Precision SM calculations and theoretical interests beyond the SM in K_{l2} & K_{l3} decays

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Introduction and Motivation

- Two ways for testing the Standard Model and finding New Physics
  - Direct searches of heavier particles (Higgs bosons, SUSY particles, Z',W'…): by Collider physics (Tevatron, LHC…)
  - Indirect searches in Flavour Physics by precision physics: measuring Low Energy observables ⇨ Indication of Λ, the New Physics scale and sensitive to effects of the underlying theory and particles at higher energy…

Decoupling scenario :

$$\boxed{\mathcal{L}_{eff} = \mathcal{L}_{SM} + \sum_{D>4} \frac{1}{\Lambda^{D-4}} \mathcal{O}_D}$$

with D: mass dimension.

SM degrees of freedom

New Physics

- Studying K_{l3} & K_{l2} decays ⇨ indirect searches of New Physics, several high-precision tests possible.

Outline

K_{l3} decays and K_{l2} decays $\Rightarrow$ stringent tests of the SM and New Physics probe

1. K_{l3} decays, extraction of V_{us}
   - Theoretical inputs : EW, EM and isospin breaking corrections
   - Form factor shapes $\Rightarrow$ determination of the phase space integrals IKs
   - $f_+(0)$ calculations

2. V_{us} and the CKM unitarity test using K_{l3} and K_{l2} decays

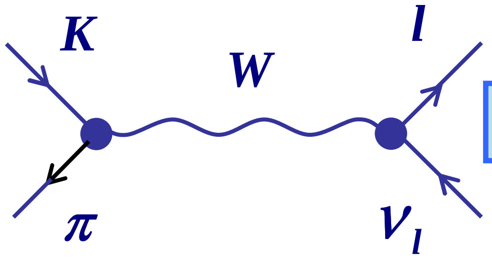
3. Test of lepton universality with K_{l3} decays

4. Test of the SM EW couplings via the CT theorem using K_{l2} & K_{l3} decays and probe of new physics

5. Lepton Flavour Universality test with K_{l2} decays

1. K_{l3} decays, extraction of V_{us}

- K_{l3} decays $\quad K \rightarrow \pi l \nu_l$



$$\langle \pi(p_\pi) | \bar{s}\gamma_\mu u | K(p_K) \rangle = F\left(f_+(t), f_0(t) \right)$$

- Decay rate formula for K_{l3}

$$l = (e, \mu)$$

$$\Gamma_{K^{+/0}l3} = \frac{Br_{K^{+/0}l3}}{\tau_{K^{+/0}}} = \frac{C_K^2 G_F^2 m_{K^{+/0}}^5}{192\pi^3} S_{EW} \left(1 + 2\Delta_{EM}^{K^{+/0}l} + 2\Delta_{SU(2)}\right) \left| f_+^{K^{+/0}}(0) V_{us} \right|^2 I_{K^{+/0}}^l$$

½ for K⁺, 1 for K⁰

$$I_{K^{+/0}}^l = \int dt \frac{1}{m_{K^{+/0}}^8} \lambda^{3/2} F\left(t, \bar{f}_+(t), \bar{f}_0(t)\right)$$

- Experimental inputs: → $\boxed{Br_{K^{+/0}l3}, \tau_{K^{+/0}}}$ K_{l3} branching ratios, Kaon life time, with good treatment of radiative corrections

  → $\boxed{I_{K^{+/0}}^l}$ Phase space integrals, need form factor shapes extracted from Dalitz plot, from **NA48, KTeV, KLOE** and **ISTRA+**

- Theoretical inputs: → $\boxed{S_{EW}}$ Short distance EW corrections

  → $\boxed{\Delta_{EM}^{K^{+/0}l}}$ Long distance EM corrections

  → $\boxed{\Delta_{SU(2)}}$ Isospin breaking corrections

- For the experimental inputs, see the Flavianet review, talk by M.Palutan

- Theoretical inputs :

→ $\boxed{S_{EW}}$ Short distance EW corrections, universal factor

$$\boxed{S_{EW} = 1 + \frac{2\alpha}{\pi}\left(1 + \frac{\alpha_S}{4\pi}\right)\log\frac{M_Z}{M_\rho} + O\left(\frac{\alpha\alpha_S}{\pi^2}\right)} \Rightarrow \boxed{S_{EW} = 1.0232(5)}$$

[Sirlin'82]

→ $\boxed{\Delta_{EM}^{K^{+/0}l}}$ Long distance EM corrections

- First analyses of EM corrections for K_{l3} by **Ginsberg'67, '70**, then by **Andre'04:** hard UV cutoff in the loops (used in the KTeV analysis).
- Description of the EM interaction within an effective theory

⇒ ChPT with photons **[Urech'95, Neufeld & Rupertsberger'95],**
EM LECs K_i **[Ananthanaryan & Moussallam'04]**

⇒ ChPT with photons and Leptons **[Knecht et al'00]**
Additional LECs X_i **[Descotes-Genon & Moussallam'05]**

→ $\boxed{\Delta_{EM}^{K^{+/0}l}}$ Long distance EM corrections : Calculations at $\mathcal{O}(p^2e^2)$

K_{e3} [Cirigliano et al'01, Cirigliano, Neufeld, Pichl'04, Cirigliano, Giannotti, Neufeld'08]

$K_{\mu 3}$ [Cirigliano, Giannotti, Neufeld'08]

New !

In this recent analysis, fully inclusive prescription of real photon emission, update of structure-dependent EM contributions, take the most recent estimates of the LECs

+ Errors: estimates of higher order corrections

Results : (NB: A part depends on the IK values)

$\Delta_{EM}^{K^{+/0}l}(\%)$	K_{e3}^{0}	$K_{e3}^{\pm}$	$K_{\mu 3}^{0}$	$K_{\mu 3}^{\pm}$
CGN'08	0.50 ± 0.11	0.05 ± 0.13	0.70 ± 0.11	0.008 ± 0.13
Andre'04	0.65 ± 0.15	-	0.95 ± 0.15	-

Larger effects in K^0 due to Coulomb final state interactions

Reliable calculations to use in the experimental analyses + in the same analysis estimates of the EM corrections for the decay distribution, crucial role !

→ $\boxed{\Delta_{SU(2)}}$ Isospin breaking corrections, studied up to O(p^4) in ChPT

$\boxed{\Delta_{SU(2)} = 0}$ for K^0, Corrections only hold for K$^+$

- Leading contribution (O(p^2)), due to π^0-η mixing in the final state

⇨ small denominators $\boxed{O\left((m_d - m_u)/m_s\right)}$

$$\boxed{\Delta_{SU(2)} = \frac{3}{4}\frac{1}{R}}$$ depend of the quark mass ratio $$\boxed{R = \frac{m_s - \hat{m}}{m_d - m_u}}$$ $$\hat{m} = \frac{m_u + m_d}{2}$$

- At NLO (O(p^4)),

$$\boxed{\Delta_{SU(2)} = \frac{3}{4}\frac{1}{R}\left(1 + \chi_{p^4} + \frac{4}{3}\frac{M_K^2 - M_\pi^2}{M_\eta^2 - M_\pi^2}\Delta_M + O\left(m_q^2\right)\right)}$$

Chiral correction

$$\chi_{p^4} = 0.219$$

related to the ratio of quark masses

$$\frac{M_K^2}{M_\pi^2} = \frac{m_s + \hat{m}}{m_u + m_d}\left(1 + \Delta_M + O\left(m_q^2\right)\right)$$

- Use quark mass ratios as input ⇨ $\boxed{\Delta_{SU(2)}}$

or determination of $\boxed{\Delta_{SU(2)} = \frac{f_+^{K^+}(0)}{f_+^{K^0}(0)} - 1}$ from measurements ⇨ $\boxed{R}$

Hard to pin down precisely !

See talk by V. Cirigliano at Kaon'07

- Theoretical prediction for $\Delta_{SU(2)}$:

Use $\boxed{Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2} = R\,\frac{m_s/\hat{m}+1}{2}}$, can be extracted from :

– $\eta \to \pi^+ \pi^- \pi^0$ decays ⇨ $Q = 22.7 \pm 0.8$ [Anisovitch & Leutwyler'95, Leutwyler'96]

$Q = 22.4 \pm 0.9$ [Kambor, Wiesendanger, Wyler'95]

$Q = 23.2$ [Bijnens & Ghorbani'07]

⇨ $\boxed{\begin{array}{l} R = 40.8 \pm 3.2 \\ \Delta_{SU(2)} = 2.36(22)\% \end{array}}$

– From kaon mass splitting [Gasser & Leutwyler'85]

$\boxed{Q^2 = \frac{M_K^2}{M_\pi^2}\,\frac{M_K^2 - M_\pi^2}{M_{K^0}^2 - M_{K^+}^2}}$ ⇨ $Q = 20.7 \pm 1.2$ [Kastner & Neufeld'08]

Based on Ananthanarayan & Moussallam'04

Large deviation of the Dashen's limit

⇨ $\boxed{\begin{array}{l} R = 33.5 \pm 4.3 \\ \Delta_{SU(2)} = 2.9(4)\% \end{array}}$ New !

Slight disagreement between the 2 approaches (~1.2σ) ⇨ analysis based on new KLOE data for $\eta \to \pi^+ \pi^- \pi^0$ decays underway.

- Determination of $\Delta_{SU(2)}$ from K_{l3} data :

$$\boxed{\Delta_{SU(2)} = \frac{f_+^{K^+}(0)}{f_+^{K^0}(0)} - 1} \Rightarrow \boxed{\Delta_{SU(2)} = \frac{\Gamma_{K^+l3}}{\Gamma_{K^+l3}} \frac{I^l_{K^0}}{I^l_{K^+}} \left(\frac{M_{K^0}}{M_{K^+}} \right)^5 - \frac{1}{2} - \left[\Delta^{K^+l}_{EM} - \Delta^{K^0l}_{EM} \right]}$$

Good precision from the data + from the EM correction estimates

⇨ phenomenological estimate from K_{l3} data with a very good precision possible

$\boxed{\Delta_{SU(2)} = 2.7(4)\%}$ [FIT **Flavianet Kaon WG,** see talk by **M.Palutan**]

Very good agreement with the recent result from **Kastner & Neufeld'08**

The tension between the phenomenological estimate $\boxed{\Delta^{pheno}_{SU(2)} = 3.21(38)\%}$ and the theoretical estimate existing at **Kaon'07** (see talk by **V. Cirigliano**) has disappeared with the new experimental value (1σ lower) and the recent estimate of R !

⇨ A small value of R (R~33) seems to be favoured.

Has to be confirmed by the analysis from $\eta \to \pi^+ \pi^- \pi^0$ using the new KLOE data, underways.

