Precision SM calculations and theoretical interests beyond the SM in $K_{12}$ & $K_{13}$ decays

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Introduction and Motivation

• Two ways for testing the Standard Model and finding New Physics
  – Direct searches of heavier particles (Higgs bosons, SUSY particles, Z’,W’…): by Collider physics (Tevatron, LHC…)
  – Indirect searches in Flavour Physics by precision physics: measuring Low Energy observables  
    Indication of $\Lambda$, the New Physics scale and sensitive to effects of the underlying theory and particles at higher energy…

Decoupling scenario:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \sum_{D>4} \frac{1}{\Lambda^{D-4}} \mathcal{O}_D$$

with $D$: mass dimension.

SM degrees of freedom  New Physics

• Studying $K_{l3}$ & $K_{l2}$ decays  indirect searches of New Physics, several high-precision tests possible.
Outline

$K_{l3}$ decays and $K_{l2}$ decays → stringent tests of the SM and New Physics probe

1. $K_{l3}$ decays, extraction of $V_{us}$
   - Theoretical inputs: EW, EM and isospin breaking corrections
   - Form factor shapes → determination of the phase space integrals IKs
   - $f_+(0)$ calculations

2. $V_{us}$ and the CKM unitarity test using $K_{l3}$ and $K_{l2}$ decays

3. Test of lepton universality with $K_{l3}$ decays

4. Test of the SM EW couplings via the CT theorem using $K_{l2}$ & $K_{l3}$ decays and probe of new physics

5. Lepton Flavour Universality test with $K_{l2}$ decays
1. $K_{l3}$ decays, extraction of $V_{us}$

- $K_{l3}$ decays $K \rightarrow \pi l \nu_l$

- Decay rate formula for $K_{l3}$

\[
\Gamma_{K^{+/0}l3} = \frac{B r_{K^{+/0}l3}}{\tau_{K^{+/0}}} = \frac{C^2 G_F^2 m_{K^{+/0}}^5}{192 \pi^3} S_{EW} \left( 1 + 2 \Delta_{EM}^{K^{+/0}} + 2 \Delta_{SU(2)} \right) \left| f_+^{K^{+/0}}(0) V_{us} \right|^2 I_{K^{+/0}}^l
\]

$\frac{1}{2}$ for $K^+$, 1 for $K^0$

- Experimental inputs: $\rightarrow$ $B r_{K^{+/0}l3}$, $\tau_{K^{+/0}}$, $K_{l3}$ branching ratios, Kaon life time, with good treatment of radiative corrections

$\rightarrow$ $I_{K^{+/0}}^l$ Phase space integrals, need form factor shapes extracted from Dalitz plot, from NA48, KTeV, KLOE and ISTRA+

- Theoretical inputs: $\rightarrow$ $S_{EW}$ Short distance EW corrections

$\rightarrow$ $\Delta_{EM}^{K^{+/0}}$ Long distance EM corrections

$\rightarrow$ $\Delta_{SU(2)}$ Isospin breaking corrections
• For the experimental inputs, see the Flavianet review, talk by M. Palutan

• Theoretical inputs:

→ **$S_{EW}$** Short distance EW corrections, universal factor

\[ S_{EW} = 1 + \frac{2\alpha}{\pi} \left( 1 + \frac{\alpha_s}{4\pi} \right) \log \frac{M_Z}{M_\rho} + O \left( \frac{\alpha \alpha_s}{\pi^2} \right) \]

\[ S_{EW} = 1.0232(5) \quad \text{[Sirlin’82]} \]

→ **$\Delta_{EM}^{K+/0}$** Long distance EM corrections
  
  - First analyses of EM corrections for $K_{l3}$ by Ginsberg’67, ’70, then by Andre’04: hard UV cutoff in the loops (used in the KTeV analysis).
  
  - Description of the EM interaction within an effective theory
    
    ChPT with photons [Urech’95, Neufeld & Rupertsberger’95],
    EM LECs $K_i$ [Ananthanaryan & Moussallam’04]
    
    ChPT with photons and Leptons [Knecht et al’00]
    Additional LECs $X_i$ [Descotes-Genon & Moussallam’05]
Long distance EM corrections: Calculations at $O(p^2 e^2)$

$K_{e3}$ [Cirigliano et al'01, Cirigliano, Neufeld, Pichl'04, Cirigliano, Giannotti, Neufeld'08]

$K_{\mu3}$ [Cirigliano, Giannotti, Neufeld'08]  New!

In this recent analysis, fully inclusive prescription of real photon emission, update of structure-dependent EM contributions, take the most recent estimates of the LECs

+ Errors: estimates of higher order corrections

Results: (NB: A part depends on the IK values)

<table>
<thead>
<tr>
<th>$\Delta_{EM}^{K+/0}$ (%)</th>
<th>$K_{e3}^0$</th>
<th>$K_{e3}^\pm$</th>
<th>$K_{\mu3}^0$</th>
<th>$K_{\mu3}^\pm$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CGN'08</td>
<td>0.50 ± 0.11</td>
<td>0.05 ± 0.13</td>
<td>0.70 ± 0.11</td>
<td>0.008 ± 0.13</td>
</tr>
<tr>
<td>Andre’04</td>
<td>0.65 ± 0.15</td>
<td>-</td>
<td>0.95 ± 0.15</td>
<td>-</td>
</tr>
</tbody>
</table>

Larger effects in $K^0$ due to Coulomb final state interactions

Reliable calculations to use in the experimental analyses + in the same analysis estimates of the EM corrections for the decay distribution, crucial role!
Isospin breaking corrections, studied up to $O(p^4)$ in ChPT

$\Delta_{SU(2)} = 0$ for $K^0$, Corrections only hold for $K^+$

- Leading contribution ($O(p^2)$), due to $\pi^0$-$\eta$ mixing in the final state
  - small denominators $O\left(\frac{(m_d - m_u)}{m_s}\right)$
  - $\Delta_{SU(2)} = \frac{3}{4} \frac{1}{R}$ depend on the quark mass ratio $R = \frac{m_s - \hat{m}}{m_d - m_u} \frac{m_u + m_d}{2}$

- At NLO ($O(p^4)$),
  \[
  \Delta_{SU(2)} = \frac{3}{4} \frac{1}{R} \left( 1 + \chi_{p^4} + \frac{4}{3} \frac{M_K^2}{M_\pi^2} - \frac{M_\pi^2}{M_\eta^2} \Delta_M + O\left(m_q^2\right) \right)
  \]
  - Chiral correction $\chi_{p^4} = 0.219$
  - related to the ratio of quark masses $\frac{M_K^2}{M_\pi^2} = \frac{m_s + \hat{m}}{m_u + m_d} \left(1 + \Delta_M + O\left(m_q^2\right)\right)$

- Use quark mass ratios as input or determination of $\Delta_{SU(2)} = \frac{f_{K^+}^0(0)}{f_{K^0}^+(0)} - 1$ from measurements $\Rightarrow R$
  - Hard to pin down precisely!

