Studies of the QCD and QED effects on Isospin breaking

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for Riken-BNL-Columbia/UKQCD collaboration
Electromagnetic Splittings

**QED + QCD simulations**


[R. Zhou, T.Blum, T.Doi, M.Hayakawa, T.Izubuchi, and N.Yamada],


[T. Blum, T. Doi, M. Hayakawa, Tl, N. Yamada],

“Determination of light quark masses from the electromagnetic splitting of pseudoscalar meson masses computed with two flavors of domain wall fermions” Phys. Rev.D76 (2007) 114508 (38 pages)

“The isospin breaking effect on baryons with Nf=2 domain wall fermions” PoS(LAT2006) 174 (7 pages)

“Electromagnetic properties of hadrons with two flavors of dynamical domain wall fermions” PoS(LAT2005) 092 (6 pages)

“Hadronic light-by light scattering contribution to the muon g-2 from lattice QCD: Methodology” PoS(LAT2005) 353(6 pages)
Isospin Breaking Effects

• The first principle calculations of isospin breaking effects due to electromagnetic (EM) and the up, down quark mass difference are necessary for accurate hadron spectrum, quark mass determination.

• Isospin breaking’s are measured very accurately:
  \[ m_{\pi^+} - m_{\pi^0} = 4.5936(5)\text{MeV}, \]
  \[ m_N - m_P = 1.2933317(5)\text{MeV} \]

• From \( \Gamma(\pi^+ \to \mu^+\nu_\mu, \mu^+\nu_\mu\gamma) + V_{ud}(\text{exp}) \)
  \[ f_{\pi^+} = 130.7 \pm 0.1 \pm 0.36\text{MeV} \quad \text{PDG 2004} \]

• the last error is due to the uncertainty in the part of \( O(\alpha) \) radiative corrections that depends on the hadronic structure of the \( \pi \) meson.

  \[ \Gamma(PS^+ \to \mu^+\nu_\mu, \mu^+\nu_\mu\gamma) \propto [1 + C_{PS}^{\alpha}]\text{had. struc.} \]
  \[ C_\pi \sim 0 \pm 0.24, \quad C_\pi - C_K = 3.0 \pm 1.5 \]

  \text{c.f. Marciano 2004, MILC, RBC/UKQCD : } V_{us} \text{ from } f_\pi/f_K \text{ (Lattice) + } \Gamma(\pi l_2)/\Gamma(K l_2). \]
Isospin Breaking Effects (contd.)

- PS meson spectrum and quark masses.
  - Asymmetry due to Quark mass differences:  
    \[ m_u \neq m_d \neq m_s \]
  - Asymmetry due to QED interactions:  
    \[ Q_u = \frac{2}{3}e, \quad Q_d = Q_s = -\frac{1}{3}e \]
  - QCD axial anomaly makes \( m'_\eta \) heavy.

- Could \( m_u \approx 0 \), which would explain the very small Neutron EDM? (Strong CP problem)  

- Positive mass difference between Neutron (\( udd \)) and Proton (\( uud \)) stabilizes proton thus make our world as it is.  
  \[ m_N - m_P = 1.2933317(5)\text{MeV} \]

- \( m^+_\rho - m^0_\rho, \Gamma^+_{\rho}, \Gamma^0_{\rho} \) are related to the conversion of \( \Gamma(\tau \rightarrow \text{Hadrons}) \) to \( \Gamma(e^+e^- \rightarrow \text{Hadrons}) \) to determine leading QCD correction to muon \( g - 2 \).
EM splittings

- Axial WT identity with EM for massless quarks $(N_F = 3)$,

\[ \mathcal{L}_{\text{em}} = e A_{\text{em} \mu}(x) \bar{q} Q_{\text{em}} \gamma_\mu q(x), \quad Q_{\text{em}} = \text{diag}(2/3, -1/3, -1/3) \]

\[ \partial^\mu A^a_\mu = ie A_{\text{em} \mu} \bar{q} [T^a, Q_{\text{em}}] \gamma^\mu \gamma_5 q - \frac{\alpha}{2\pi} tr \left( Q_{\text{em}}^2 T^a \right) F_{\text{em} \mu \nu} F^\mu_\nu, \]

neutral currents, four $A^a_\mu(x)$, are conserved (ignoring $O(\alpha^2)$ effects): $\pi^0, K^0, \bar{K}^0, \eta_8$ are still a NG bosons.