Determination of the phase space integrals and the form factor shapes

- Decay rate formula for K_{l3}

$$\Gamma_{K^{+/0}l3} = \frac{Br_{K^{+/0}l3}}{\tau_{K^{+/0}}} = \frac{C_K^2 G_F^2 m_{K^{+/0}}^5}{192\pi^3} S_{EW}\left(1 + 2\Delta_{EM}^{K^{+/0}l} + 2\Delta_{SU(2)}\right)\left|f_+^{K^{+/0}}(0)V_{us}\right|^2 I_{K^{+/0}}^l$$

$$I_{K^{+/0}}^l = \int dt \frac{1}{m_{K^{+/0}}^8} \lambda^{3/2} F\left(t, \bar{f}_+(t), \bar{f}_0(t)\right)$$

To extract $\boxed{f_+(0)\left|V_{us}\right|}$, one needs to calculate the phase space integrals I_K and determine $f_+(0)$ ⇨ Need to measure 2 form factors : $f_+(t)$ and $f_0(t)$

- The hadronic matrix element of K_{l3} decays

$$\left\langle \pi(p_\pi)\right| \bar{s}\gamma_\mu u \left|K(p_K)\right\rangle = f_+(t)(p_K + p_\pi)_\mu + f_-(t)(p_K - p_\pi)_\mu$$

→ $f_+(t)$, $f_-(t)$: form factors

→ $t = q^2 = (p_\mu + p_{\nu_\mu})^2 = (p_K - p_\pi)^2$

$$f_0(t) = f_+(t) + \frac{t}{m_K^2 - m_\pi^2} f_-(t)$$

NB:

$$f_+(0) = f_0(0)$$

- Impossible to measure $f_+(0)$ from experiment, has to be determined from theory

- Determination of $\bar{f}_{+,0}(t) = \dfrac{f_{+,0}(t)}{f_+(0)}$ by a fit to the measured K_{l3} decay distribution

How to measure the form factor shapes ?

- Data available from **KTeV**, **NA48** and **KLOE** for K^0 and from **ISTRA+**, **NA48** and **KLOE** for K^+.
- Necessity to parametrize the 2 form factors $\bar{f}_+(t)$ and $\bar{f}_0(t)$ to fit the measured distributions.
- Different parametrizations available, 2 classes of parametrizations :

→ 1rst class: parametrizations based on mathematical rigourous expansion, the slope and the curvature are free parameters :

- Taylor expansion

$$\bar{f}_{+,0}(t) = 1 + \lambda'_{+,0} \frac{t}{m_\pi^2} + \frac{1}{2} \lambda''_{+,0} \frac{t}{m_\pi^2} + \ldots$$

- Z-parametrization, conformal mapping from t to z variable with |z|<1 improve the convergence of the series

$$f_{+,s}(t) = f_{+,s}(t_0) \frac{\phi(t_0,t_0,Q^2)}{\phi(t,t_0,Q^2)} \sum_{k=0}^{n} a_k \left(t_0,Q^2\right) z\left(t,t_0\right)^k$$

[Hill'06]

Theoretical error can be estimated : for a specific choice of ϕ, $\sum_{k=0}^{n} a_k^2$ bounded ⇨ use of some high-energy inputs (τ data …).

Work on the scalar FF by **[Bourrely & Caprini'05], [Abbas, Ananthanaryan'09]** and on the vector FF by **[Hill'06]**.

→ 2nd class: parametrizations which by using physical inputs impose specific relations between the slope and the curvature

⇨ reduce the correlations, only one parameter fit.

- Pole parametrization, the dominance of a resonance is assumed

$$\boxed{\bar{f}_{+,0}(t) = \frac{m_{V,S}^2}{m_{V,S}^2 - t}}$$

$m_{V,S}$ is the parameter of the fit

- Dispersive parametrization: use of the low energy $K\pi$ scattering data and presence of resonances to contrain by dispersion relations the higher order terms of the expansion. Analysis from **[Jamin, Oller, Pich'04], [Bernard, Oertel, E.P, Stern'06],** for the scalar form factor and from **[Moussallam'07], [Jamin, Pich & Portoles'08], [Boito, Escribano & Jamin'08]** for the vector form factor using τ data.

- Requirements in the measurements of the form factor shapes from the K_{l3} data
  - Try to measure the form factor shapes from the data with the best accuracy for determination of the IKs.
  - Measurement of $\overline{f}_0(\Delta_{K\pi}) \equiv C$ to test the Standard Model via the CT theorem

$$\boxed{C = \overline{f}_0(\Delta_{K\pi}) = \underbrace{\frac{F_K}{F_\pi f_+^{K^0}(0)} + \Delta_{CT}}}$$

$$\Delta_{K\pi} = m_K^2 - m_\pi^2$$

Can be determined very precisely assuming the SM EW couplings from BRs measurements + ChPT estimate for Δ_{CT}

A measurement of C allows for a test of the SM EW couplings and new physics effects.

  - The slope and curvature of the scalar form factor are of special interest to allow for a matching with the 2 loop ChPT calculations.

- Experimental constraints: if one uses a parametrization from 1rst class, for example a Taylor expansion,
  - Only two parameters measurable for the vector form factor, λ'_+ and λ_+''
  - Only one parameter accessible for the scalar form factor λ'_0

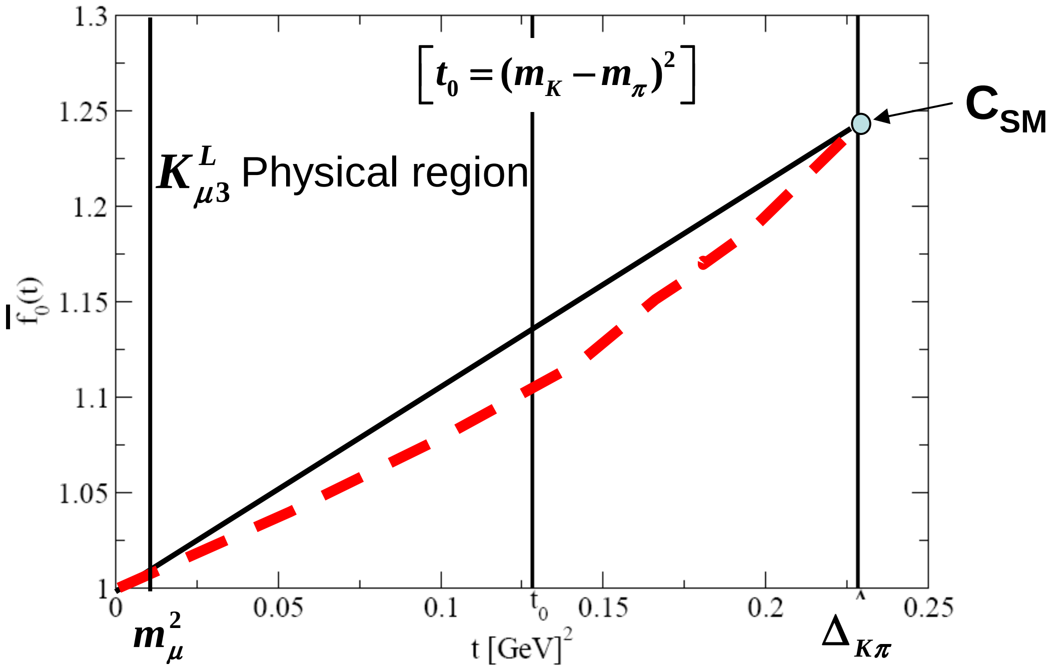
    ⇨ see Flavianet Kaon WG result (talk by **M. Palutan**).
  - The correlations are strong,

λ'_0	1	-0.9996	-0.97	0.91
λ''_0		1	0.98	-0.92
λ'_+			1	-0.98
λ''_+				1

[Franzini, Kaon'07]

- Necessity to use a second class parametrization which reduces the correlation, only one parameter is fitted.
  - For the vector form factor ⇨ pole parametrization with dominance of the K*(892) in good agreement with the data.
  - For the scalar form factor, not a such obvious dominance ⇨ necessity to use a dispersive parametrization to improve the extraction of the ff parameters and to reach the CT point.

- Impossible to use the linear parametrization to extrapolate with a good precision up to the CT point



- Dispersive parametrization for the scalar and the vector FFs, for the scalar:

$$\bar{f}_0(t) = \exp\left[\frac{t}{\Delta_{K\pi}}\left(\ln C - G(t)\right)\right]$$ with $$G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi}\int_{(M_K+M_\pi)^2}^{\infty}\frac{ds}{s}\frac{\phi(s)}{(s-\Delta_{K\pi})(s-t)}$$

elastic region

- – ϕ(t) phase of the form factor $t < \Lambda: \; \phi_0(t) = \phi_{K\pi}(t) = \delta^{s,\frac{1}{2}}_{\pi,K}(t) \pm \Delta\delta^{s,\frac{1}{2}}_{\pi,K}(t)$

  $t > \Lambda: \; \phi_0(t) = \phi_{as}(t) = \pi \pm \pi$ [Watson theorem]

- – 2 subtractions ⇨ Rapid convergence of G(t)

- Dispersive parametrization for the scalar FF **[Bernard, Oertel, E.P., Stern'06]**

$$\bar{f}_0(t) = \exp\left[\frac{t}{\Delta_{K\pi}} (\ln C - G(t))\right]$$ with $$G(t) = \frac{\Delta_{K\pi}(\Delta_{K\pi} - t)}{\pi} \int_{(M_K+M_\pi)^2}^{\infty} \frac{ds}{s} \frac{\phi_0(s)}{(s-\Delta_{K\pi})(s-t)}$$

- Impose a relation between slope and curvature

$$\lambda_0'' = \lambda_0'^2 - 2\frac{m_\pi^4}{\Delta_{K\pi}} G'(0) > \lambda_0'^2$$

$$\lambda_0' = \frac{m_\pi^2}{\Delta_{K\pi}}(\ln C - G(0))$$

$\Rightarrow$ One parameter lnC to fit to determine the shape of $\bar{f}_0(t)$

- Dispersive parametrization for the vector FF **[Bernard, Oertel, E.P., Stern'09]**

$$\bar{f}_+(t) = \exp\left[\frac{t}{m_\pi^2} (\Lambda_+ + H(t))\right]$$ with $$H(t) = \frac{m_\pi^2 t}{\pi} \int_{(M_K+M_\pi)^2}^{\infty} \frac{ds}{s^2} \frac{\phi_+(s)}{(s-t)}$$

- Also one parameter to fit Λ_+ to determine the shape of $\bar{f}_+(t)$

$$\lambda_+' = \Lambda_+$$ and $$\lambda_+'' = \lambda_+'^2 + 2m_\pi^2 H'(0) > \lambda_+'^2$$