See talk by V. Cirigliano at Kaon'07
• Theoretical prediction for $\Delta_{SU(2)}$:

Use $Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} = R \frac{m_s / \hat{m} + 1}{2}$, can be extracted from:

- $\eta \to \pi^+ \pi^- \pi^0$ decays
  
  $Q = 22.7 \pm 0.8$ \cite{Anisovitch & Leutwyler'95, Leutwyler'96}
  $Q = 22.4 \pm 0.9$ \cite{Kambor, Wiesendanger, Wyler'95}
  $Q = 23.2$ \cite{Bijnens & Ghorbani'07}

  $R = 40.8 \pm 3.2$
  $\Delta_{SU(2)} = 2.36(22)\%$

- From kaon mass splitting

  $Q^2 = \frac{M_K^2}{M_{\pi}^2} - \frac{M_{K^0}^2 - M_K^2}{M_{K^0}^2 - M_{K^+}^2}$

  $Q = 20.7 \pm 1.2$ \cite{Kastner & Neufeld'08}

  Based on Ananthanarayan & Moussallam'04
  Large deviation of the Dashen’s limit

  $R = 33.5 \pm 4.3$
  $\Delta_{SU(2)} = 2.9(4)\%$

Slight disagreement between the 2 approaches ($\sim 1.2\sigma$) analysis based on new KLOE data for $\eta \to \pi^+ \pi^- \pi^0$ decays underway.
• Determination of $\Delta_{SU(2)}$ from $K_{i3}$ data:

$$\Delta_{SU(2)} = \frac{f_{+}^{K^{+}}(0)}{f_{+}^{K^{0}}(0)} - 1$$

$$\Delta_{SU(2)} = \frac{\Gamma_{K^{+}l3} I_{K^{0}}^l}{\Gamma_{K^{+}l3} I_{K^{+}}^l} \left( \frac{M_{K^{0}}}{M_{K^{+}}} \right)^5 - \frac{1}{2} - \left[ \Delta_{EM}^{K^{+}l} - \Delta_{EM}^{K^{0}l} \right]$$

Good precision from the data + from the EM correction estimates

phenomenological estimate from $K_{i3}$ data with a very good precision possible

$$\Delta_{SU(2)} = 2.7(4)\%$$  [FIT Flavianet Kaon WG, see talk by M.Palutan]

Very good agreement with the recent result from Kastner & Neufeld’08

The tension between the phenomenological estimate and the theoretical estimate existing at Kaon’07 (see talk by V. Cirigliano) has disappeared with the new experimental value (1σ lower) and the recent estimate of R!

A small value of R ($R \sim 33$) seems to be favoured.

Has to be confirmed by the analysis from $\eta \rightarrow \pi^+ \pi^- \pi^0$ using the new KLOE data, underways.
Determination of the phase space integrals and the form factor shapes

- Decay rate formula for $K_{l3}$

$$\Gamma_{K^{+/0}_{l3}} = \frac{B r_{K^{+/0}_{l3}}}{\tau_{K^{+/0}}} = \frac{C_k^2 G_F^2 m_{K^{+/0}}^5}{192 \pi^3} S_{EW} \left(1 + 2 \Delta_{EM}^{K^{+/0}} + 2 \Delta_{SU(2)}^{K^{+/0}}\right) \left| f_{+}^{K^{+/0}}(0) V_{us} \right|^2 I^{l}_{K^{+/0}}$$

To extract $\left| f_{+}(0) V_{us} \right|$, one needs to calculate the phase space integrals $I_K$ and determine $f_{+}(0)$.

- The hadronic matrix element of $K_{l3}$ decays

$$\langle \pi(p_\pi) | \bar{s} \gamma_\mu u | K(p_K) \rangle = f_{+}(t)(p_K + p_\pi)_\mu + f_{-}(t)(p_K - p_\pi)_\mu$$

$\Rightarrow f_{+}(t), f_{-}(t)$: form factors

$\Rightarrow t = q^2 = (p_\mu + p_{v_\mu})^2 = (p_K - p_\pi)^2$

$$f_0(t) = f_{+}(t) + \frac{t}{m_K^2 - m_\pi^2} f_{-}(t)$$

NB:

$$f_{+}(0) = f_0(0)$$

- Impossible to measure $f_{+}(0)$ from experiment, has to be determined from theory

- Determination of $\bar{f}_{+0}(t) = \frac{f_{+0}(t)}{f_{+}(0)}$ by a fit to the measured $K_{l3}$ decay distribution
How to measure the form factor shapes?

- Data available from KTeV, NA48 and KLOE for $K^0$ and from ISTRA+, NA48 and KLOE for $K^+$.
- Necessity to parametrize the 2 form factors $\bar{f}'(t)$ and $\bar{f}_0(t)$ to fit the measured distributions.
- Different parametrizations available, 2 classes of parametrizations:
  - $1^{\text{st}}$ class: parametrizations based on mathematical rigorous expansion, the slope and the curvature are free parameters:
    - Taylor expansion
      \[
      \bar{f}'_{+,0}(t) = 1 + \lambda'_{+,0} \frac{t}{m_\pi^2} + \frac{1}{2} \lambda''_{+,0} \frac{t}{m_\pi^2} + \ldots
      \]
    - Z-parametrization, conformal mapping from $t$ to $z$ variable with $|z|<1$ improve the convergence of the series
      \[
      f_{+,s}(t) = f_{+,s}(t_0) \frac{\phi(t_0,t_0,Q^2)}{\phi(t,t_0,Q^2)} \sum_{k=0}^{n} a_k(t_0,Q^2) z(t,t_0)^k
      \]  
      [Hill’06]

Theoretical error can be estimated: for a specific choice of $\phi$, $\sum_{k=0}^{n} a_k^2$ bounded $\implies$ use of some high-energy inputs ($\tau$ data ...).

Work on the scalar FF by [Bourrely & Caprini’05], [Abbas, Ananthanaryan’09] and on the vector FF by [Hill’06].
→ 2\textsuperscript{nd} class: parametrizations which by using physical inputs impose specific relations between the slope and the curvature reduce the correlations, only one parameter fit.

- Pole parametrization, the dominance of a resonance is assumed

\[ f_{+,0}(t) = \frac{m_{V,S}^2}{m_{V,S}^2 - t} \]

\( m_{V,S} \) is the parameter of the fit

- Dispersive parametrization: use of the low energy K\pi scattering data and presence of resonances to contrain by dispersion relations the higher order terms of the expansion. Analysis from [Jamin, Oller, Pich’04], [Bernard, Oertel, E.P, Stern’06], for the scalar form factor and from [Moussallam’07], [Jamin, Pich & Portoles’08], [Boito, Escribano & Jamin’08] for the vector form factor using \( \tau \) data.
• Requirements in the measurements of the form factor shapes from the $K_{l3}$ data
  
  – Try to measure the form factor shapes from the data with the best accuracy for determination of the IKs.

