} + Y_5 q^2_{13}\bar{\chi}_1$$
  $$+\chi_{13}\log(\chi_{13}/\Lambda^2_\chi)q^2_{13} + \bar{B}(\chi_\gamma, \chi_{13}, \chi_{13})q^2_{13}\chi_{13} - \bar{B}_1(\chi_\gamma, \chi_{13}, \chi_{13})q^2_{13}\chi_{13} + \cdots$$

- QED LO adds mass to $\pi^\pm$ at $m_q = 0$, QED NLO changes slope, B_0, in m_q.

- Partially quenched formula ($m_{\rm sea} \neq m_{\rm val}$) SU(3)$_{N_F}$ [Bijnens Danielsson, PRD75 (07)] SU(2)$_{N_F}$+heavy Kaon+FiniteV [Hayakawa Uno, PTP 120(08) 413] [RBC/UKQCD; P.Boyle's talk] (also [C. Haefeli, M. A. Ivanov and M. Schmid, EPJ C53(08)549])

SU(3) PQChPT fit in $N_F = 2 + 1$ QCD+QED simulations

- SU(3) PQChPT fit.



- $a^{-1} \sim$ 1.8 GeV from Ω^- baryon mass (no log in NLO).
- Four degenerate up/down quark masses in the simulation: $\sim$ 25, 40, 70, 100 MeV.
- One strange quark point ($\sim$ 10% heavier than the physical).
- Two volumes: $(1.8\ \text{fm})^3$ and $(2.7\ \text{fm})^3$

- Determine 3 QCD LEC + 5 QED LEC (also 3 QCD LEC for f_π)
- In total 234 charge,quark mass combinations are measured. $M_{\text{PS}}(m_1, q_1; m_2, q_2; m_l)$

SU(3)+EM ChPT LEC (preliminary)

[R. Zhou] [Bijnens Danielsson, PRD75 (07)]

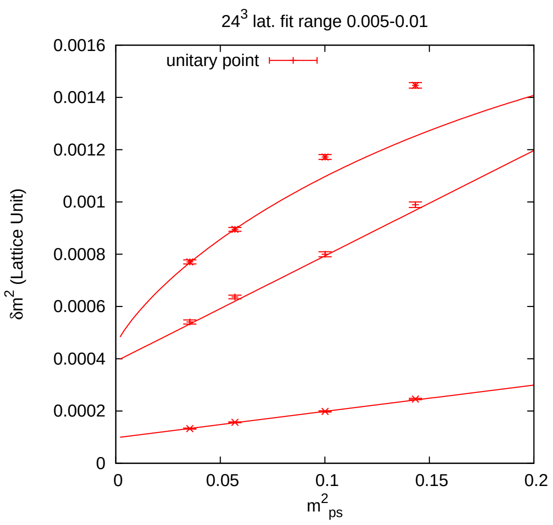
- By fitting charge splitting

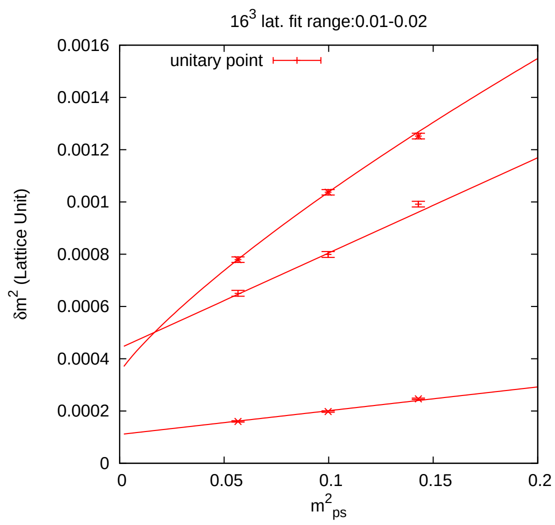
$$\delta M^2 = M^2_{\mathsf{PS}}(m_1, q_1; m_2, q_2; m_l) - M^2_{\mathsf{PS}}(m_1, 0; m_2, 0; m_l)$$

  by SU(3) ChPT+EM formula at NLO, 3 QCD LECs (1 LO + 2 NLO), 5 QED LECs (1 LO + 4 NLO) are determined.