Results of the dispersive analyses

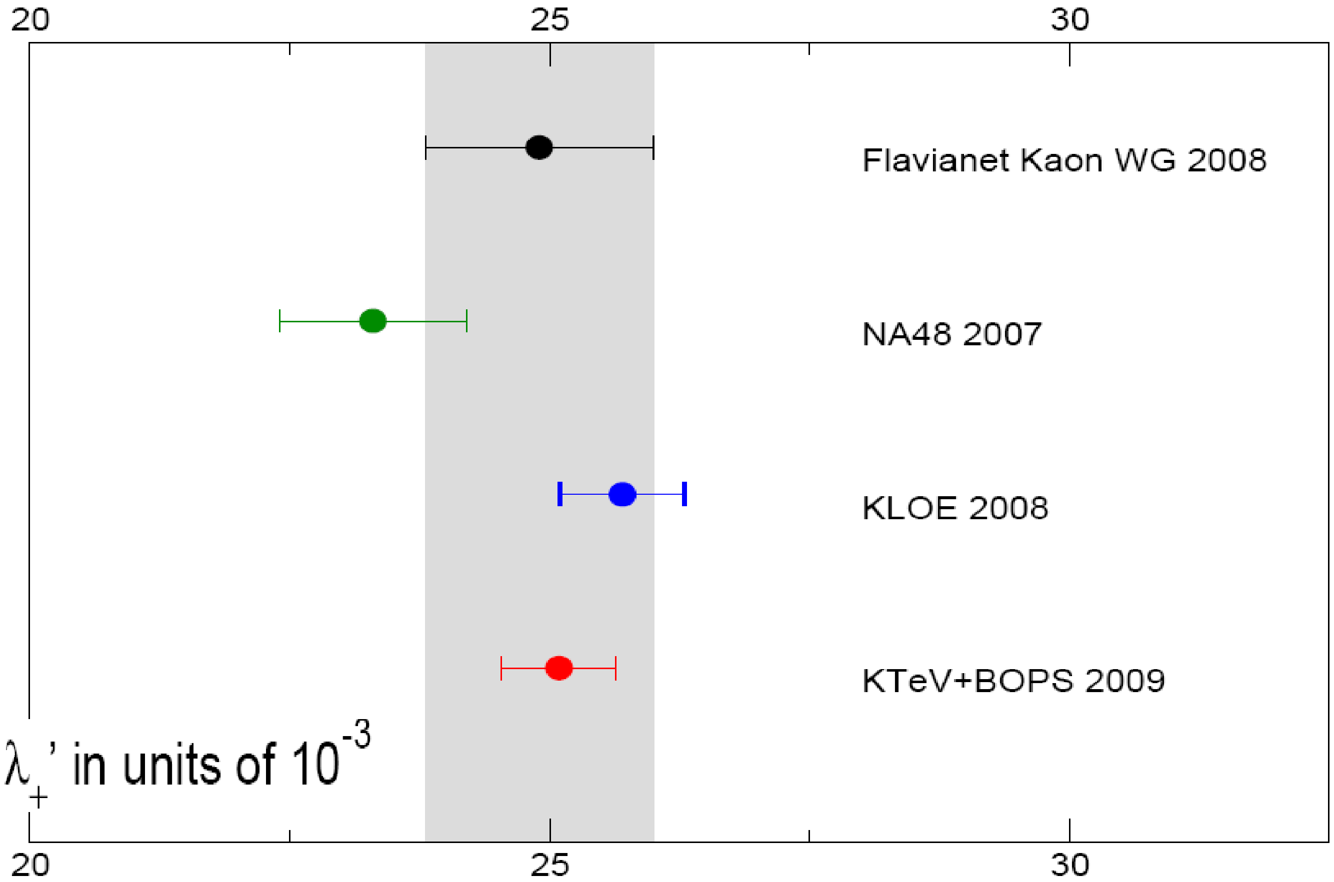
- Since Kaon'07 in addition of **NA48'07**, dispersive analyses by **KLOE'08** and **KTeV + Bernard,Oertel, E.P., Stern'09** with old KTeV data (submitted to PRD)

NA48	$K_{\mu 3}$
$\Lambda_+ \times 10^3$	23.3 ± 0.9
$\ln C$	0.1438 ± 0.0138
$\rho(\Lambda_+, \ln C)$	-0.44
χ^2/dof	595/582
$\lambda'_+ \times 10^3$	23.33 ± 0.9
$\lambda''_+ \times 10^3$	1.3 ± 0.1
$\lambda'_0 \times 10^3$	8.9 ± 1.2
$\lambda''_0 \times 10^3$	0.50 ± 0.05

KLOE	K_{e3} and $K_{\mu 3}$ combined
$\Lambda_+ \times 10^3$	25.7 ± 0.6
$\ln C$	0.204 ± 0.025
$\rho(\Lambda_+, \ln C)$	-0.27
χ^2/dof	2.6/3
$\lambda'_+ \times 10^3$	25.7 ± 0.6
$\lambda''_+ \times 10^3$	1.1 ± 0.1
$\lambda'_0 \times 10^3$	14.0 ± 2.1
$\lambda''_0 \times 10^3$	0.50 ± 0.06

KTeV	K_{e3}	$K_{\mu 3}$	K_{e3} and $K_{\mu 3}$ combined
$\Lambda_+ \times 10^3$	25.17 ± 0.58	24.57 ± 1.10	25.09 ± 0.55
$\ln C$	-	0.1947 ± 0.0140	0.1915 ± 0.0122
$\rho(\Lambda_+, \ln C)$	-	-0.557	-0.269
χ^2/dof	66.6/65	193/236	0.48/2
$\lambda'_+ \times 10^3$	25.17 ± 0.58	24.57 ± 1.10	25.09 ± 0.55
$\lambda''_+ \times 10^3$	1.22 ± 0.10	1.19 ± 0.11	1.21 ± 0.10
$\lambda'_0 \times 10^3$	-	13.22 ± 1.20	12.95 ± 1.04
$\lambda''_0 \times 10^3$	-	0.59 ± 0.05	0.58 ± 0.05

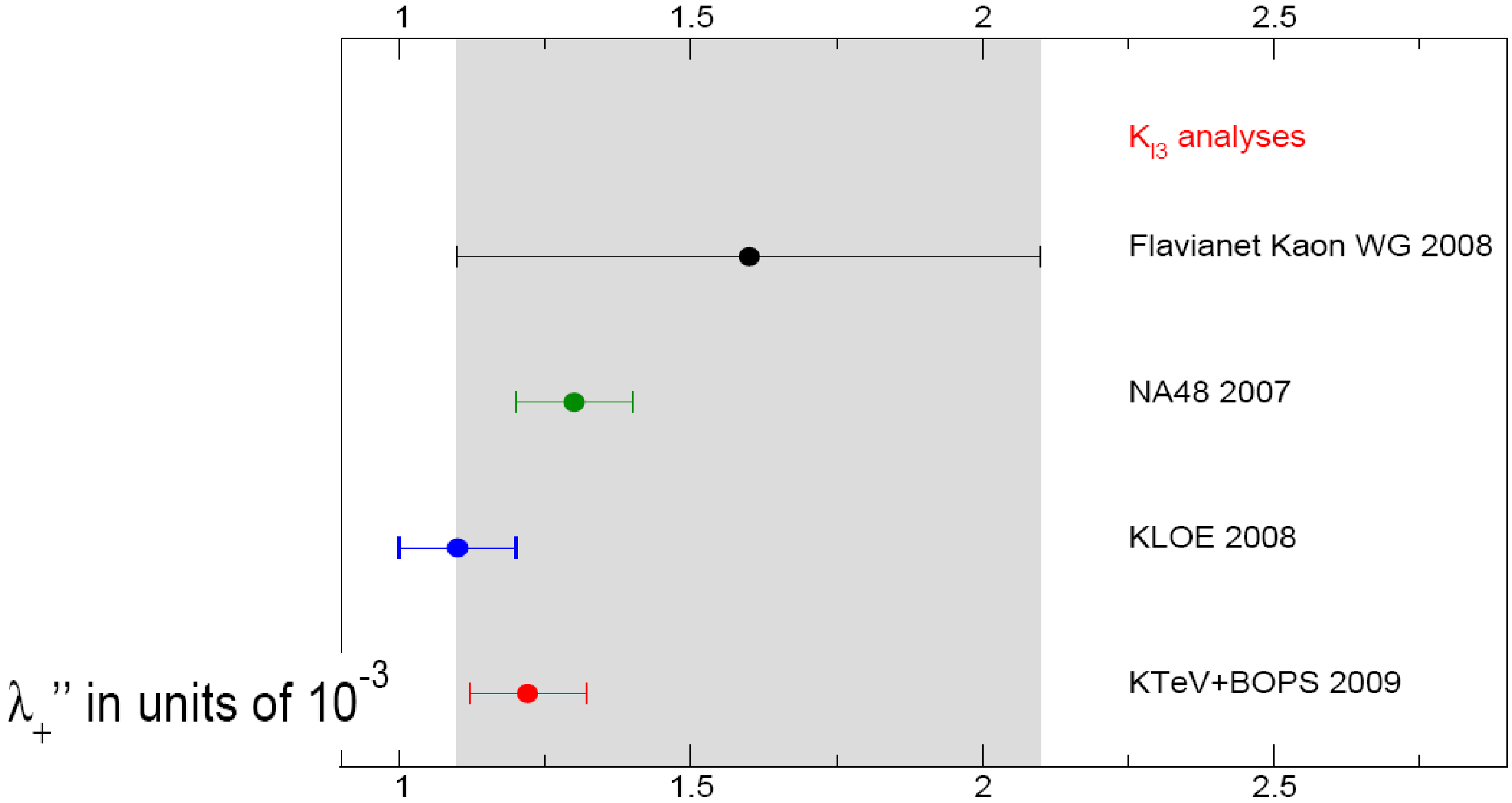
Vector FF results



- A good agreement between the Flavianet Kaon WG average using a quadratic parametrisation and the dispersive results.
- KLOE and KTeV in perfect agreement, NA48 1σ away.
- The dispersive results twice more precise than the average !

⇨ has to be used by the Flavianet Kaon WG for the IKs calculations.

Vector FF results



- The precision reached for the curvature using a dispersive parame-trisation is much more precise than the one using the quadratic parametrization by a factor 4 !
- Good agreement between the dispersive results.

⇨ The dispersive results have really to be used by the Flavianet Kaon WG for the IKs calculations !