  – Measurement of $f_0(\Delta_{K\pi}) \equiv C$ to test the Standard Model via the CT theorem

\[
C = \bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi f^{k_+}(0)} + \Delta_{CT}
\]

\[
\Delta_{K\pi} = m_K^2 - m_\pi^2
\]

Can be determined very precisely assuming the SM EW couplings from BRs measurements + ChPT estimate for $\Delta_{CT}$

A measurement of $C$ allows for a test of the SM EW couplings and new physics effects.

  – The slope and curvature of the scalar form factor are of special interest to allow for a matching with the 2 loop ChPT calculations.
• Experimental constraints: if one uses a parametrization from 1st class, for example a Taylor expansion,
  – Only two parameters measurable for the vector form factor, \( \lambda'_+ \) and \( \lambda''_+ \)
  – Only one parameter accessible for the scalar form factor \( \lambda'_0 \)
    ➔ see Flavianet Kaon WG result (talk by M. Palutan).
  – The correlations are strong,

\[
\begin{array}{ccccc}
\lambda'_0 & 1 & -0.9996 & -0.97 & 0.91 \\
\lambda''_0 & 1 & 0.98 & -0.92 & \\
\lambda'_+ & 1 & -0.98 & \\
\lambda''_+ & 1 & &
\end{array}
\]

[Franzini, Kaon’07]

• Necessity to use a second class parametrization which reduces the correlation, only one parameter is fitted.
  – For the vector form factor ➔ pole parametrization with dominance of the \( K^*(892) \) in good agreement with the data.
  – For the scalar form factor, not a such obvious dominance ➔ necessity to use a dispersive parametrization to improve the extraction of the ff parameters and to reach the CT point.
• Impossible to use the linear parametrization to extrapolate with a good precision up to the CT point

\[ t_0 = (m_K - m_\pi)^2 \]

• Dispersive parametrization for the scalar and the vector FFs, for the scalar:

\[ \bar{f}_0(t) = \exp \left[ \frac{t}{\Delta_{K\pi}} \left( \ln C - G(t) \right) \right] \]

with

\[ G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi} \int_0^\infty \frac{ds}{s} \frac{\phi(s)}{(s - \Delta_{K\pi})(s - t)} \]

- \( \phi(t) \) phase of the form factor

  \( t < \Lambda : \phi_0(t) = \phi_{K\pi}(t) = \delta^{s,\frac{1}{2}}_{\pi,K}(t) \pm \Delta \delta^{s,\frac{1}{2}}_{\pi,K}(t) \)

  \( t > \Lambda : \phi_0(t) = \phi_{as}(t) = \pi \pm \pi \) [Watson theorem]

- 2 subtractions \( \Rightarrow \) Rapid convergence of \( G(t) \)
• Dispersive parametrization for the scalar FF [Bernard, Oertel, E.P., Stern’06]

\[
\overline{f}_0(t) = \exp \left[ \frac{t}{\Delta_{K\pi}} \left( \ln C - G(t) \right) \right]
\]

with

\[
G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi} \int_0^\infty ds \frac{\phi_0(s)}{(s - \Delta_{K\pi})(s - t)}
\]

• Impose a relation between slope and curvature

\[
\lambda''_0 = \frac{\lambda_0^2}{\Delta_{K\pi}} - 2 \frac{m^4_\pi G'(0)}{\Delta_{K\pi}} > \lambda_0^2
\]

\[
\lambda'_0 = \frac{m^2_\pi}{\Delta_{K\pi}} \left( \ln C - G(0) \right)
\]

One parameter $\ln C$ to fit to determine the shape of $f_0(t)$

• Dispersive parametrization for the vector FF [Bernard, Oertel, E.P., Stern’09]

\[
\overline{f}_+(t) = \exp \left[ \frac{t}{m^2_\pi} \left( \Lambda_+ + H(t) \right) \right]
\]

with

\[
H(t) = \frac{m^2_\pi t}{\pi} \int_0^\infty ds \frac{\phi_+(s)}{(s - \Lambda_+)(s - t)}
\]

• Also one parameter to fit $\Lambda_+$ to determine the shape of $f_+(t)$

\[
\lambda'_+ = \Lambda_+ \quad \text{and} \quad \lambda''_+ = \lambda'_+^2 + 2m^2_\pi H'(0) > \lambda'_+^2
\]
Results of the dispersive analyses

- Since Kaon’07 in addition of NA48’07, dispersive analyses by KLOE’08 and KTeV + Bernard, Oertel, E.P., Stern’09 with old KTeV data (submitted to PRD)

<table>
<thead>
<tr>
<th>NA48</th>
<th>$\bar{K}_{\mu 3}$</th>
<th>KLOE</th>
<th>$K_{\epsilon 3}$ and $K_{\mu 3}$ combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_+ \times 10^3$</td>
<td>$23.3 \pm 0.9$</td>
<td>$\Lambda_+ \times 10^3$</td>
<td>$25.7 \pm 0.6$</td>
</tr>
<tr>
<td>ln $C'$</td>
<td>$0.1438 \pm 0.0138$</td>
<td>ln $C'$</td>
<td>$0.204 \pm 0.025$</td>
</tr>
<tr>
<td>$\rho(\Lambda_+, \ln C')$</td>
<td>-0.44</td>
<td>$\rho(\Lambda_+, \ln C')$</td>
<td>-0.27</td>
</tr>
<tr>
<td>$\chi^2$/dof</td>
<td>595/582</td>
<td>$\chi^2$/dof</td>
<td>2.6/3</td>
</tr>
</tbody>
</table>

| $\lambda'_+ \times 10^3$ | $23.33 \pm 0.9$ | $\lambda'_+ \times 10^3$ | $25.7 \pm 0.6$ |
| $\lambda''_+ \times 10^3$ | $1.3 \pm 0.1$   | $\lambda''_+ \times 10^3$ | $1.1 \pm 0.1$ |
| $\lambda'_0 \times 10^3$  | $8.9 \pm 1.2$   | $\lambda'_0 \times 10^3$     | $14.0 \pm 2.1$ |
| $\lambda''_0 \times 10^3$ | $0.50 \pm 0.05$ | $\lambda''_0 \times 10^3$   | $0.50 \pm 0.06$ |