- ChPT with EM at $O(p^4, p^2 e^2)$:

\[ M_{\pi^\pm}^2 = 2mB_0 + 2e^2 C f_0^2 + O(m^2 \log m, m^2) + I_0 e^2 m \log m + K_0 e^2 m \]

\[ M_{\pi^0}^2 = 2mB_0 + O(m^2 \log m, m^2) + I_\pm e^2 m \log m + K_\pm e^2 m \]

**Dashen’s theorem**:
The difference of squared pion mass is independent of quark mass up to $O(e^2 m)$,

\[ \Delta M_{\pi}^2 \equiv M_{\pi^\pm}^2 - M_{\pi^0}^2 = 2e^2 C f_0^2 + (I_\pm - I_0) e^2 m \log m + (K_\pm - K_0) e^2 m \]

$C, K_\pm, K_0$ is a new low energy constant. $I_\pm, I_0$ is known in terms of them.
QCD+QED lattice simulation

- In 1996, Duncan, Eichten, Thacker carried out SU(3)×U(1) simulation to do the EM splittings for the hadron spectroscopy using quenched Wilson fermion on $a^{-1} \sim 1.15$ GeV, $12^3 \times 24$ lattice. [Duncan, Eichten, Thacker PRL76(96) 3894, PLB409(97) 387]

- Using $N_F = 2 + 1$ Dynamical DWF ensemble (RBC/UKQCD) would have benefits of chiral symmetry, such as better scaling and smaller quenching errors.

- Especially smaller systematic errors due to the the quark massless limits, 
  $m_f \rightarrow -m_{res}(Q_i)$, has smaller $Q_i$ dependence than that of Wilson fermion, $\kappa \rightarrow \kappa_c(Q_i)$ (PCAC).

- Generate Coulomb gauge fixed (quenched) non-compact U(1) gauge action with $\beta_{QED} = 1$. $U^{EM}_\mu = \exp[-iA_{em\mu}(x)]$.

- Quark propagator, $S_{qi}(x)$ with EM charge $Q_i = q_i e$ with Coulomb gauge fixed wall source

\[
D[(U^{EM}_\mu)^Q_i \times U^{SU(3)}_\mu] S_{qi}(x) = b_{src}, \quad (i = \text{up, down})
\]

\[
q_{\text{up}} = 2/3, \quad q_{\text{down}} = -1/3
\]
• non-compact $U(1)$ gauge is generated by using Fast Fourier Transformation (FFT). Coulomb gauge $\partial_j A_{em,j}(x) = 0, \tilde{A}_{em,\mu}(p_0,0) = 0$ with eliminating zero modes. ($N_F = 2 + 1$: Feynman gauge)

• static lepton potential on $16^3 \times 32$ lattice ($\beta_{QED} = 100$, 4,000 confs) vs lattice Coulomb potential.

• L=16 has significant finite volume effect for $ra > 6 \sim 1.5r_0 \sim 0.75$ fm. It would be worth considering for generation of U(1) on a larger lattice and cutting it off.
ChPT+EM at NLO

- Double expansion of \( M_{PS}^2(m_1, q_1; m_3, q_3) \) in \( \mathcal{O}(\alpha), \mathcal{O}(m_q) \).

**QCD LO:**
\[
M_{PS}^2 = \chi_{13} = B_0(m_1 + m_3)
\]

**QCD NLO:**
\[
(1/F_0^2 \times)
\]

\[
(2L_6 - L_4)\chi_{13}^2 + (2L_5 - L_8)\chi_{13}\bar{\chi}_1 + \chi_{13} \sum_{I=1,3,\pi,\eta} R_{I\chi_I} \log(\chi_I/\Lambda_\chi^2),
\]

**QED LO:**  (Dashen’s term)
\[
2C \frac{F_0^2}{q_1 - q_3}^2
\]

**QED NLO:**  (\( \bar{Q}_2 = \sum q_{sea-i}^2 \), no \( \bar{Q}_1 \) in SU(3)\( _{NF} \))
\[
- Y_1 \bar{Q}_2 \chi_{13} + Y_2(q_1^2 \chi_1 + q_3^2 \chi_3) + Y_3 q_{13}^2 \chi_{13} - Y_4 q_1 q_3 \chi_{13} + Y_5 q_{13}^2 \bar{\chi}_1
\]
\[
+ \chi_{13} \log(\chi_{13}/\Lambda_\chi^2) q_{13}^2 + \bar{B}(\chi_\gamma, \chi_{13}, \chi_{13}) q_{13}^2 \chi_{13} - \bar{B}_1(\chi_\gamma, \chi_{13}, \chi_{13}) q_{13}^2 \chi_{13} + \cdots
\]

- QED LO adds mass to \( \pi^\pm \) at \( m_q = 0 \), QED NLO changes slope, \( B_0 \), in \( m_q \).