The extraction of the vector form factor from K_{l3} can be tested from $\tau \to K\pi\nu_\tau$ decays

- Tau decay width

Kinematic factor

$$\frac{d\Gamma_{K\pi}(t)}{d\sqrt{t}} = \frac{|V_{us}f_+(0)|^2 G_F^2 M_\tau^3}{128\pi^3} S_{EW}\, q_{K\pi}(t) \left(1-\frac{t}{M_\tau^2}\right)^2 \times$$
$$\left[\left(1+\frac{2t}{M_\tau^2}\right)\frac{4q_{K\pi}^2(t)}{t}|\bar{f}_+(t)|^2 + \frac{3(M_K^2-M_\pi^2)^2}{t^2}|\bar{f}_0(t)|^2\right],$$

- In the Tau energy range at much higher energy than K_{l3} decays, vector contribution (K*(892)) dominates $\Rightarrow$ precise determination of $\bar{f}_+(t)$

- Fit to **Belle** data using different representations for $\bar{f}_+(t)$
  - A coupled-channel DR using the Kπ scattering data **[Moussallam'08]**
  - A representation using RChPT, 2 resonances K*(892) and K*(1414) **[Jamin, Pich, Portoles'08]**
  - A three-time subtracted DR with a RChPT description of the phase $\Rightarrow$ 2 resonances K*(892) and K*(1414) **[Boito, Escribano, Jamin'08]**

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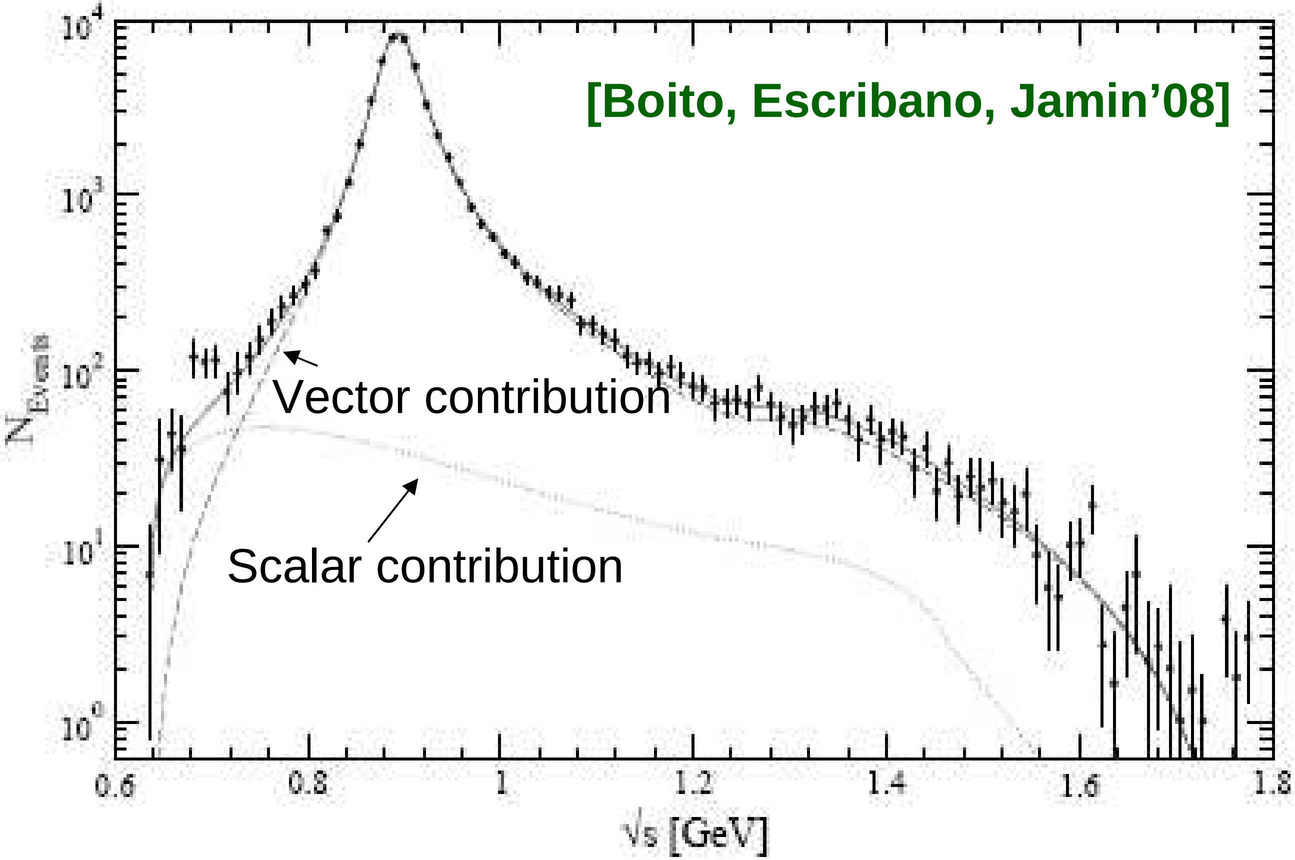
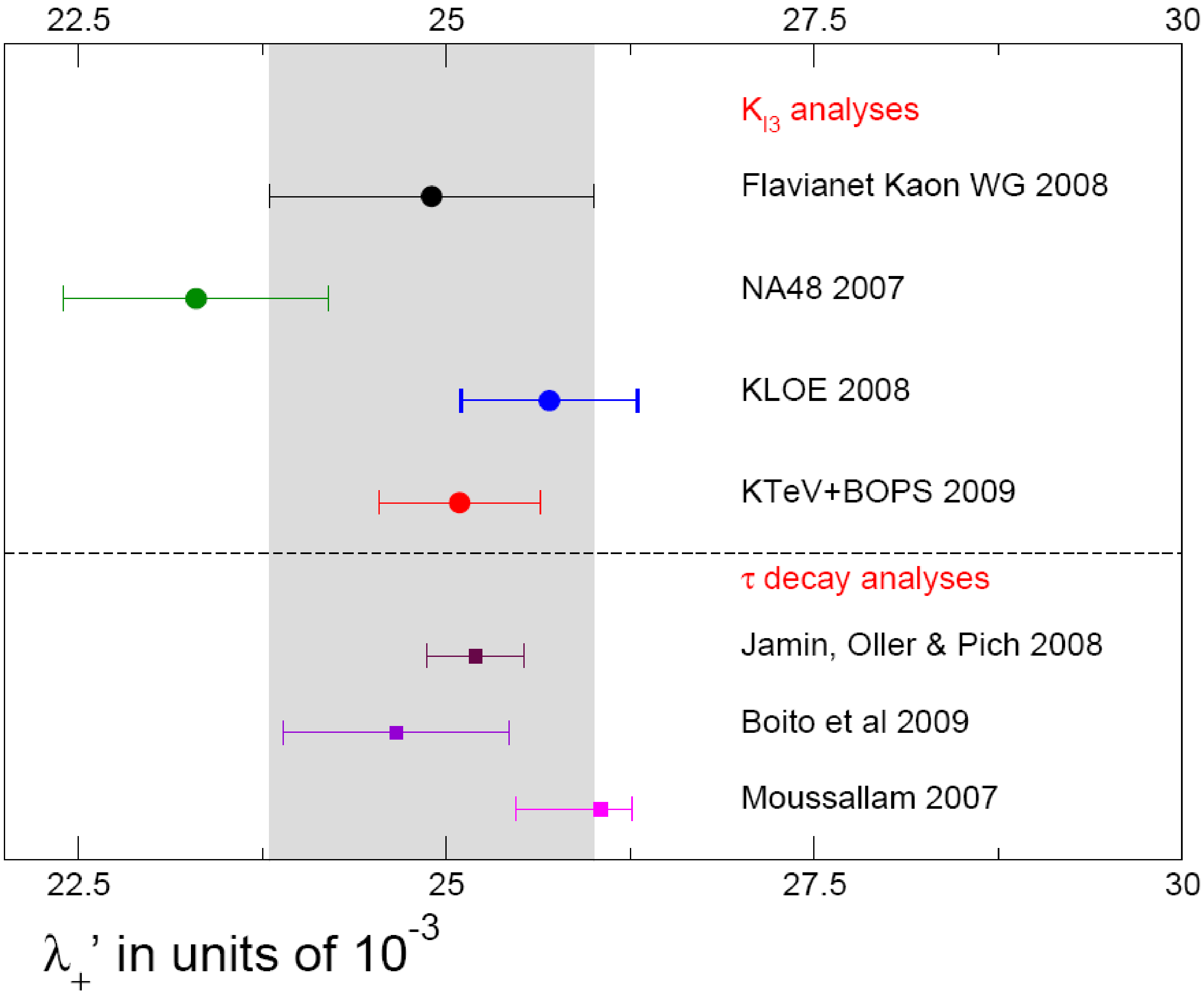
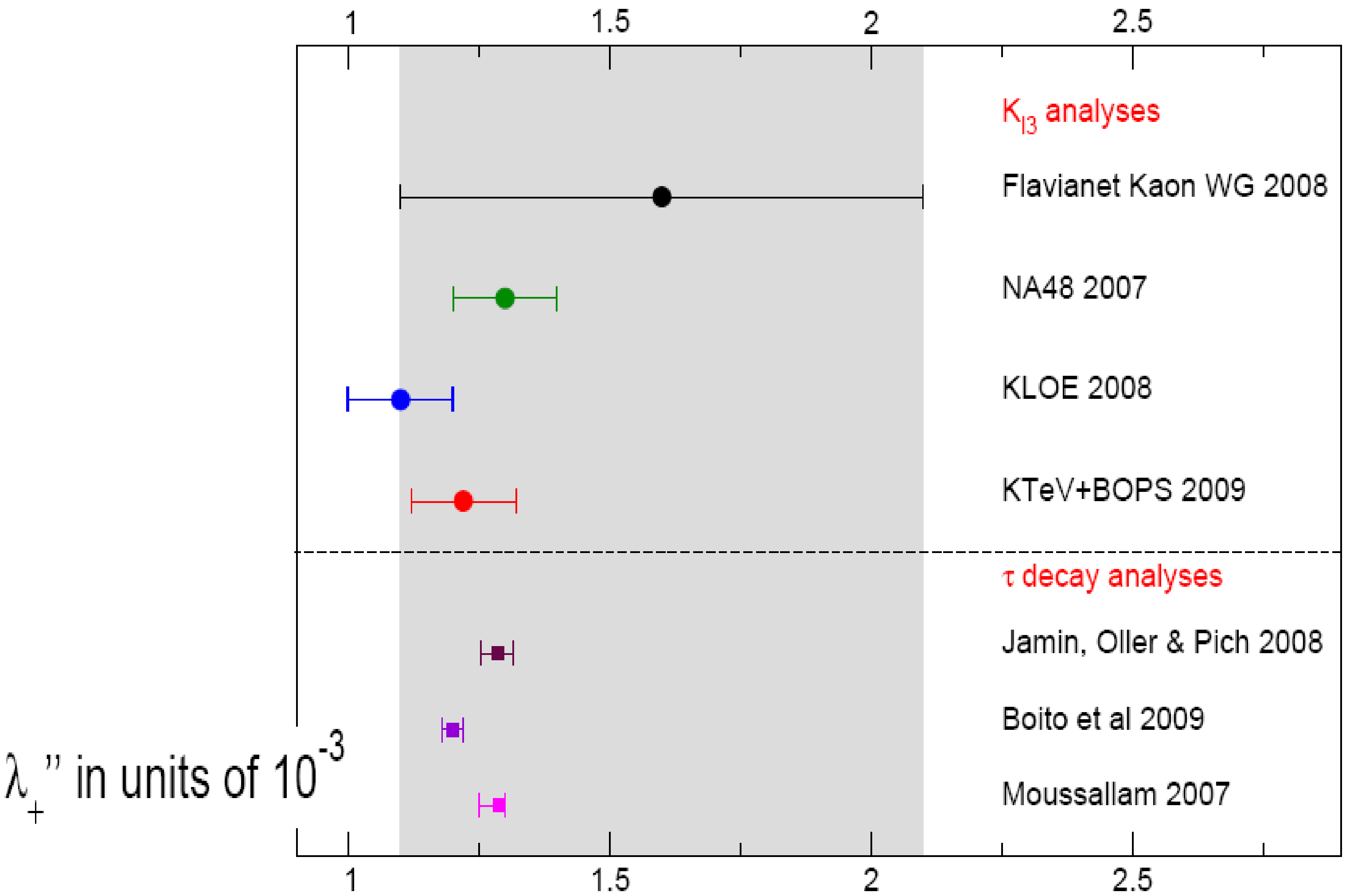