<table>
<thead>
<tr>
<th>KTeV</th>
<th>$K_{\epsilon 3}$</th>
<th>$K_{\mu 3}$</th>
<th>$K_{\epsilon 3}$ and $K_{\mu 3}$ combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Lambda_+ \times 10^3$</td>
<td>$25.17 \pm 0.58$</td>
<td>$24.57 \pm 1.10$</td>
<td>$25.09 \pm 0.55$</td>
</tr>
<tr>
<td>ln $C'$</td>
<td>-</td>
<td>$0.1947 \pm 0.0140$</td>
<td>$0.1915 \pm 0.0122$</td>
</tr>
<tr>
<td>$\rho(\Lambda_+, \ln C')$</td>
<td>-</td>
<td>-0.557</td>
<td>-0.269</td>
</tr>
<tr>
<td>$\chi^2$/dof</td>
<td>66.6/65</td>
<td>193/236</td>
<td>0.48/2</td>
</tr>
</tbody>
</table>

| $\lambda'_+ \times 10^3$ | $25.17 \pm 0.58$ | $24.57 \pm 1.10$     | $25.09 \pm 0.55$                          |
| $\lambda''_+ \times 10^3$ | $1.22 \pm 0.10$  | $1.19 \pm 0.11$      | $1.21 \pm 0.10$                          |
| $\lambda'_0 \times 10^3$  | -                 | $13.22 \pm 1.20$    | $12.95 \pm 1.04$                          |
| $\lambda''_0 \times 10^3$ | -                 | $0.59 \pm 0.05$     | $0.58 \pm 0.05$                          |
Vector FF results

- A good agreement between the Flavianet Kaon WG average using a quadratic parametrisation and the dispersive results.
- KLOE and KTeV in perfect agreement, NA48 $1\sigma$ away.
- The dispersive results twice more precise than the average!

has to be used by the Flavianet Kaon WG for the IKs calculations.
Vector FF results

- The precision reached for the curvature using a dispersive parametrisation is much more precise than the one using the quadratic parametrization by a factor 4!
- Good agreement between the dispersive results.
  - The dispersive results have really to be used by the Flavianet Kaon WG for the IKs calculations!
The extraction of the vector form factor from $K_{l3}$ can be tested from $\tau \to K \pi \nu_\tau$ decays

- Tau decay width

\[
\frac{d\Gamma_{K\pi}(t)}{d\sqrt{t}} = \frac{|V_{us}f_+(0)|^2 G_F^2 M_\tau^3}{128\pi^3} S_{EW} q_{K\pi}(t) \left(1 - \frac{t}{M_\tau^2}\right)^2 \times \\
\left[ \left(1 + \frac{2t}{M_\tau^2}\right) \frac{4q_{K\pi}(t)}{t} |f_+(t)|^2 + \frac{3(M_K^2 - M_\pi^2)^2}{t^2} |f_0(t)|^2 \right],
\]

- In the Tau energy range at much higher energy than $K_{l3}$ decays, vector contribution ($K^*(892)$) dominates \(\Rightarrow\) precise determination of $f_+(t)$

- Fit to Belle data using different representations for $f_+(t)$
  - A coupled-channel DR using the $K\pi$ scattering data \[\text{[Moussallam’08]}\]
  - A representation using RChPT, 2 resonances $K^*(892)$ and $K^*(1414)$ \[\text{[Jamin, Pich, Portoles’08]}\]
  - A three-time subtracted DR with a RChPT description of the phase \(\Rightarrow\) 2 resonances $K^*(892)$ and $K^*(1414)$ \[\text{[Boito, Escribano, Jamin’08]}\]
The extraction of the vector form factor from $K_{l3}$ can be tested from $\tau \rightarrow K\pi\nu_{\tau}$ decays

- Tau decay width

\[
\frac{d\Gamma_{K\pi}(t)}{d\sqrt{t}} = \frac{|V_{us} f_+(0)|^2 G_F^2 M_\tau^3}{128\pi^3} S_{EW} q_{K\pi}(t) \left(1 - \frac{t}{M_\tau^2}\right)^2 \times \\
\left[ \left(1 + \frac{2t}{M_\tau^2}\right) \frac{4q_{K\pi}^2(t)}{t} |f_+(t)|^2 + \frac{3(M_K^2 - M_\pi^2)^2}{t^2} |f_0(t)|^2 \right],
\]

[Boito, Escribano, Jamin’08]
Vector FF determination from $K_{l3}$ decays/$\tau \rightarrow K \pi \nu_\tau$ decays

$\lambda_+^*$ in units of $10^{-3}$

- $K_{l3}$ analyses
  - Flavianet Kaon WG 2008
  - NA48 2007
  - KLOE 2008
  - KTeV+BOPS 2009

- $\tau$ decay analyses
  - Jamin, Oller & Pich 2008
  - Boito et al 2009
  - Moussallam 2007
Vector FF determination from $K_{l3}$ decays/$\tau \to K\pi\nu\tau$ decays

The results are in very good agreement. Very precise measurements for $\lambda_+''$ can be reached from $\tau$ decays $\Rightarrow$ combined $K_{l3}$ & $\tau$ analyses are underway. This will allow to improve the precision + test of the dispersive relation between $\lambda_+'$ and $\lambda_+''$. 
The results are in agreement except for the NA48 one which disagrees with the others.

One can access with the DR to the curvature. The curvature is in this case very small (~$5 \times 10^{-4}$) but needed if one wants reach a high level of precision.
Extraction of $f_+(0) V_{us}$

- From $\Delta_{EM} + \Delta_{SU(2)} + IKs + \text{Experimental measurements}$

$$\left| f_+^{K^{\pm/0}}(0) V_{us} \right|^2 = \frac{Br_{K^{\pm/0}l3} / \tau_{K^{\pm/0}}}{C_K^2 G_F^2 m_{K^{\pm/0}}^5 \frac{S_{EW}}{192\pi^3} \left(1 + 2\Delta_{EM}^{K^{\pm/0}} + 2\Delta_{SU(2)}^K\right) I_{K^{\pm/0}}^l}$$

See Talk by M. Palutan

$IKs$ still used from the linear/quadratic fit
dispersive results have to be used!

The $K^l$ are more precise.

$K^+$ measurements underways

Average: $|V_{us}| f_+(0) = 0.21660(47)$
$\chi^2/\text{ndf} = 3.03/4 (55\%)$
Extraction of Vus : \( f_+(0) \)?

- **ChPT**:
  \[
  f_+(0) = 1 + f_2 + f_4 + \ldots
  \]
  \[
  f_2 = \mathcal{O}(m_s - \hat{m})^2
  \]
  SU(3) breaking \[\text{[Ademollo-Gatto theorem]}\]

  \[
  f_2 = -0.023
  \]
  \[
  \rightarrow \text{no contribution from the } O(p^4) \text{ Li’s}
  
  \rightarrow \text{NLO chiral logs fully determined in terms of } M_K, M_\pi \text{ anf } F_\pi
  
  \]

- **1\text{rst higher order estimate } (\mathcal{O}(p^6))**
  \[
  f_+(0) - 1 - f_2 = -0.016(8) \] by quark model

  \[
  f_+(0) = 0.961(8)
  \] \[\text{[Leutwyler&Roos’84]}\]

- **Analytic estimates at 2 loops in the isospin limit** \[\text{[Post-Schicher’02], [Bijnens &Talavera’03]}\]

  \[
  f_+(0) = 1 + \Delta(0) - \frac{8}{F_\pi^4} \left( C_{12}^r + C_{34}^r \right) \Delta_{K\pi}^2
  \]
  Only 2 \( O(p^6) \) LECs \( C_{12} \) and \( C_{34} \) appear

  In \( \Delta(0) \), no dependence on the \( L_i \) at \( p^4 \), only via \( p^6 \)

  \[
  \Delta(0) = -0.0080 \pm 0.0057 [loops] \pm 0.0028 [L_i^r]
  \] \[\text{[Bijnens &Talavera’03]}\]

  \[
  \rightarrow \text{To be updated with the new experimental inputs } (K_{14})
  \]
Extraction of Vus : \( f_+ (0) \)?