- Partially quenched formula \( (m_{sea} \neq m_{val}) \) SU(3)\( _{NF} \) [Bijnens Danielsson, PRD75 (07)]

SU(2)\( _{NF} \)+heavy Kaon+FiniteV  [Hayakawa Uno, PTP 120(08) 413]  [RBC/UKQCD; P.Boyle’s talk]  (also [C. Haefeli, M. A. Ivanov and M. Schmid, EPJ C53(08)549] )
• SU(3) PQChPT fit.

\[ N_F = 2 + 1 \] QCD+QED simulations

- \( \alpha^{-1} \sim 1.8 \) GeV from \( \Omega^- \) baryon mass (no log in NLO).
- Four degenerate up/down quark masses in the simulation:
  \( \sim 25, 40, 70, 100 \) MeV.
- One strange quark point
  \( \sim 10\% \) heavier than the physical.
- Two volumes:
  \((1.8 \text{ fm})^3 \) and \((2.7 \text{ fm})^3\)

- Determine 3 QCD LEC + 5 QED LEC (also 3 QCD LEC for \( f_\pi \))
- In total 234 charge, quark mass combinations are measured.
  \[ M_{PS}(m_1, q_1; m_2, q_2; m_l) \]
SU(3)+EM ChPT LEC (preliminary)

[R. Zhou]  [Bijnens Danielsson, PRD75 (07)]

• By fitting charge splitting

\[ \delta M^2 = M_{PS}^2(m_1, q_1; m_2, q_2; m_l) - M_{PS}^2(m_1, 0; m_2, 0; m_l) \]

by SU(3) ChPT+EM formula at NLO, 3 QCD LECs (1 LO + 2 NLO), 5 QED LECs (1 LO + 4 NLO) are determined.

• Requiring \( m_1, m_3, m_l \leq 40 \text{ MeV} (70 \text{ MeV}) \), 48 (120) partially quenched data for \( M_{PS}(m_1, q_1; m_2, q_2; m_l) \) are used in the fit (to see NNLO effects).

• Finite volume effects are observed by repeating the fit on \((1.8 \text{ fm})^3 \) and \((2.7 \text{ fm})^3 \).
SU(2)-heavy Kaon+EM ChPT Fit (preliminary)

[S.Uno] [Hayakawa Uno, PTP 120(08) 413]

• Treating Kaon as heavy particle (no chiral log from $\eta$).

• Finite volume analysis is done.

• Ultimately should give our main quote.

• EM splitting NLO/LO is still large ($\sim 50\%$ at $m_q = 40$ MeV) for Pion but small ($\sim 10\%$ at $m_q = 70$ MeV) for Kaon.
Quark mass determination

- Using the LECs, $B_0, F_0, L_i, C_0, Y_i$, from the fit, we could determine the quark masses $m_{\text{up}}, m_{\text{dwn}}, m_{\text{str}}$ by the solving equations [PDG08]:

$$M_{PS}(m_{\text{up}}, 2/3, m_{\text{dwn}}, -1/3) = 139.57018(35)\text{MeV}$$

$$M_{PS}(m_{\text{up}}, 2/3, m_{\text{str}}, -1/3) = 493.673(14)\text{MeV}$$

$$M_{PS}(m_{\text{dwn}}, -1/3, m_{\text{str}}, -1/3) = 497.614(24)\text{MeV}$$

- $(m_{\text{up}} - m_{\text{dwn}})$ is mainly determined by Kaon charge splittings,

$$M_{K^\pm}^2 - M_{K^0}^2 = B_0(m_{\text{up}} - m_{\text{dwn}}) + \frac{2C}{F_0^2}(q_1 - q_3)^2 + \text{NLO}$$

- $\pi^0$ mass is not used for now (disconnected quark loops).

- The term proportional to sea quark charge, $-Y_1 \bar{Q}_2 \chi_{13}$, is omitted. We will estimate the systematics by varying $Y_1$. 
Quark mass results *(Preliminary)*

[R. Zhou, S.Uno]

- $\overline{MS}$ at 2 GeV. Non-perturbative technique for the mass renormalization constant is used. [RBC/UKQCD, PRD78(08)054510; P.Boyle’s talk]

- Quark mass have small finite size volume effects. $SU(3)_{NF}$ and $SU(2)_{NF}$ in infinite volume.