- Requiring $m_1, m_3, m_l \le$ 40 MeV (70 MeV), 48 (120) partially quenched data for $M_{\mathsf{PS}}(m_1, q_1; m_2, q_2; m_l)$ are used in the fit (to see NNLO effects).

- Finite volume effects are observed by repeating the fit on (1.8 fm)^3 and and (2.7 fm)^3.

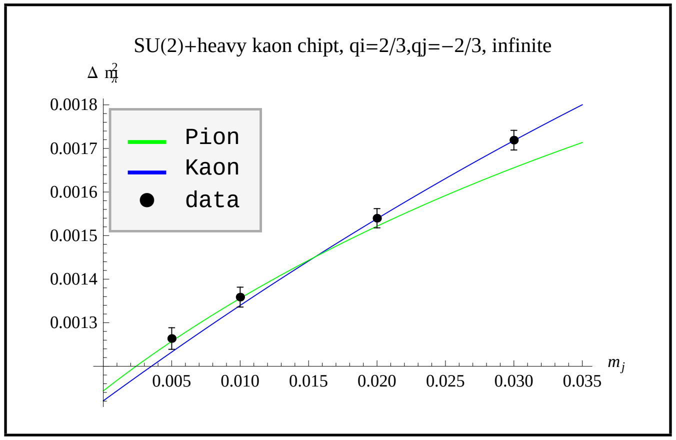


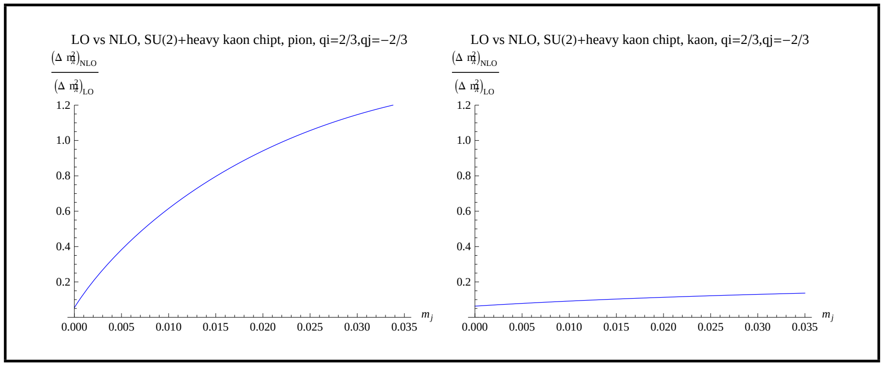


SU(2)-heavy Kaon+EM ChPT Fit (preliminary))

[S.Uno] [Hayakawa Uno, PTP 120(08) 413]

- Treating Kaon as heavy particle (no chiral log from η).
- Finite volume analysis is done.
- Ultimately should give our main quote.
- EM splitting NLO/LO is still large ($\sim$ 50% at $m_q = 40$ MeV) for Pion but small ($\sim$ 10% at $m_q = 70$ MeV) for Kaon.





Quark mass determination

- Using the LECs, B_0, F_0, L_i, C_0, Y_i, from the fit, we could determine the quark masses $m_{\mathrm{up}}, m_{\mathrm{dwn}}, m_{\mathrm{str}}$ by the solving equations [PDG08] :

$$M_{\mathrm{PS}}(m_{\mathrm{up}}, 2/3, m_{\mathrm{dwn}}, -1/3) = 139.57018(35)\mathrm{MeV}$$
$$M_{\mathrm{PS}}(m_{\mathrm{up}}, 2/3, m_{\mathrm{str}}, -1/3) = 493.673(14)\mathrm{MeV}$$
$$M_{\mathrm{PS}}(m_{\mathrm{dwn}}, -1/3, m_{\mathrm{str}}, -1/3) = 497.614(24)\mathrm{MeV}$$

- $(m_{\mathrm{up}} - m_{\mathrm{dwn}})$ is mainly determined by Kaon charge splittings,

$$M_{K^\pm}^2 - M_{K^0}^2 = B_0(m_{\mathrm{up}} - m_{\mathrm{dwn}}) + \frac{2C}{F_0^2}(q_1 - q_3)^2 + \mathrm{NLO}$$

- π^0 mass is not used for now (disconnected quark loops).

- The term proportional to sea quark charge, $-Y_1 \bar{Q}_2 \chi_{13}$, is omitted. We will estimate the systematics by varying Y_1.