- Analytic estimates at 2 loops in the isospin limit [Post-Schicher’02], [Bijnens & Talavera’03]

\[
f_+ (0) = 1 + \Delta (0) - \frac{8}{F_\pi^4} (C_{12}^r + C_{34}^r) \Delta_{K\pi}^2
\]

Only 2 O(p\(^6\)) LECs \( C_{12} \) and \( C_{34} \) appear

In \( \Delta (0) \), no dependence on the \( L_i \) at \( p^4 \), only via \( p^6 \)

\[
\Delta (0) = -0.0080 \pm 0.0057 \text{[loops]} \pm 0.0028 \text{[} L_i \text{]} \quad [\text{Bijnens & Talavera’03}]
\]

Estimate of the two O(p\(^6\)) LECs \( C_{12} \) and \( C_{34} \)
- By resonance exchange estimates [Cirigliano et al’05], [Kastner & Neufeld’08]
- Matching with dispersive representation or parametrisation [Jamin et al’05], [Bernard, E.P’08]
  
  {
  \[\text{Possibility to use the scalar ff dispersive measurements}\]

- Estimate on the lattice, see Talks by P. Boyle, G. Colangelo, F. Mescia

Only one published result in \( N_f=2+1 \), more results are awaited!
The analytical results based on resonance model estimates for $C_{12}$ and $C_{34}$ give larger results for $f_+(0)$ than the lattice calculations.

Possibility from $f_0(t)$ dispersive measurements to test these estimates.
Matching of the 2 loop ChPT with the DR

\[ f_S(t) = f_+(0) + \Delta(t) + \frac{F_K/F_\pi - 1}{\Delta_{K\pi}} t + \frac{8}{F_\pi} \left( 2C_{12}^r + C_{34}^r \right) \left( m_K^2 + m_\pi^2 \right) t - \frac{8}{F_\pi} t^2 C_{12}^r \]

- Taking the derivative:
  \[ \lambda_0' f_+(0) = \frac{m_\pi^2}{\Delta_{K\pi}} \left( \frac{F_K}{F_\pi} - 1 \right) + \frac{8m_\pi^2 \Sigma_{K\pi}}{F_\pi^4} \left( 2C_{12}^r + C_{34}^r \right) + m_\pi^2 \Delta'(0) \]

- And derivate 2 times:
  \[ \lambda_0'' f_+(0) = -\frac{16m_\pi^4}{F_\pi^4} C_{12}^r + m_\pi^4 \Delta''(0) \quad (1) \]

- Combine with the two loop result for \( f_+(0) \)
  \[ f_+(0) = 1 + \Delta(0) - \frac{8}{F_\pi^4} \left( C_{12}^r + C_{34}^r \right) \Delta_{K\pi}^2 \quad (2) \]

- From (1)+(2)
  \[ 2C_{12}^r + C_{34}^r \]

- From DR
  \[ \lambda_0'' = \lambda_0'^2 - 2 \frac{m_\pi^4}{\Delta_{K\pi}} G'(0) \]
  \[ = \lambda_0'^2 + (4.16 \pm 0.50) \times 10^{-4} \]

\[ \lambda_0' = f \left( \frac{F_K}{F_\pi}, f_+(0) \right) \]

[Bijnens & Talavera’03]
Results

• We will present trends and not exact results: use of $\Delta(0)$ and $\Delta(t)$ from [Bijnens & Talavera] determined with $F_K/F_\pi = 1.22$ and $F_\pi = 92.4$ MeV

  Redo the fit varying $F_K/F_\pi$ and $F_\pi$.

• We vary $\Delta(0)$ in its error bars, give the largest uncertainty.
• For instance, take the published most recent and precise value for $F_K/F_\pi$ from lattice ($N_f=2+1$) \rightarrow In the future use the FLAG average

\[ \frac{F_K}{F_\pi} = 1.189 \pm 0.007 \]  

[HPQCD-UKQCD'07]

<table>
<thead>
<tr>
<th></th>
<th>$\lambda_0^\prime$</th>
<th>$f_+(0)$</th>
<th>$C_{12}$</th>
<th>$C_{34}$</th>
<th>$\Delta CT$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$10^{-3}$</td>
<td>$10^{-6}$</td>
<td>$10^{-6}$</td>
<td>$10^{-2}$</td>
<td></td>
</tr>
<tr>
<td>KLOE</td>
<td>14.0 ± 2.1</td>
<td>0.9700(218)</td>
<td>0.463(537)</td>
<td>3.387(4.226)</td>
<td>0.028(1.011)</td>
</tr>
<tr>
<td>KTeV</td>
<td>12.95 ± 1.04</td>
<td>0.9803(127)</td>
<td>0.720(251)</td>
<td>1.323(2.233)</td>
<td>-0.180(933)</td>
</tr>
<tr>
<td>NA48</td>
<td>8.88 ± 1.24</td>
<td>1.0212(149)</td>
<td>1.523(200)</td>
<td>-6.634(2.586)</td>
<td>-0.963(905)</td>
</tr>
</tbody>
</table>

• Uncertainties from $\Delta(0)$, $F_K/F_\pi$ and $\lambda_0^\prime$.
• Uncertainties on $f_+(0)$ between 1.5% and 2%, not competitive with the most recent lattice result (uncertainties of $\sim 0.5\%$).
• Limiting uncertainty from $\lambda_0^\prime$, average of the dispersive results.
• Uncertainties on $\Delta(0)$ and $\overline{\Delta}(t)$ should decrease with new fits.

Promising
2. $V_{us}$ and the CKM unitarity test using $K_{l3}$ and $K_{l2}$ decays (Flavianet Kaon WG)

- From $K_{l3}$ decays

$$\Gamma_{K^{+/0}_{l3}} = \frac{B r_{K^{+/0}_{l3}}}{\tau_{K^{+/0}}} = \frac{C_K^2 G_F^2 m_{K^{+/0}}^5}{192 \pi^3} S_{EW} \left(1 + 2 \Delta_{EM}^{K^{+/0}_{l3}} + 2 \Delta_{SU(2)}^{K^{+/0}_{l3}}\right) \left| f_+^{K^{+/0}_{l3}}(0) V_{us} \right|^2 I^{l}_{K^{+/0}}$$

$$f_+(0) |V_{us}| = 0.21660(47) \quad \Rightarrow \quad |V_{us}| = 0.2246(12)$$

- From $K_{l2}/\pi_{l2}$ decays [Marciano’04]

$$\Gamma_{K^{\pm}_{\mu2}(\gamma)} = \frac{M_{K}}{M_{\pi}} \left(1 - m_{\mu}^2 / M_{K}^2\right) \frac{\left|V_{us} F_{K}\right|^2}{\left|V_{ud} F_{\pi}\right|^2} \left(1 + \delta_{em}\right)$$

$$\frac{V_{us} F_{K}}{V_{ud} F_{\pi}} = 0.2760(6) \quad \Rightarrow \quad \frac{F_{K}}{F_{\pi}} = 1.189(7)$$