- Uncertainties in QED LEC have small effect to quark mass. ($\pi^0$ is excluded)

- **Statistical error only.** For now, we didn’t use $F_{PS}$ in fitting LEC, and inflate error by statistical uncertainties of $a^{-1}$ and $f_0$. Our $SU(2)$ $m_s$ is smaller than $m_s = 107.3(4.4)$ MeV [RBC/UKQCD PRD78(08) 114509 ] due to this difference in two analyzes.

<table>
<thead>
<tr>
<th>lat</th>
<th>$m_q$ range</th>
<th>$m_u$</th>
<th>$m_d$</th>
<th>$m_s$</th>
<th>$m_u/m_d$</th>
<th>$m_s/m_{ud}$</th>
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<tr>
<td>$(2.7 \text{ fm})^3 SU(3)_\infty$</td>
<td>$\leq 40$ MeV</td>
<td>2.48(18)</td>
<td>4.77(30)</td>
<td>95(7)</td>
<td>0.52(3)</td>
<td>26.3(6)</td>
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<tr>
<td>$(2.7 \text{ fm})^3$</td>
<td>$\leq 70$ MeV</td>
<td>2.50(18)</td>
<td>4.81(30)</td>
<td>95(8)</td>
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<td>26.1(6)</td>
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<tr>
<td>$(1.8 \text{ fm})^3$</td>
<td>$\leq 70$ MeV</td>
<td>2.64(19)</td>
<td>4.81(32)</td>
<td>95(9)</td>
<td>0.55(4)</td>
<td>25.5(8)</td>
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<tr>
<td>$(2.7 \text{ fm})^3 SU(2)_\infty$</td>
<td>$\leq 70$ MeV</td>
<td>2.24(16)</td>
<td>4.62(24)</td>
<td>101(5)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Quark masses

red \( N_F = 2 + 1 \) DWF
blue \( N_F = 2 \) DWF (2.7 fm)³

(Only statistical errors are shown).
Components of Kaon masses splittings

• Reason why the iso doublet, \((K^+, K^0)\), has the mass splitting

\[ M_{K^\pm} - M_{K^0} = -3.937(29) \text{ MeV} \]  [PDG08]

\( \triangleright (m_{dwn} - m_{up}) : \text{makes } M_{K^+} - M_{K^0} \text{ negative.} \)
\( \triangleright (q_u - q_d) : \text{makes } M_{K^+} - M_{K^0} \text{ positive.} \)

• Using the determined quark masses and SU(3) LEC, we could isolate (to \(O((m_{up} - m_{dwn})\alpha)\)) each of contributions,

\[
M_{PS}^2(m_{up}, 2/3, m_{str}, -1/3) - M_{PS}^2(m_{dwn}, -1/3, m_{str}, -1/3) \\
\approx M_{PS}^2(m_{up}, 0, m_{str}, 0) - M_{PS}^2(m_{dwn}, 0, m_{str}, 0) \quad [\Delta M(m_{up} - m_{dwn})] \\
+ M_{PS}^2(\bar{m}_{ud}, 2/3, \bar{m}_{ud}, -1/3) - M_{PS}^2(\bar{m}_{ud}, -1/3, m_{str}, -1/3) \quad [\Delta M(q_u - q_d)]
\]

\( \triangleright \Delta M(m_{up} - m_{dwn}) = -5.7(1) \text{ MeV} \quad [145\% \text{ in } \Delta M^2(m_{up} - m_{dwn})] \)
\( \triangleright \Delta M(q_u - q_d) = 1.8(1) \text{ MeV} \quad [-45\% \text{ in } \Delta M^2(q_u - q_d)] \)

Also SU(2) ChPT, \(\Delta M(m_{up} - m_{dwn}) = -5.3(7) \text{ MeV} \) and \(\Delta M(q_u - q_d) = 1.4(7) \text{ MeV.} \)

• Similar analysis for \(\pi\) is possible, but facing a difficulty of isolating sea strange quark terms. SU(2) analysis gives a reasonable value.
Nucleon mass splitting in $N_F = 2, 2 + 1$ (Preliminary)

[R.Zhou, T.Doi]

$(q_u - q_d)$ effect

\[
\begin{align*}
\text{Cottingham formula} & \\
\text{Nf}=2 & (1.9 \text{ fm})^3 \\
\text{Nf}=2+1 & (1.8 \text{ fm})^3 \\
\text{Nf}=2+1 & (2.7 \text{ fm})^3
\end{align*}
\]

$(m_{u} - m_{d})$ effect

\[\sim 2 \text{ MeV at } m_{u} - m_{d}\]

- Only EM effect, $m_u = m_d$ case, are shown. c.f. [Gasser Leutwyler, PR87(82)77]

\[
M_N - M_P|_{\text{EM}} = -0.76(30) \text{ MeV}
\]

\[
M_N - M_P|_{\text{quark mass}} = 2.05(30) \text{ MeV}
\]
Systematic errors

- Chiral extrapolation: $m_q \leq 40$ or $70$ MeV.