Quark mass results (Preliminary)

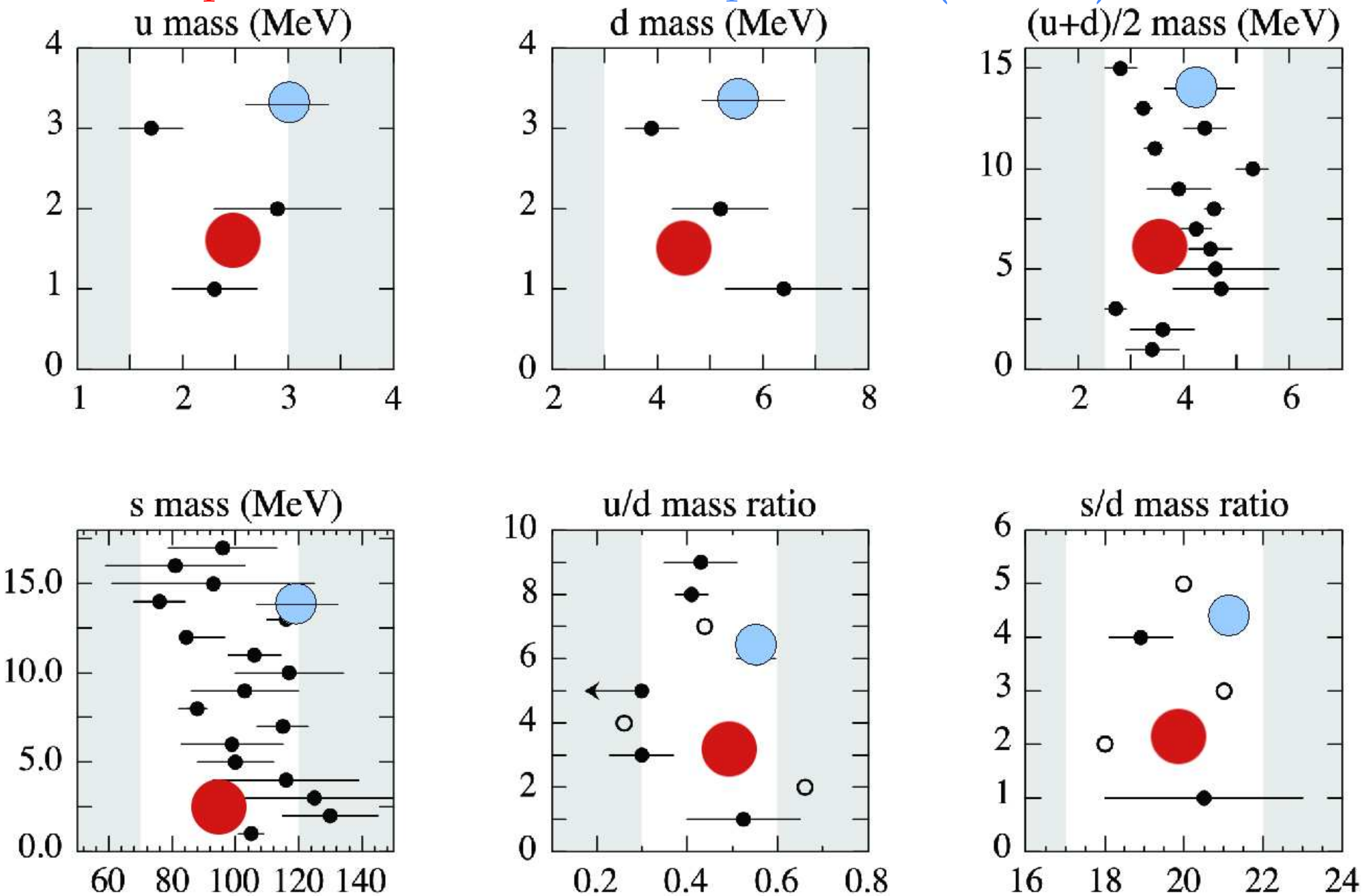
[R. Zhou , S.Uno]