[HPQCD-UKQCD’07]

$$V_{us} = 0.2319(15)$$

- Test of the CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

$0^+ \rightarrow 0^+ \beta$ decays \hspace{1cm} $K_{l3}$ decays \hspace{1cm} Negligible (B decays)

E. Passemar

Kaon’09, Tsukuba
• Put everything in a fit with $|V_{ud}| = 0.97424(22)$ [Towner & Hardy’09]

\[ V_{ud} = 0.97424(22) \]
\[ V_{us} = 0.2252(9) \]
\[ \chi^2/\text{ndf} = 0.52/1 \ (47\%) \]

Unitarity test:

\[ 1 - |V_{us}|^2 - |V_{ud}|^2 = 0.00003(60) \]

Incredible precision!

Stringent test for New Physics models
Implications of CKM unitarity on New Physics models [Marciano, Kaon’07]

• CKM unitarity can be converted in a test of the universality of the gauge coupling $G_F$

\[
G_F^{\text{CKM}} = 1.16626(30) \times 10^{-5} \text{ GeV}^2
\]

[Flavianet Kaon WG average]

• More precise determination after $\mu$ decays $G_\mu = 1.166371(6) \times 10^{-5} \text{ GeV}^2$

[Mulan’07]

• A lot of NP effects absorbed in $G_\mu$ (Top bottom loop, Higgs loop, $W^*$, $WZ'$, box, SUSY loops, Technicolor, exotic $\mu$ decays)

A comparison of $G_\mu$ with other measurements allows to constrain new physics effects

→ No sign of SUSY in CKM, no sign of technicolor, constraint on $Z'$ boson mass from SO(10) GUT :

\[
G_\mu = G_{\text{CKM}} \left[ 1 - 0.007Q_{el} (Q_{\mu\ell} - Q_{dl}) \frac{2\ln\left(M_{Z'}/M_W\right)}{M_{Z'}^2/M_W^2 - 1} \right]
\]

\[Q_{el} = Q_{\mu\ell} = -3Q_{dl} = 1 \rightarrow M_{Z'} \geq 700 \text{ GeV}\]

→ Competitive with direct searches

No sign of SUSY in CKM, no sign of technicolor, constraint on $Z'$ boson mass from SO(10) GUT : $M_{Z'} \geq 700$ GeV
3. Test of lepton $\mu/e$ universality in $K_{l3}$ decays

$$\left( \frac{G_\mu}{G_e} \right)^2 = \frac{\Gamma_{K^{+/0}\mu}^3}{\Gamma_{K^{+/0}e}^3} \cdot \frac{I_{K^{+/0}\mu}^\mu}{I_{K^{+/0}e}^e} \left[ 1 + 2 \Delta EM_{K^{+/0}\mu} \right] \left[ 1 + 2 \Delta EM_{K^{+/0}e} \right]$$

1 in the SM  \hspace{1cm} \text{Exp inputs from Flavianet}  \hspace{1cm} \text{Theoretical inputs}

- For an average of $K^L$ and $K^+$ results (see Flavianet Kaon WG review)

$$r_{\mu e} = \left( \frac{G_\mu}{G_e} \right) = 1.008 \pm 0.005 \quad (r_{\mu e} = 1.002 \pm 0.005 \text{ without NA48 } K_{\mu3} \text{ result})$$

- Result in good agreement with lepton universality.

- With 0.5% precision, test competitive with $\tau$, almost with $\pi$ decay analyses

- $\pi \rightarrow l\nu \quad r_{\mu e} = 1.0042 \pm 0.0033 \quad [\text{Ramsey-Muslof, Su, Tulin’07}]$

- $\tau \rightarrow l\nu \quad r_{\mu e} = 1.0000 \pm 0.0004 \quad [\text{Davier, Hoecker, Zhang’06}]$
4. Test of the SM EW couplings via the CT theorem and the $K_{l2}$ & $K_{l3}$ decays measurements

- Callan-Treiman Theorem: SU(2) x SU(2) theorem

$$C = \bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_{\pi} f_+(0)} + \Delta_{CT}$$

$$\Delta_{K\pi} = m_k^2 - m_\pi^2$$

$$\Delta_{CT} \sim \mathcal{O}\left(\frac{m_{u,d}}{4\pi F_\pi}\right)$$

$\Delta_{CT}^{NLO} \sim -3.5 \times 10^{-3}$

Test of the Standard Model EW couplings:

$C_{SM} = \bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi} \left( V^{us}_u - V^{us}_d \right) + \Delta_{CT}$

$\Delta_{CT} = (-3.5 \pm 8) \times 10^{-3}$

Higher order terms $\mathcal{O}\left(m_{u,d}, m_s\right)$

C is predicted in the Standard Model using the measured Brs:

$Br(K_{l2}/\pi_{l2}), \Gamma(K_{e3})$ and $|V_{ud}|$. ($|V_{us}|$ not needed in this prediction.)

$$B_{exp} = 1.2446 \pm 0.0041$$

and

$$C_{SM} = 1.2411(90)$$
In the Standard Model: \( r = 1 \)

In presence of new physics, new couplings: \( r \neq 1 \)

- Right handed quark currents appearing at NLO of an EW low energy effective theory as a signature of exchange of new particles \( (W_R, \ldots) \) at high energy. [Bernard, Oertel, E.P., Stern’06]

\[
C = f_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi} \begin{pmatrix} V_{us}^u \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix} \begin{pmatrix} V_{ud}^u \end{pmatrix} r + \Delta_{CT}
\]

\[
B_{exp} = 1.2446 \pm 0.0041
\]

Effects expected on the % level, can reached several % depending on \( V_R (V_L \sim V_{CKM}) \)

Right-handed currents also in Extra-Dimension scenario, L-R symmetric models \( \rightarrow \) similar effects, test the coupling of \( W_R \) with fermions
In the Standard Model: $r = 1$

In presence of new physics, new couplings: $r \neq 1$
- Scalar couplings, exchange of a charged Higgs $H^\pm$ in two Higgs doublet models (MSSM + large tan\(\beta\) …) [Hou’92, Isidori & Paradisi’06]

$B_{\text{exp}} = 1.2446 \pm 0.0041$

Effects expected of several 0.1% up to 1%,
Ex: $\epsilon_0 = 10^{-2}$, $M_{H^\pm}^2 = 400$ GeV and $\tan\beta = 40 \implies r = 0.2\%$

N.B: Modify the extraction of the FFs.