Vector FF determination from K_{l3} decays/$\tau \rightarrow K \pi \nu_\tau$ decays

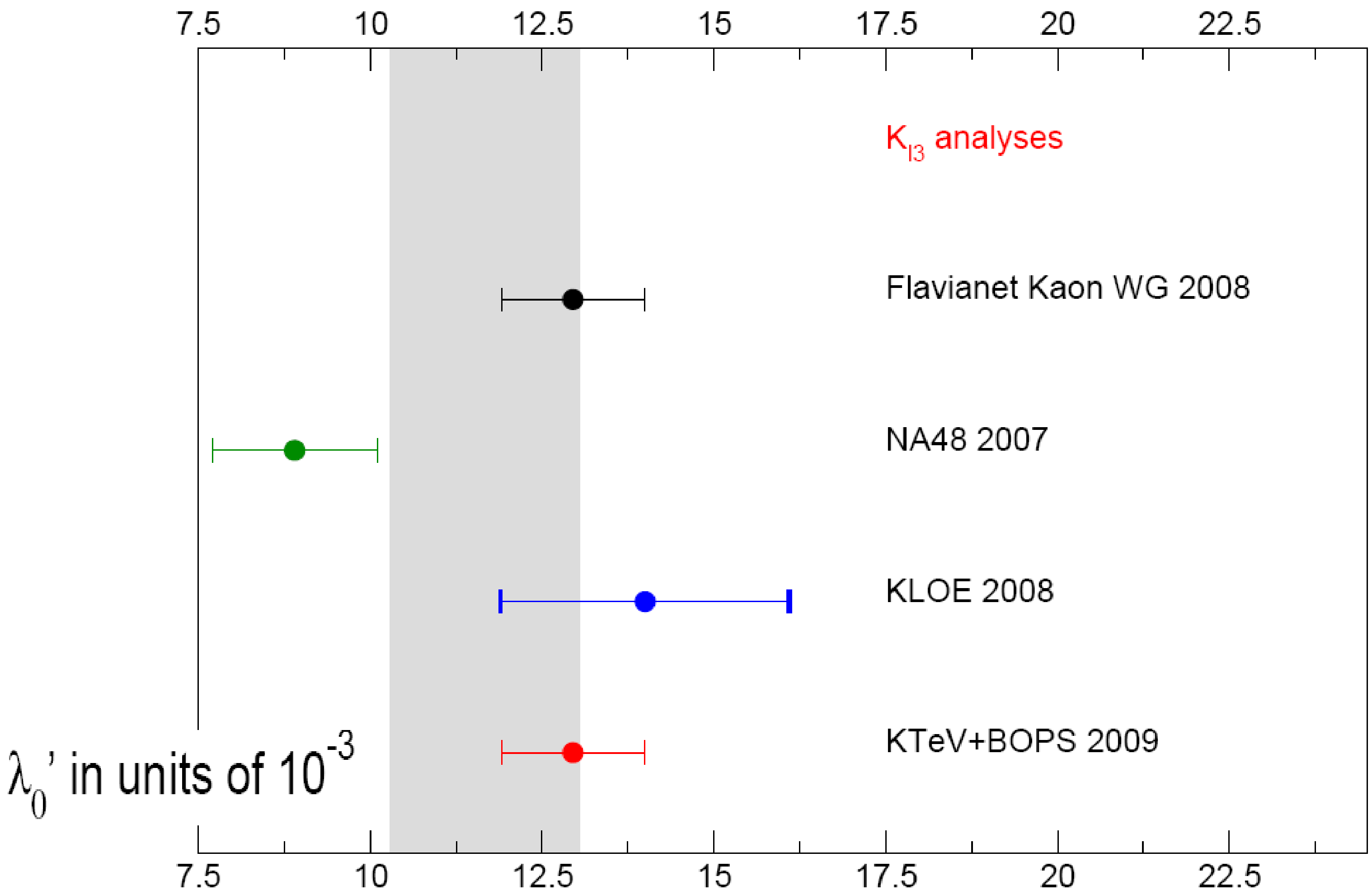


Vector FF determination from K_{l3} decays/$\tau \rightarrow K\pi\nu_\tau$ decays



- The result are in very good agreement. Very precise measurements for λ_+'' can be reached from τ decays ⇨ combined K_{l3} & τ analyses are underway. This will allow to improve the precision + test of the dispersive relation between $\lambda+'$ and $\lambda+''$.

Scalar FF results



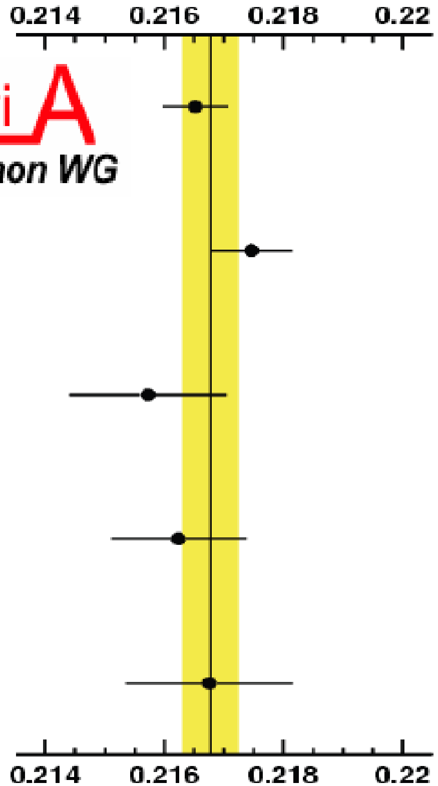
- The result are in agreement except for the NA48 one which disagrees with the others.
- One can access with the DR to the curvature. The curvature is in this case very small (~5 10^{-4}) but needed if one wants reach a high level of precision.

Extraction of $f_+(0)$ Vus

- From Δ_{EM} + $\Delta_{SU(2)}$ + IKs + Experimental measurements ⇨ $f_+(0)|V_{us}|$

$$\left|f_+^{K^{+/0}}(0)V_{us}\right|^2 = \frac{Br_{K^{+/0}l3}/\tau_{K^{+/0}}}{\frac{C_K^2 G_F^2 m_{K^{+/0}}^5}{192\pi^3} S_{EW}\left(1+2\Delta_{EM}^{K^{+/0}l}+2\Delta_{SU(2)}^{K}\right) I_{K^{+/0}}^{l}}$$

$|V_{us}|f_+(0)$



FlaviA net Kaon WG

K_Le3	0.21652(56)
$K_L\mu3$	0.21746(69)
K_Se3	0.21572(132)
$K^{\pm}e3$	0.21624(113)
$K^{\pm}\mu3$	0.21676(141)

See Talk by M. Palutan

I_Ks still used from the linear/quadratic fit ⇨ dispersive results have to be used !

The K^L are more precise. ⇨ K^+ measurements underways

Average: $|V_{us}|f_+(0)$ = 0.21660(47) **χ^2/ndf = 3.03/4 (55%)**

Extraction of Vus : $f_+(0)$?

- ChPT : $\boxed{f_+(0) = 1 + f_2 + f_4 + ...}$

$\boxed{f_2 = \mathcal{O}(m_s - \hat{m})^2}$ SU(3) breaking **[Ademollo-Gatto theorem]**

⇨ $f_2 = -0.023$ → no contribution from the O(p^4) Li's
→ NLO chiral logs fully determined in terms of M_K, M_π anf F_π

- 1rst higher order estimate ($\mathcal{O}(p^6)$) $f_+(0) - 1 - f_2 = -0.016(8)$ by quark model

⇨ $\boxed{f_+(0) = 0.961(8)}$ **[Leutwyler&Roos'84]**

- Analytic estimates at 2 loops in the isospin limit [Post-Schicher'02], [Bijnens &Talavera'03]

$$\boxed{f_+(0) = 1 + \Delta(0) - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\Delta_{K\pi}^2}$$

Only 2 O(p^6) LECs C_{12} and C_{34} appear

In $\Delta(0)$, no dependence on the L_i at p^4, only via p^6

$$\Delta(0) = -0.0080 \pm 0.0057\left[loops\right] \pm 0.0028\left[L_i^r\right]$$ [Bijnens &Talavera'03]

⇨ To be updated with the new experimental inputs (K_{l4})

Extraction of Vus : $f_+(0)$?

- Analytic estimates at 2 loops in the isospin limit [Post-Schicher'02], [Bijnens &Talavera'03]

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[Bijnens &Talavera'03]