- $\overline{MS}$ at 2 GeV.Non-perturbative technique for the mass renormalization constant is used.
  [RBC/UKQCD, PRD78(08)054510; P.Boyle's talk]
- Quark mass have small finite size volume effects. SU(3)$_{N_F}$ and SU(2)$_{N_F}$ in infinite volume.
- Uncertainties in QED LEC have small effect to quark mass. (π^0 is excluded)
- Statistical error only. For now, we didn't use F_{PS} in fitting LEC, and inflate error by statistical uncertainties of a^{-1} and f_0.
  Our SU(2) m_s is smaller than m_s =107.3(4.4) MeV [RBC/UKQCD PRD78(08) 114509] due to this difference in two analyzes.

lat	m_q range	m_u	m_d	m_s	m_u/m_d	m_s/m_{ud}
(2.7 fm)^3 SU(3)$_\infty$	$\leq$ 40 MeV	2.48(18)	4.77(30)	95(7)	0.52(3)	26.3(6)
(2.7 fm)^3	$\leq$ 70 MeV	2.50(18)	4.81(30)	95(8)	0.52(3)	26.1(6)
(1.8 fm)^3	$\leq$ 70 MeV	2.64(19)	4.81(32)	95(9)	0.55(4)	25.5(8)
(2.7 fm)^3 SU(2)$_\infty$	$\leq$ 70 MeV	2.24(16)	4.62(24)	101(5)		

Quark masses

red $N_F = 2+1$ DWF   blue N_F=2 DWF (2.7 fm)^3



(Only statistical errors are shown).

Components of Kaon masses splittings

- Reason why the iso doublet, (K^+, K^0), has the mass splitting

$$M_{K^\pm} - M_{K^0} = -3.937(29) \quad \text{MeV}, \qquad \text{[PDG08]}$$

  ▷ $(m_{\text{dwn}} - m_{\text{up}})$ *: makes* $M_{K^+} - M_{K^0}$ ***negative.***
  ▷ $(q_u - q_d)$ *: makes* $M_{K^+} - M_{K^0}$ ***positive.***

- Using the determined quark masses and SU(3) LEC, we could isolate (to $\mathcal{O}((m_{\text{up}} - m_{\text{dwn}})\alpha)$) each of contributions,

$$\begin{aligned}
&M^2_{\text{PS}}(m_{\text{up}}, 2/3, m_{\text{str}}, -1/3) - M^2_{\text{PS}}(m_{\text{dwn}}, -1/3, m_{\text{str}}, -1/3) \\
\simeq& M^2_{\text{PS}}(m_{\text{up}}, 0, m_{\text{str}}, 0) - M^2_{\text{PS}}(m_{\text{dwn}}, 0, m_{\text{str}}, 0) && [\Delta M(m_{\text{up}} - m_{\text{dwn}})] \\
+& M^2_{\text{PS}}(\bar{m}_{ud}, 2/3, \bar{m}_{ud}, -1/3) - M^2_{\text{PS}}(\bar{m}_{ud}, -1/3, m_{\text{str}}, -1/3) && [\Delta M(q_u - q_d)]
\end{aligned}$$

- ▷ $\Delta M(m_{\text{up}} - m_{\text{dwn}})$ *= -5.7(1) MeV* *[145% in $\Delta M^2(m_{\text{up}} - m_{\text{dwn}})$]*
  ▷ $\Delta M(q_u - q_d)$ *= 1.8(1) MeV* *[-45% in $\Delta M^2(q_u - q_d)$]*