Flavianet Kaon WG’08
Scalar FF results: Test of the SM EW couplings

<table>
<thead>
<tr>
<th>Experiment</th>
<th>InC</th>
<th>r</th>
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<td>0.144(14)</td>
<td>1.072(16)</td>
</tr>
<tr>
<td>SM</td>
<td>0.2160(73)</td>
<td>1</td>
</tr>
</tbody>
</table>

- KLOE and KTeV in agreement and in agreement with the SM. NA48 4.5σ away!
- A deviation from the SM prediction can be explained by New Physics (new couplings) in different scenarios
  Ex: NA48 result, 4.5% effect → Indication of an inverted hierarchy for $V_R$ but hard to explain with scalar couplings, effects expected on permille level. but also by the existence of a complex zero and its complex conjugate for $f_0(t)$ (not very probable) [Bernard, Oertel, E.P., Stern’09]
- To confirm this result NA48 $K^+$ analysis underway
• Other low energy theorem that allows to test for physics beyond the Standard Model and to constrain the scalar FF

The soft-kaon analog to the CT theorem [Bernard, Oertel, E.P., Stern’09]

\[ f_0(\Delta_{\pi K}) = \frac{F_\pi}{F_K f_+^0(0)} + \tilde{\Lambda}_{CT} \]

\[ \Delta_{\pi K} = m_\pi^2 - m_K^2 \]

Less precise, indeed \( \tilde{\Delta}_{CT} = 0.03 \) in the isospin limit is an SU(3) correction but rather small for a first order SU(3) x SU(3) breaking effect [Gasser&Leutwyler’85]

\[ C = f_0(\Delta_{\pi K}) = \frac{\hat{F}_\pi}{\hat{F}_K \hat{f}_+^0(0)} r' + \tilde{\Lambda}_{CT} \]

\[ -0.035 < \tilde{\Delta}_{CT} < 0.11 \]

\[ 0.8752 \pm 0.0020 \]

1 in the SM

Provide an other interesting test of NP effects knowing the scalar FF from lattice QCD (t<0) but \( \tilde{\Delta}_{CT} \) has to be better known

• If there is physics beyond the SM via a modification of the couplings, the values of \( F_K/F_\pi, f_+(0) \)…extracted from semileptonic, leptonic decays will change compared to their determination assuming the SM couplings. [Bernard, E.P.,’08]
5. Lepton Flavour Universality Tests via $R_K$

- See Talks by E. Goudzovski, B. Sciascia this morning and by P. Paradisi on Friday

$$R_K = \frac{\Gamma(K^+ \rightarrow e^+\nu)}{\Gamma(K^+ \rightarrow \mu^+\nu)}$$  

sensitive to LFU breaking

- In the SM, ratio very precisely determined with a 0.04% precision, cancellation of hadronic uncertainties
  - First systematic calculation at $\mathcal{O}(e^2p^4)$  
  - Only diagrams with photon connected to lepton lines contribute to the ratio
  - Relevant counterterms determined by matching with large $N_c$ QCD
  - Inclusion of real photon corrections
  - Summation of leading logs

$R_K = 2.477(1) \times 10^{-5}$  

[Discrepancy]  

$R_K = 2.472(1) \times 10^{-5}$  

[Cirigliano & Rosell’07]

[Improves the previous calculation]  

$R_K = 2.472(1) \times 10^{-5}$  

[Finkemeir]  

Discrepancy!
5. Lepton Flavour Universality Tests via $R_K$

- $R_K = \frac{\Gamma(K^+ \rightarrow e^+\nu)}{\Gamma(K^+ \rightarrow \mu^+\nu)}$ sensitive to LFU breaking

- In the SM, ratio very precisely determined with a 0.04% precision, cancellation of hadronic uncertainties

- Sizeable contribution of LFV terms (% level) in a SUSY scenario with a two Higgs doublet + large $\tan\beta$ in the slepton sector [Masiero, Paradisi, Petronzio’06, ’08]

$$R_K = R_K^{SM} \left[ 1 + \frac{M^4_K}{M^4_H} \left( \frac{m^2_\tau}{m^2_e} \right) |\Delta^{31}_R|^2 \tan^6 \beta \right]$$

0.013 for $M^2_{H^\pm} = 500$ GeV

$\tan\beta = 40$

$\Delta^{31}_R = 5 \cdot 10^{-4}$

$slepton$ flavour mixing angle

$\Delta^{31}_R \sim \frac{\alpha_2}{4\pi} \delta^{31}_R$
5. Lepton Flavour Universality Tests via $R_K$

Very impressive experimental improvements since Kaon’07, see next talks. Can still be improved to reach the SM precision! – NA62

World Average  $R_K = 2.468(25) \cdot 10^{-5}$

- NA48/2 ‘04
- NA48/2 ‘03
- KLOE ‘09
- SM Prediction

[T. Spadaro, Moriond March’09]

\[
\begin{align*}
R_K &= (2.468 \pm 0.025) \cdot 10^{-5} \\
M_H (\text{GeV}) &= \Delta_{13} = 10^{-3} \\
\beta &= 80
\end{align*}
\]
Conclusion and outlook

• The charged current analyses using $K_{l3}$ and $K_{l2}$ data have entered an era of very high precision
  – Improvements on the theoretical side: EM, isospin breaking corrections, dedicated dispersive parametrizations to analyse the FFs with the best precision.
  – On the experimental side, very precise data on $K_{l3}$ and $K_{l2}$ decays
    Flavianet Kaon WG

• This allows for very precise tests of the SM (test of unitarity of the 1$^{\text{st}}$ line of CKM matrix, universality, quark mass ratios…) and New Physics scenarios (Charged right-handed currents, scalar couplings, Lepton flavour violation…)

• But still on the experimental side, need K+ measurements (FFs..). Experimental puzzle on $f_0(t)$ (NA48 doesn’t agree with the other experiments).

• On theoretical side, $f_+(0)$ determination should be improved
  disagreement between analytical and lattice determinations. Lattice improvements are promising.
Additional slides
Requirements in the measurements of the form factor shapes from the $K_{l3}$ data

- Try to measure the form factor shapes from the data with the best accuracy for determination of the $K$.

- Measurement of $f_0(\Delta_{K\pi})$ to test the Standard Model via the CT theorem

\[
C = f_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+^0(0)} + \Delta_{CT}
\]

\[
\Delta_{K\pi} = m_K^2 - m_\pi^2
\]

\[
\Delta_{CT} \approx O\left(\frac{m_{ud}}{4\pi F_\pi}\right)
\]

measured very precisely assuming the SM EW couplings from $\text{Br}(K_{l2}/\pi l2)$, $\Gamma(Ke3)$ and $|V_{ud}|$

\[
\ln C_{SM} = 0.2188(35) + \Delta_{CT}
\]

- Relation which tests the Standard Model very accurately for $K^0$.
  If physics beyond the SM: ~1% difference between $C$ and $B_{exp}$. Uncertainties from $\Delta_{CT}$ and $B_{exp}$ on the permile level opportunity to see a possible effect.