Estimate of the two O(p^6) LECs C_{12} and C_{34}

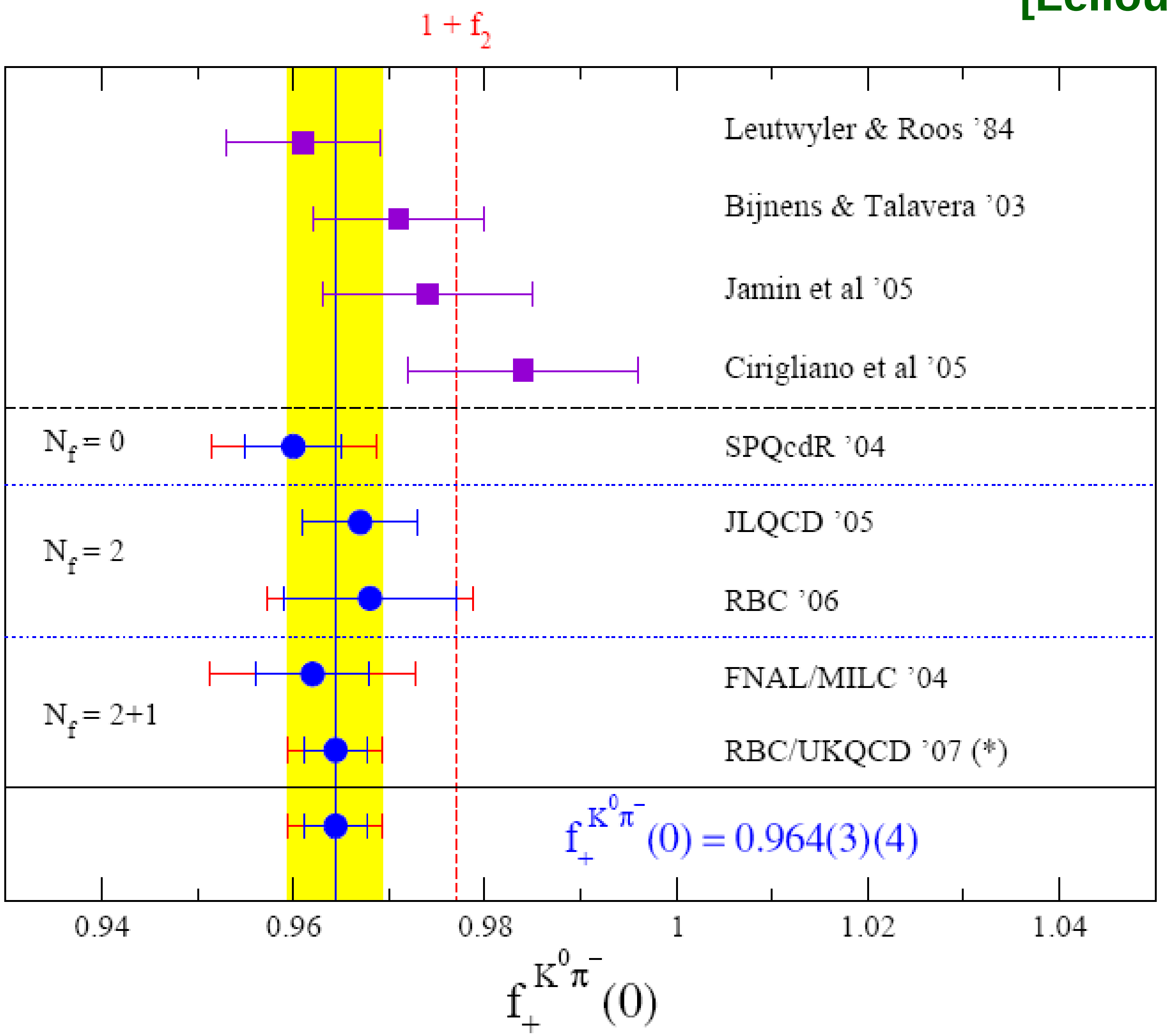
- By resonance exchange estimates [Cirigliano et al'05], [Kastner & Neufeld'08]
- Matching with dispersive representation or parametrisation [Jamin et al'05], [Bernard, E.P'08]

$\Rightarrow$ Possibility to use the scalar ff dispersive measurements

- Estimate on the lattice, see Talks by P. Boyle, G. Colangelo , F. Mescia

Only one published result in N_f=2+1, more results are awaited !

[Lellouch, Lattice'08]



- The analytical results based on resonance model estimates for C_{12} and C_{34} give larger results for $f_+(0)$ than the lattice calculations
- Possibility from $\overline{f}_0(t)$ dispersive measurements to test these estimates

Matching of the 2 loop ChPT with the DR

[Bernard & E.P'08]

$$f_S(t) = f_+(0) + \bar{\Delta}(t) + \frac{F_K/F_\pi - 1}{\Delta_{K\pi}} t + \frac{8}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right)t - \frac{8}{F_\pi^4} t^2 C_{12}^r$$

[Bijnens &Talavera'03]

- Taking the derivative:

$$\Longrightarrow \quad \lambda_0' f_+(0) = \frac{m_\pi^2}{\Delta_{K\pi}}\left(\frac{F_K}{F_\pi} - 1\right) + \frac{8 m_\pi^2 \Sigma_{K\pi}}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right) + m_\pi^2 \bar{\Delta}'(0)$$

- And derivate 2 times:

$$\Longrightarrow \quad \lambda_0'' f_+(0) = -\frac{16 m_\pi^4}{F_\pi^4} C_{12}^r + m_\pi^4 \bar{\Delta}''(0) \qquad (1)$$

- Combine with the two loop result for $f_+(0)$

$$\Longrightarrow \quad f_+(0) = 1 + \Delta(0) - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\Delta_{K\pi}^2 \qquad (2)$$

- From (1)+(2) $\Longrightarrow$ $2C_{12}^r + C_{34}^r$ $\Longrightarrow$ $\boxed{f_+(0) = f\left(\frac{F_K}{F_\pi}, \lambda_0'\right)}$

or

$$\boxed{\lambda_0' = f\left(\frac{F_K}{F_\pi}, f_+(0)\right)}$$

- From DR

$$\lambda_0'' = \lambda_0'^2 - 2\frac{m_\pi^4}{\Delta_{K\pi}} G'(0)$$
$$= \lambda_0'^2 + \left(4.16 \pm 0.50\right) \times 10^{-4}$$

Results

- We will present trends and not exact results: use of $\Delta(0)$ and $\overline{\Delta}(t)$ from **[Bijnens &Talavera]** determined with F_K/F_π =1.22 and F_π =92.4 MeV

  ⇒ Redo the fit varying F_K/F_π and F_π.

- We vary $\Delta(0)$ in its error bars, give the largest uncertainty.

- For instance, take the published most recent and precise value for F_K/F_π from lattice (N_f=2+1) ⇨ In the future use the FLAG average

See Talk by G. Colangelo

$$\boxed{\frac{F_K}{F_\pi} = 1.189 \pm 0.007}$$

[HPQCD-UKQCD'07]

	λ_0' 10^{-3}	$f_+(0)$	C_{12} 10^{-6}	C_{34} 10^{-6}	Δ_{CT} 10^{-2}
KLOE	14.0 ± 2.1	0.9700(218)	0.463(537)	3.387(4.226)	0.028(1.011)
KTeV	12.95 ± 1.04	0.9803(127)	0.720(251)	1.323(2.233)	-0.180(933)
NA48	8.88 ± 1.24	1.0212(149)	1.523(200)	-6.634(2.586)	-0.963(905)

- Uncertainties from $\Delta(0)$, F_K/F_π and λ_0'
- Uncertainties on $f_+(0)$ between 1.5% and 2%, not competitive with the most recent lattice result (uncertainties of ~ 0.5%)
- Limiting uncertainty from λ_0', average of the dispersive results ?
- Uncertainties on $\Delta(0)$ and $\bar{\Delta}(t)$ should decrease with new fits.

⇨ Promising

2. V_{us} and the CKM unitarity test using K_{l3} and K_{l2} decays (Flavianet Kaon WG)

- From K_{l3} decays

$$\Gamma_{K^{+/0}l3} = \frac{Br_{K^{+/0}l3}}{\tau_{K^{+/0}}} = \frac{C_K^2 G_F^2 m_{K^{+/0}}^5}{192\pi^3} S_{EW} \left(1 + 2\Delta_{EM}^{K^{+/0}l} + 2\Delta_{SU(2)}^{K^{+/0}l}\right) \left| f_+^{K^{+/0}}(0) V_{us} \right|^2 I_{K^{+/0}}^l$$

$f_+(0) = 0.964(5)$ [RBC-UKQCD'07]

$$\boxed{f_+(0)\left|V_{us}\right| = 0.21660(47)} \longrightarrow \boxed{\left|V_{us}\right| = 0.2246(12)}$$

- From $K_{l2}/\,\pi_{l2}$ decays [Marciano'04]

$$\frac{\Gamma_{K^\pm_{\mu 2}(\gamma)}}{\Gamma_{\pi^\pm_{\mu 2}(\gamma)}} = \frac{M_K \left(1 - m_\mu^2 / M_K^2\right)^2}{M_\pi \left(1 - m_\mu^2 / M_\pi^2\right)^2} \frac{\left|V_{us} F_K\right|^2}{\left|V_{ud} F_\pi\right|^2} \left(1 + \delta_{em}\right) \Rightarrow \boxed{\frac{\left|V_{us} F_K\right|}{\left|V_{ud} F_\pi\right|} = 0.2760(6)}$$

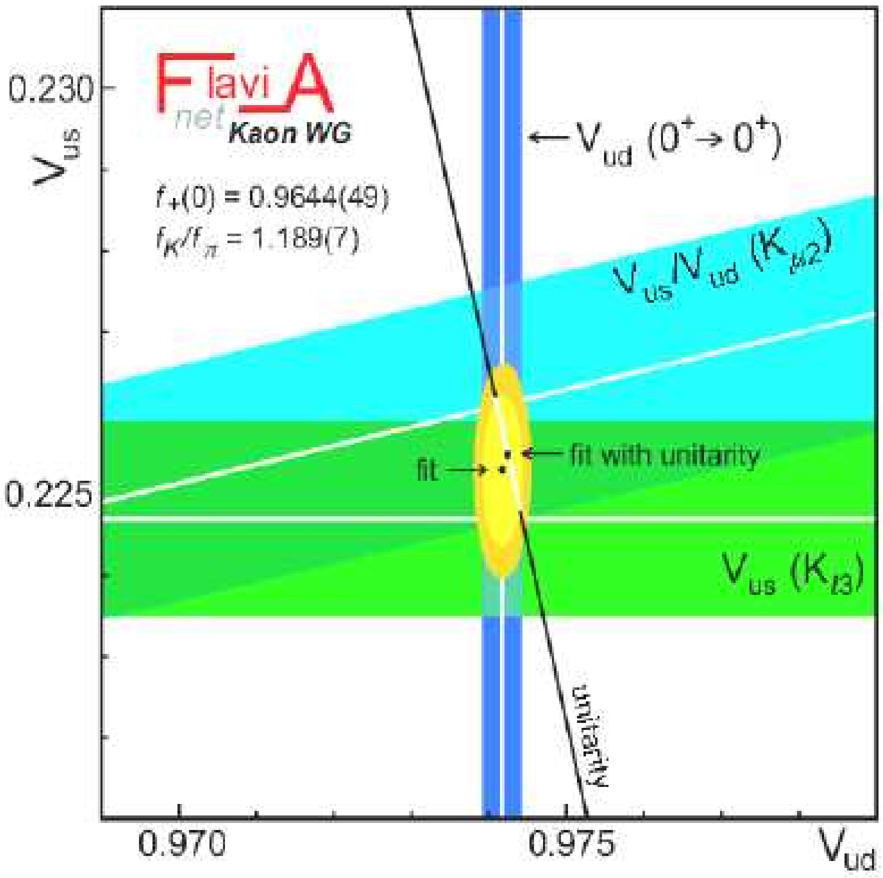
$\frac{F_K}{F_\pi} = 1.189(7)$ [HPQCD-UKQCD'07]

$$\boxed{\frac{V_{us}}{V_{ud}} = 0.2319(15)}$$

- Test of the CKM unitarity $\boxed{\left|V_{ud}\right|^2 + \left|V_{us}\right|^2 + \left|V_{ub}\right|^2 \stackrel{?}{=} 1}$

$0^+ \rightarrow 0^+$ β decays | K_{l3} decays | Negligible (B decays)

- Put everything in a fit with $\boxed{|V_{ud}| = 0.97424(22)}$ [Towner & Hardy'09]



V_{ud} = **0.97424(22)**

V_{us} = **0.2252(9)**

χ^2/ndf = 0.52/1 (47%)

Unitarity test :

$$\boxed{1 - |V_{us}|^2 - |V_{ud}|^2 = 0.00003(60)}$$

Incredible precision !