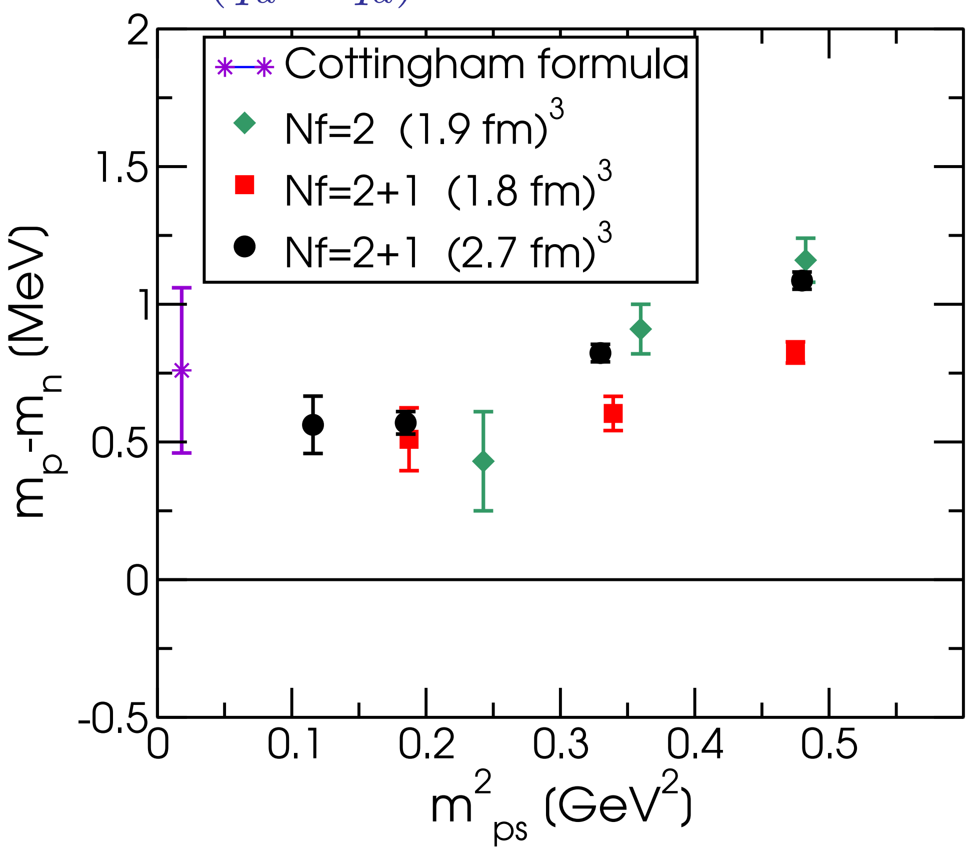
  Also SU(2) ChPT, $\Delta M(m_{\text{up}} - m_{\text{dwn}})$=-5.3(7) MeV and $\Delta M(q_u - q_d)$=1.4(7) MeV.

- Similar analysis for π is possible, but facing a difficulty of isolating sea strange quark terms. SU(2) analysis gives a reasonable value.

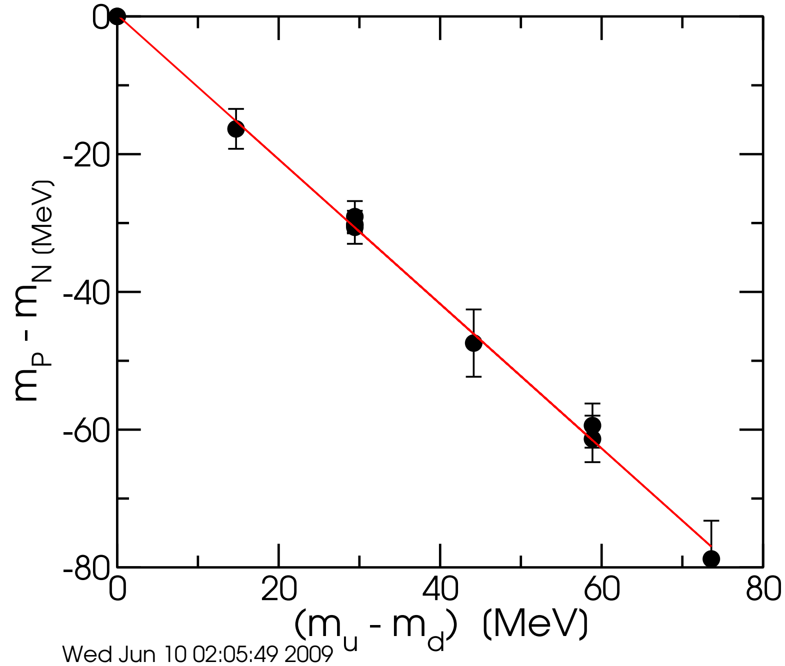
Nucleon mass splitting in $N_F = 2, 2+1$ (Preliminary)

[R.Zhou, T.Doi]