- QCD’s $Z_m$: $\Lambda_{QCD} = 250 - 300$ MeV, $\mathcal{O}(\alpha) \sim 1\%$.

- $\pi^0$: disconnected loops ( $\eta'$ from DWF [K. Hashimoto TI PTP (08) ] )

- Quenched QED $\mathcal{O}(\alpha\alpha_S)$: ChPT and a clever combinations of masses [Bijnens Danielsson, PRD75 (07) 014505 ]

- One lattice spacing results, $\mathcal{O}(a^2)$.

- Finite Size Effect from vector-saturation model: $\Delta_{\pi,EM} = m_{\pi^+}^2 - m_{\pi^0}^2$, to be

$$\Delta_{\pi,EM}(L) = \frac{3\alpha}{4\pi} \frac{1}{a^2} \frac{2^4 \cdot \pi^2}{N} \sum_{q \in \tilde{\Gamma}'} \frac{(am_{\rho})^2(am_{A})^2}{\tilde{q}^2 (\tilde{q}^2 + (am_{\rho})^2) (\tilde{q}^2 + (am_{A})^2)},$$

$$\frac{\Delta_{\pi,EM}(\infty)}{\Delta_{\pi,EM}(L \approx 1.9 \text{ fm})} = 1.10.$$

Generally quark masses are stable against $\Delta_{\pi,EM} \sim 10\%$, Finite volume for P-N case may be larger [1/3 closer by $(1.8 \text{ fm})^3 \rightarrow (2.7 \text{ fm})^3$.]
Effect of the residual chiral symmetry breaking’s in $N_F = 2 + 1$ QCD+QED simulations

- $\delta m^2 = M_{PS0}^2(e \neq 0) - M_{PS0}^2(e = 0)$, $L_s = 16$ and 32 consistent with PCAC.
Other works of isospin breaking effects on lattice

- [MILC Collaboration (S. Basak et al.) Lattice08 arXiv:0812.4486] EM spectrum using staggered ensemble to get the breaking of Dashen’s theorem

\[ \Delta M_D^2 = \left( M_{K\pm}^2 - M_{K0}^2 \right)_{\text{em}} - \left( M_{\pi\pm}^2 - M_{\pi0}^2 \right)_{\text{em}} \]

- [McNeile, Michael, Urbach (ETMC) PLB674(09) 286] \(\rho - \omega\) mass splitting using twisted Wilson fermion. Discussed \(\rho - \omega\) mixing from \(m_{\text{up}} - m_d\). Measure disconnected quark loop correlation.

- [JLQCD PRL 101(08) 242001, PRD79(09)] Calculate \(\Pi_V - \Pi_A\), derive the EM contribution to the pion’s charge splittings in quark massless limit and the S-parameter using overlap fermion.
Summary and Future perspective

- **DWF QCD** is now a practical tool for the accurate determination of important quantities for QCD and SM. Continuum-like chiral behavior and renormalization allow us to compute indispensable SM parameters, Hadronic matrix elements and form factor computation etc. that relate experiments and theory [P.Boyle, N.Christ’s talk].

- **Isospin breaking effects** are interesting and inevitable for precise understanding of hadron physics, which could now be addressed by QCD+QED simulations from the first principle: quark masses, \( m_{up} \simeq 0 \), \( m_N - m_P > 0 \), ...

Future plans

- Systematical errors are being estimated.

- Analysis on the finer lattice, \( a \sim 0.08 \) fm or larger volume. [AuxDet N.Christ’s talk]

- EM splittings using the direct calculation of the QED diagrams [JLQCD OPE].

- Dynamical QED effects by reweighting

- \( \mathcal{O}(\alpha) \) contribution to \( g_\mu - 2 \) (pure QED). \( \mathcal{O}(\alpha^3) \) contribution (light-by-light) to \( g_\mu - 2 \). Chiral magnetic effect in QGP. [T.Blum]