Stringent test for New Physics models

Implications of CKM unitarity on New Physics models [Marciano, Kaon'07]

- CKM unitarity can be converted in a test of the universality of the gauge coupling G_F $G_F^{CKM} = 1.16626(30)\times 10^{-5}\ \text{GeV}^2$ [Flavianet Kaon WG average]
- More precise determination after μ decays $G_\mu = 1.166371(6)\times 10^{-5}\ \text{GeV}^2$ [Mulan'07]

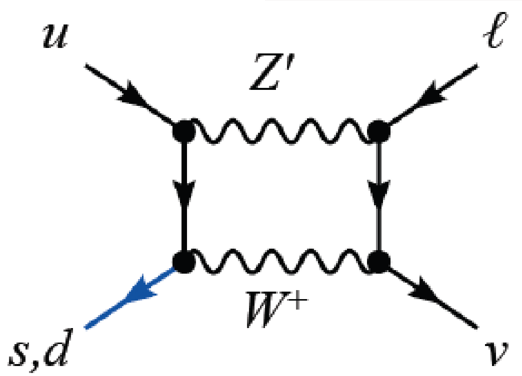
- A lot of NP effects absorbed in G_μ (Top bottom loop, Higgs loop, W*, WZ', box, SUSY loops, Technicolor, exotic μ decays)

  A comparison of G_μ with other measurements allows to constrain new physics effects

  ⇨ No sign of SUSY in CKM, no sign of technicolor, constraint on Z' boson mass from SO(10) GUT :

$$\boxed{G_\mu = G_{CKM}\left[1 - 0.007 Q_{el}\left(Q_{\mu l} - Q_{dl}\right)\frac{2\ln\left(M_{Z'}/M_W\right)}{M_{Z'}^2/M_W^2 - 1}\right]}$$

$$Q_{el} = Q_{\mu l} = -3Q_{dl} = 1 \Rightarrow \boxed{M_{Z'} \geq 700\ \text{GeV}}$$



⇨ Competitive with direct searches

3. Test of lepton μ/e universality in K_{l3} decays

$$\left(\frac{G_\mu}{G_e}\right)^2 = \frac{\Gamma_{K^{+/0}\mu 3}}{\Gamma_{K^{+/0}e 3}} \frac{I^{\mu}_{K^{+/0}}}{I^{e}_{K^{+/0}}} \left[\frac{1+2\Delta^{EM}_{K^{+/0}\mu}}{1+2\Delta^{EM}_{K^{+/0}e}}\right]$$

1 in the SM

Exp inputs from Flavianet

Theoretical inputs

- For an average of K^L and K^+ results (see **Flavianet Kaon WG review**)

$$\boxed{r_{\mu e} = \left(\frac{G_\mu}{G_e}\right) = 1.008 \pm 0.005}$$

($r_{\mu e} = 1.002 \pm 0.005$ without NA48 $K_{\mu 3}$ result)

- Result in good agreement with lepton universality.
- With 0.5% precision,test competitive with τ, almost with π decay analyses
  - π → lν $\quad r_{\mu e} = 1.0042 \pm 0.0033$ [Ramsey-Muslof, Su, Tulin'07]
  - τ → llν $\quad r_{\mu e} = 1.000 \pm 0.004$ [Davier, Hoecker, Zhang'06]

4. Test of the SM EW couplings via the CT theorem and the K_{l2} & K_{l3} decays measurements

- Callan-Treiman Theorem: SU(2) x SU(2) theorem

$$\boxed{C = \overline{f_0}(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+^{K^0}(0)} + \Delta_{CT}}$$

$$\Delta_{K\pi} = m_K^2 - m_\pi^2 \qquad \Delta_{CT} \sim \mathcal{O}\left(m_{u,d}/4\pi F_\pi\right)$$

$$\boxed{\Delta_{CT}^{NLO} \sim -3.5.10^{-3}}$$

[Gasser & Leutwyler'82]

⇨ Test of the Standard Model EW couplings :

$$C_{SM} = \overline{f}_0(\Delta_{K\pi}) = \underbrace{\frac{F_K |V_{us}|}{F_\pi |V_{ud}|} \frac{1}{f_+(0)|V_{us}|} |V_{ud}|}_{B_{exp}} + \Delta_{CT}$$

$$\Delta_{CT} = (-3.5 \pm 8).10^{-3}$$

Higher order terms $\mathcal{O}\left(m_{u,d} . m_s\right)$

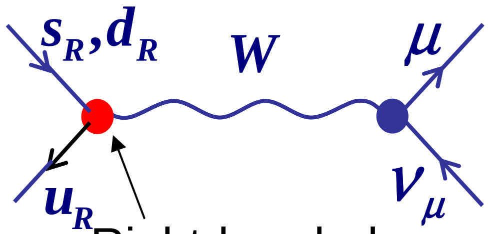
C is predicted in the Standard Model using the measured Brs:

$Br(K_{l2}/\pi_{l2})$, $\Gamma(K_{e3})$ and $|V_{ud}|$. ($|V_{us}|$ not needed in this prediction.)

⇨ $\boxed{B_{exp} = 1.2446 \pm 0.0041}$ and $C_{SM} = 1.2411(90)$

$$C=\bar{f}_0(\Delta_{K\pi})=\underbrace{\frac{F_K\left|V^{us}\right|}{F_\pi\left|V^{ud}\right|}\,\frac{1}{f_+(0)\left|V^{us}\right|}\,\left|V^{ud}\right|}_{\boxed{B_{\exp}=1.2446\pm0.0041}}\,r+\Delta_{CT}$$

- In the Standard Model : $r=1$
- In presence of new physics, new couplings : $r\neq1$
  - Right handed quark currents appearing at NLO of an EW low energy effective theory as a signature of exchange of new particules (W_R,…) at high energy. [Bernard, Oertel, E.P., Stern'06]



Right-handed coupling

$$\boxed{r=1-2\varepsilon\left(\mathrm{Re}\left(\frac{V_R^{us}}{V_L^{us}}\right)-\mathrm{Re}\left(\frac{V_R^{ud}}{V_L^{ud}}\right)\right)}$$

V_L, V_R mixing matrices

ε ~1% parameter of the model

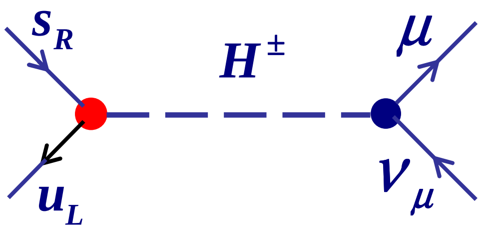
Effects expected on the % level, can reached several % depending on V_R (V_L ~ V_{CKM})

Right-handed currents also in Extra-Dimension scenario, L-R symmetric models ⇨ similar effects, test the coupling of W_R with fermions

$$C = \bar{f}_0(\Delta_{K\pi}) = \underbrace{\frac{F_K |V_{us}|}{F_\pi |V_{ud}|} \frac{1}{f_+(0)|V_{us}|} |V_{ud}|} \, r + \Delta_{CT}$$

$$\boxed{B_{\exp} = 1.2446 \pm 0.0041}$$

- In the Standard Model : $r = 1$
- In presence of new physics, new couplings : $r \neq 1$
  - Scalar couplings, exchange of a charged Higgs H± in two Higgs doublet models (MSSM +large tanβ …) [Hou’92, Isidori & Paradisi’06]



+ loop effects

$$\boxed{r = 1 - \frac{M_{K^+}^2}{M_{H^+}^2}\left(1 - \frac{m_d}{m_s}\right)\frac{\tan^2\beta}{1+\varepsilon_0 \tan\beta}}$$

$\tan\beta = \frac{v_u}{v_d}$ Ratio of the two Higgs vevs

Loop function

Effects expected of several 0.1% up to 1%,

Ex: $\varepsilon_0 = 10^{-2}$, $M_{H^\pm}^2 = 400$ GeV and $\tan\beta = 40$ ⇨ $r = 0.2\%$

N.B: Modify the extraction of the FFs.

[Flavianet Kaon WG’ 08]

Scalar FF results: Test of the SM EW couplings

Experiment Ke3+Kμ3	lnC	r
KTeV+BOPS'09	0.192(12)	1.022(15)
KLOE'08	0.204(25)	1.008(25)
NA48'07 ($K_{\mu 3}$ only)	0.144(14)	1.072(16)
SM	**0.2160(73)**	**1**

- KLOE and KTeV in agreement and in agreement with the SM. NA48 4.5σ away !
- A deviation from the SM prediction can be explained by New Physics (new couplings) in different scenarios

  Ex: NA48 result, 4.5% effect ⇨ Indication of an inverted hierarchy for V_R but hard to explain with scalar couplings, effects expected on permille level.

  but also by the existence of a complex zero and its complex conjuagate for $f_0(t)$ (not very probable) [Bernard, Oertel, E.P., Stern'09]
- To confirm this result NA48 K^+ analysis underway

- Other low energy theorem that allows to test for physics beyond the Standard Model and to constrain the scalar FF

  ⇨ The soft-kaon analog to the CT theorem **[Bernard, Oertel, E.P.,Stern'09]**

$$\boxed{\overline{f_0}(\Delta_{\pi K}) = \frac{F_\pi}{F_K f_+^{K^0}(0)} + \tilde{\Delta}_{CT}}$$

[Oehme'77]

$$\Delta_{\pi K} = m_\pi^2 - m_K^2$$

Less precise, indeed $\tilde{\Delta}_{CT} = 0.03$ in the isospin limit is an SU(3) correction but rather small for a first order SU(3) x SU(3) breaking effect

[Gasser&Leutwyler'85]

$$C = \overline{f}_0(\Delta_{\pi K}) = \underbrace{\frac{\hat{F}_\pi}{\hat{F}_K \hat{f}_+^{K^0}(0)}}_{\boxed{0.8752 \pm 0.0020}} r' + \tilde{\Delta}_{CT}, \qquad \boxed{-0.035 < \tilde{\Delta}_{CT} < 0.11}$$

1 in the SM

⇨ Provide an other interesting test of NP effects knowing the scalar FF from lattice QCD (t<0) but $\tilde{\Delta}_{CT}$ has to be better known

- If there is physics beyond the SM via a modification of the couplings, the values of F_K/F_π, $f_+(0)$…extracted from semileptonic, leptonic decays will change compared to their determination assuming the SM couplings. **[Bernard, E.P.,'08]**

5. Lepton Flavour Universality Tests via R_K

- See Talks by E. Goudzovski, B. Sciascia this morning and by P. Paradisi on Friday
- $\boxed{R_K = \frac{\Gamma(K^+ \to e^+\nu)}{\Gamma(K^+ \to \mu^+\nu)}}$ sensitive to LFU breaking
- In the SM, ratio very precisely determined with a 0.04% precision, cancellation of hadronic uncertainties ⇨ $\boxed{R_K = 2.477(1)\times 10^{-5}}$
  - First systematic calculation at $\mathcal{O}(e^2p^4)$ [Cirigliano & Rosell'07]
  - Only diagrams with photon connected to lepton lines contribute to the ratio
  - Relevant counterterms determined by matching with large N_c QCD
  - Inclusion of real photon corrections
  - Summation of leading logs

⇨ Improves the previous calculation $R_K = 2.472(1)\times 10^{-5}$ [Finkemeir]

Discrepancy !