$(q_u - q_d)$ effect



$(m_{\text{up}} - m_{\text{dwn}})$ effect



$(\sim 2$ MeV at $m_{\text{up}} - m_{\text{dwn}})$

- Only EM effect, $m_u = m_d$ case, are shown. *c.f.* [Gasser Leutwyler, PR87(82)77]

$$M_N - M_p|_{\text{EM}} = -0.76(30)\ \text{MeV}$$

$$M_N - M_p|_{\text{quark mass}} = 2.05(30)\ \text{MeV}$$

Systematic errors

- Chiral extrapolation: $m_q \leq$ 40 or 70 MeV.
- QCD's Z_m : $\Lambda_{QCD} = 250 - 300$ MeV, $\mathcal{O}(\alpha) \sim$ 1%.
- π^0: disconnected loops (η' from DWF [K. Hashimoto TI PTP (08)])
- Quenched QED $\mathcal{O}(\alpha\alpha_S)$:
  ChPT and a clever combinations of masses [Bijnens Danielsson, PRD75 (07) 014505]
- One lattice spacing results, $\mathcal{O}(a^2)$.
- Finite Size Effect from vector-saturation model: $\Delta_{\pi,EM} = m^2_{\pi^+} - m^2_{\pi^0}$, to be

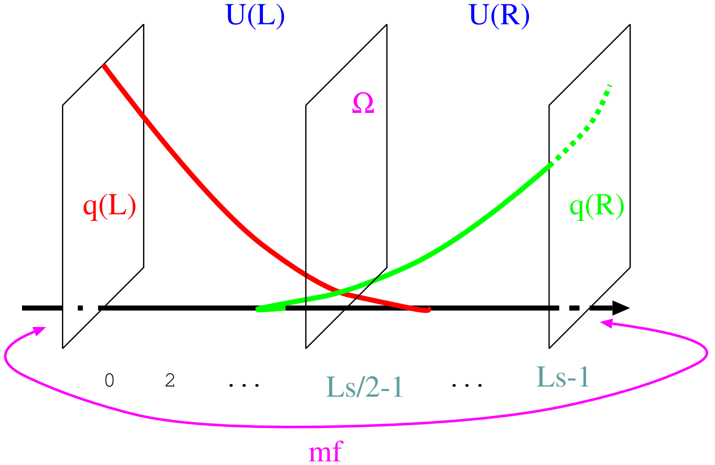
$$\Delta_{\pi,EM}(L) = \frac{3\,\alpha}{4\pi}\,\frac{1}{a^2}\,\frac{2^4 \cdot \pi^2}{N} \sum_{q \in \widetilde{\Gamma}'} \frac{(am_\rho)^2 (am_A)^2}{\widehat{q}^2\,(\widehat{q}^2 + (am_\rho)^2)\,(\widehat{q}^2 + (am_A)^2)}\,,$$

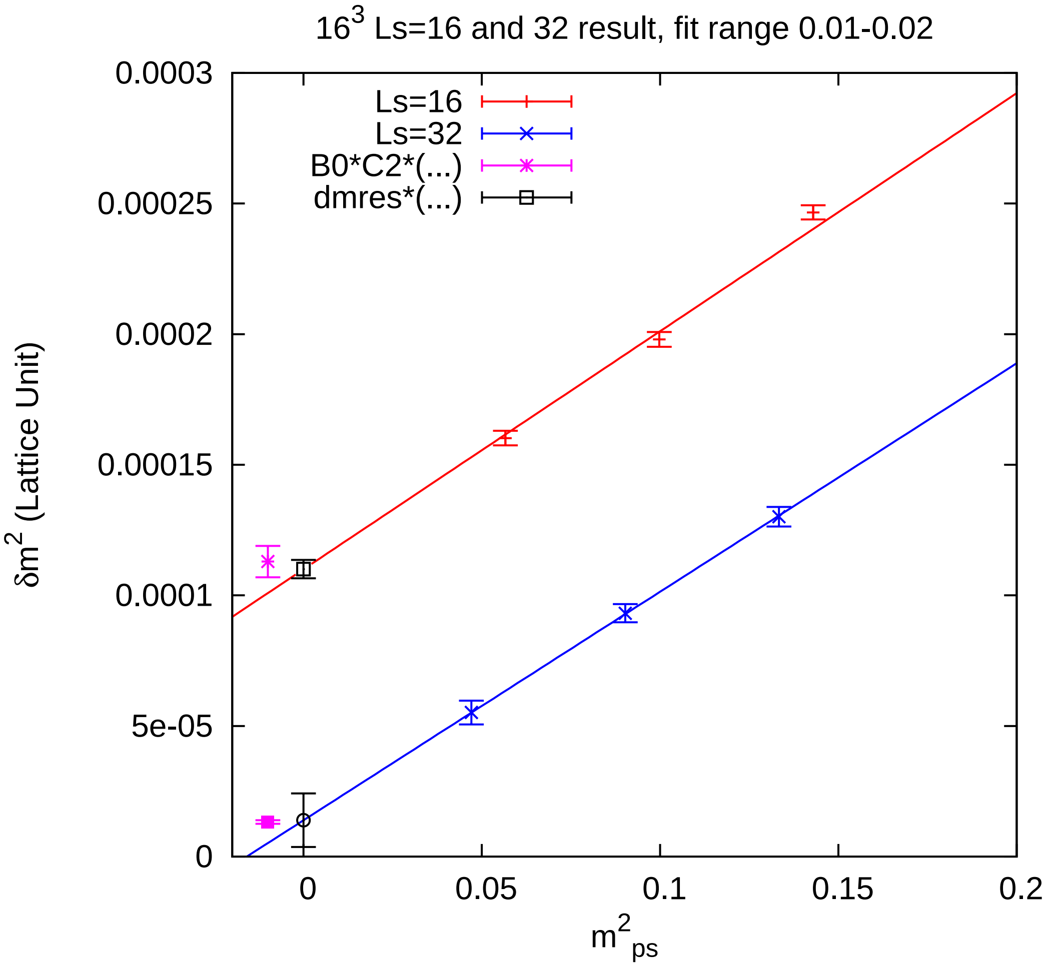
$$\frac{\Delta_{\pi,EM}(\infty)}{\Delta_{\pi,EM}(L \approx 1.9\ fm)} = 1.10\,.$$