- The slope and curvature of the scalar form factor are of special interest to allow for a matching with the 2 loop ChPT calculations.
Computation of $K_{l3}$ form factors at 2 loops in the isospin limit [Bijnens&Talavera’03]

- The scalar form factor at two loops in the isospin limit

$$f_s(t) = f_+(0) + \Delta(t) + \frac{F_K/F_\pi - 1}{\Delta_{K\pi}} t + \frac{8}{F_\pi^4} \left( 2C_{12}^r + C_{34}^r \right) \left( m_K^2 + m_\pi^2 \right) t - \frac{8}{F_\pi^4} t^2 C_{12}^r$$

- The vector form factor $f_+(0)$ at 2 loops in the isospin limit is expressed as

$$f_+(0) = 1 + \Delta(0) - \frac{8}{F_\pi^4} \left( C_{12}^r + C_{34}^r \right) \Delta_{K\pi}^2$$

- In these expressions, no dependence on the $L_i$ at $p^4$, only via $p^6$ contribution. Only 2 LECs $C_{12}$ and $C_{34}$ which can be determined by the measurement of the slope and the curvature of the scalar form factor.

- $\Delta(t)$ and $\Delta(0)$: contributions from loops: $\to F_\pi$, the LECs $L_i$ ($L_5 \leftrightarrow F_K/F_\pi$) can be calculated at $O(p^6)$ with the knowledge of the $L_i$ at $O(p^4)$ in the physical region.
Computation of $K_{l3}$ form factors at 2 loops in the isospin limit [Bijnens&Talavera’03]

- The scalar form factor at two loops in the isospin limit

$$f_s(t) = f_+(0) + \Delta(t) + \frac{F_K/F_\pi - 1}{\Delta_{K\pi}}t + \frac{8}{F_\pi^4}(2C_{12}^r + C_{34}^r)(m_K^2 + m_\pi^2)t - \frac{8}{F_\pi^4}t^2C_{12}^r$$

- The vector form factor $f_+(0)$ at 2 loops in the isospin limit is expressed as

$$f_+(0) = 1 + \Delta(0) - \frac{8}{F_\pi^4}(C_{12}^r + C_{34}^r)\Delta_{K\pi}^2$$

- \(\bar{\Delta}(t) = -0.25763t + 0.833045t^2 + 1.25252t^3\) \(K_{l3}^0\)

$$\Delta(0) = -0.0080 \pm 0.0057[\text{loops}] \pm 0.0028[\text{L}_t]$$

→ To be updated with the new experimental inputs ($K_{l4}$)
Extraction of the vector form factor from $K_{l3}$ which can be tested from tau decays

- Tau decay width

\[
\frac{d\Gamma_{K\pi}(t)}{d\sqrt{t}} = \frac{|V_{us}f_+(0)|^2 G_F^2 M_\tau^3}{128\pi^3} S_{EW} q_{K\pi}(t) \left(1 - \frac{t}{M_\tau^2}\right)^2 \times \left[\left(1 + \frac{2t}{M_\tau^2}\right) \frac{4q_{K\pi}^2(t)}{t} |f_+(t)|^2 + \frac{3(M_K^2 - M_\pi^2)^2}{t^2} |f_0(t)|^2\right],
\]

[Boito, Escribano, Jamin’08]

$\lambda^+ = (24.66 \pm 0.77) \times 10^{-3}$

$\lambda^\prime = (1.20 \pm 0.02) \times 10^{-3}$

In good agreement with the $K_{l3}$ analyses

Possibility of a combined Tau & $K_{l3}$ analysis

In progress!
• To be compared with

\[ \ln C_{SM} = 0.2160(35)(64) \]

KLOE and KTeV in agreement and in agreement with the SM. NA48 4.5\(\sigma\) away!

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• A deviation from the SM prediction can be explained:
  – Test of RHCs appearing at NLO of an EW low energy effective theory as a signature of exchange of new particles \(W_R, \ldots\) at high energy. [Bernard, Oertel, E.P., Stern’06]

  – Presence scalar couplings (charged Higgs): [Hou]
    MFV + large tan\(\beta\): hard to explain a 4.5\(\sigma\) effect (~several\% level) [Isidori, Paradisi’06]

  – Existence of a complex zero and its complex conjugate for the form factor [Bernard, Oertel, E.P., Stern, work in progress]
3.5 Matching in presence of RHCs

- Change in the values of \( \frac{F_K}{F_\pi} \) and \( f_+(0) \) compared to the SM, apparition of \( V_L \) and \( V_R \) \( \mathcal{V}_{\text{eff}} \) and \( \mathcal{A}_{\text{eff}} \)

\[
\left( \frac{F_K}{F_\pi} \right)^2 = \left( \frac{\hat{F}_K}{\hat{F}_\pi} \right)^2 \frac{1 + 2(\varepsilon_S - \varepsilon_{NS})}{1 + \frac{2}{\sin^2 \theta}(\delta + \varepsilon_{NS})}
\]

and

\[
\left[ f_+^{K^0\pi^-}(0) \right]^2 = \left[ \hat{f}_+^{K^0\pi^-}(0) \right]^2 \frac{1 - 2(\varepsilon_S - \varepsilon_{NS})}{1 + \frac{2}{\sin^2 \theta}(\delta + \varepsilon_{NS})}
\]

with \( (\delta + \varepsilon_{NS}) \) and \( (\varepsilon_S - \varepsilon_{NS}) \), combination of new physics parameters.

- Use experimental knowledge of \( \lambda_0 \) and \( \Delta \varepsilon \) obtained from dispersive fits to determine \( \frac{F_K}{F_\pi} \), \( f_+(0) \), \( C_{12} \), \( C_{34} \), \( \Delta_{CT} \)

\[
\ln C = 0.2188(35) + 2(\varepsilon_S - \varepsilon_{NS}) + \Delta_{CT} / B_{\text{exp}}
\]

\( \Delta \varepsilon \)

KLOE compatible with lattice results + no RHCs

NA48, RHCs + small \( \frac{F_K}{F_\pi} \) \( (F_K/F_\pi \sim 1.15) \)
4. Conclusion and outlook

- Dispersive parametrization very useful to analyse $K^{L}_{\mu 3}$ decays: parametrization physically motivated which allows with one parameter to determine the shape of the form factor, quite robust
  - Allows for a test of the SM electroweak couplings via the CT theorem
  - Allows for a matching with the 2 loop ChPT calculation

- Experimental results from dispersive analysis: KLOE and KTeV agree with the SM and NA48 at 4.5σ results for $K^+$

- Matching the $K_{i3}$ two loop computation + experimental results using dispersive representation offer the opportunity to determine $f_+(0)$, $C_{12}$, $C_{34}$, $\Delta_{CT}$ as a function of $F_K/F_\pi$

- Uncertainties too large at the moment to extract these quantities, need of
  - more precise and consistent fits
  - more precise lattice determinations
  - more precise scalar form factor measurements