Generally quark masses are stable against $\Delta_{\pi,EM} \sim$ 10 %,
Finite volume for P-N case may be larger [1/3 closer by (1.8 fm)^3 $\longrightarrow$ (2.7 fm)^3.]

Effect of the residual chiral symmetry breaking's in $N_F = 2+1$ QCD+QED simulations

- $\delta m^2 = M^2_{\mathsf{PS}^0}(e \neq 0) - M^2_{\mathsf{PS}^0}(e = 0)$ $L_s = 16$ and 32 consistent with PCAC.





Other works of isospin breaking effects on lattice

- [MILC Collaboration (S. Basak et al.) Lattice08 arXiv:0812.4486] EM spectrum using staggered ensemble to get the breaking of Dashen's theorem

$$\Delta M_D^2 = (M_{K^\pm}^2 - M_{K^0}^2)_{\text{em}} - (M_{\pi^\pm}^2 - M_{\pi^0}^2)_{\text{em}}$$

- [McNeile, Michael, Urbach (ETMC) PLB674(09) 286] $\rho - \omega$ mass splitting using twisted Wilson fermion. Discussed $\rho - \omega$ mixing from $m_{\text{up}} - md$. Measure disconnected quark loop correlation.

- [JLQCD PRL 101(08) 242001, PRD79(09)] Calculate $\Pi_V - \Pi_A$, derive the EM contribution to the pion's charge splittings in quark massless limit and the S-parameter using overlap fermion.

Summary and Future perspective

- DWF QCD is now a practical tool for the accurate determination of important quantities for QCD and SM. Continuum-like chiral behavior and renormalization allow us to compute indispensable SM parameters, Hadronic matrix elements and form factor computation etc. that relate experiments and theory [P.Boyle, N.Christ's talk] .

- Isospin breaking effects are interesting and inevitable for precise understanding of hadron physics, which could now be addressed by QCD+QED simulations from the first principle: quark masses, ($m_{\rm up} \simeq 0$?), $m_N - m_P$ >0, ...,

Future plans

- Systematical errors are being estimated.

- Analysis on the finer lattice, $a \sim 0.08$ fm or larger volume. [AuxDet N.Christ's talk]

- EM splittings using the direct calculation of the QED diagrams [JLQCD OPE] .

- Dynamical QED effects by reweighting

- $\mathcal{O}(\alpha)$ contribution to $g_\mu - 2$ (pure QED). $\mathcal{O}(\alpha^3)$ contribution (light-by-light) to $g_\mu - 2$. Chiral magnetic effect in QGP. [T.Blum]