5. Lepton Flavour Universality Tests via R_K

- $\boxed{R_K = \dfrac{\Gamma\left(K^+ \to e^+ \nu\right)}{\Gamma\left(K^+ \to \mu^+ \nu\right)}}$ sensitive to LFU breaking

- In the SM, ratio very precisely determined with a 0.04% precision, cancellation of hadronic uncertainties ⇨ $\boxed{R_K = 2.477(1) \times 10^{-5}}$

[Cirigliano & Rosell'07]

- Sizeable contribution of LFV terms (% level) in a SUSY scenario with a two Higgs doublet + large tanβ in the slepton sector **[Masiero, Paradisi, Petronzio'06, '08]**

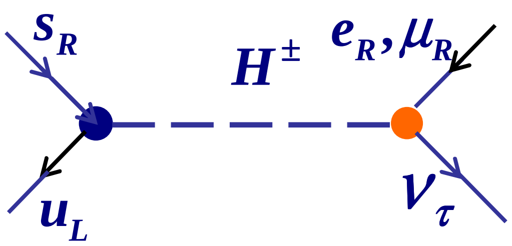
⇨ $$\boxed{R_K = \frac{\Gamma_{SM}\left(K^+ \to e^+ \nu_e\right) + \Gamma\left(K^+ \to e^+ \nu_\tau\right)}{\Gamma_{SM}\left(K^+ \to \mu^+ \nu\right)} \simeq R_K^{SM}\left[1 + \underbrace{\left(\frac{M_K^4}{M_H^4}\right)\left(\frac{m_\tau^2}{m_e^2}\right)\left|\Delta_R^{31}\right|^2 \tan^6\beta}\right]}$$

0.013 for $M^2_{H^\pm} = 500$ GeV

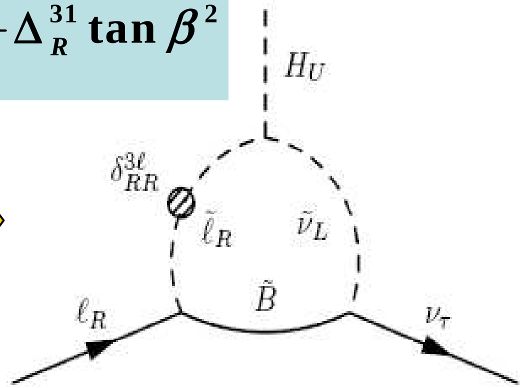
$\tan\beta = 40$

$\Delta_R^{31} = 5 \cdot 10^{-4}$

$$lH^\pm \tau \to \frac{g_2}{\sqrt{2}} \frac{m_\tau}{M_W} \Delta_R^{31} \tan\beta^2$$



⇨



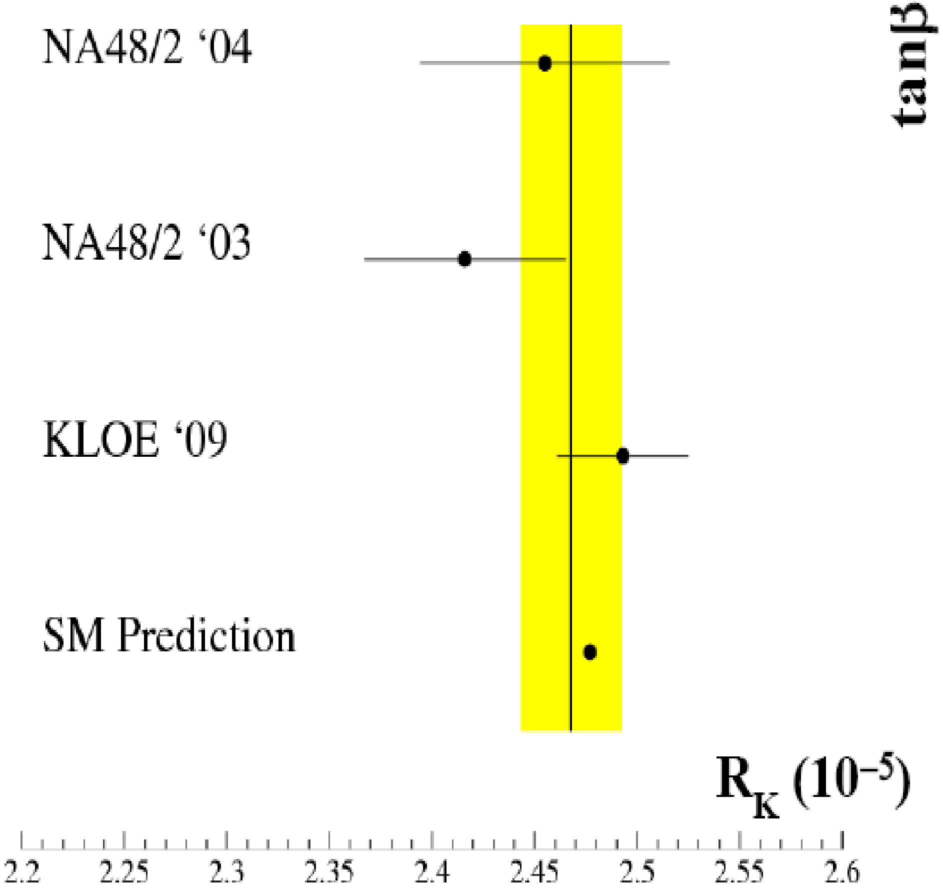
$$\Delta_R^{31} \sim \frac{\alpha_2}{4\pi} \delta_R^{31}$$

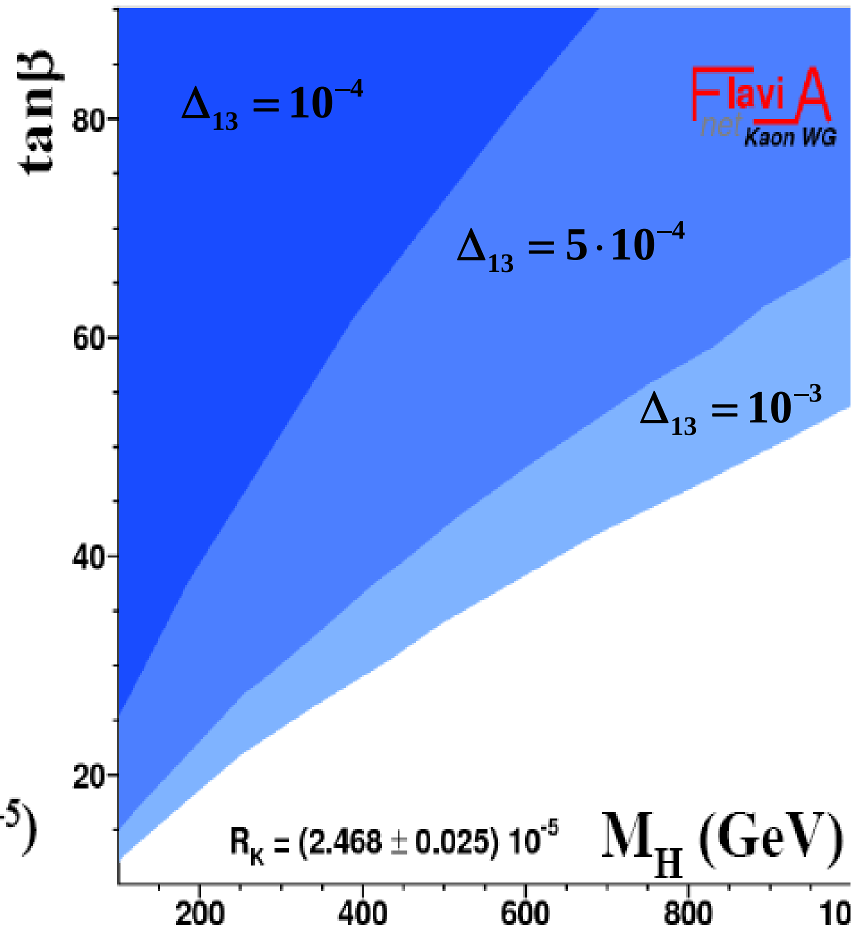
slepton flavour mixing angle

5. Lepton Flavour Universality Tests via R_K

[T. Spadaro, Moriond March'09]

World Average $R_K = 2.468(25) \cdot 10^{-5}$





- Very impresive experimental improvements since Kaon'07, see next talks
  Can still be improved to reach the SM precision ! ⇨ NA62

Conclusion and outlook

- The charged current analyses using K_{l3} and K_{l2} data have entered an era of very high precision
  - Improvements on the theoretical side: EM, isospin breaking corrections, dedicated dispersive parametrizations to analyse the FFs with the best precision.
  - On the experimental side, very precise data on K_{l3} and K_{l2} decays

    ⇨ Flavianet Kaon WG

- This allows for very precise tests of the SM (test of unitarity of the 1rst line of CKM matrix, universality, quark mass ratios…) and New Physics scenarios (Charged right-handed currents, scalar couplings, Lepton flavour violation…)

- But still on the experimental side, need K+ measurements (FFs..). Experimental puzzle on $f_0(t)$ (NA48 doesn't agree with the other experiments).

- On theoretical side, $f_+(0)$ determination should be improved

  ⇨ disagreement between analytical and lattice determinations. Lattice improvements are promising.

Additional slides

- Requirements in the measurements of the form factor shapes from the K_{l3} data
  - Try to measure the form factor shapes from the data with the best accuracy for determination of the lKs.
  - Measurement of $\bar{f}_0(\Delta_{K\pi}) \equiv C$ to test the Standard Model via the CT theorem

$$\boxed{C = \bar{f}_0(\Delta_{K\pi}) = \frac{F_K}{F_\pi f_+^{K^0}(0)} + \Delta_{CT}} \Rightarrow C_{SM} = \bar{f}_0(\Delta_{K\pi}) = \underbrace{\frac{F_K |\mathcal{V}^{us}|}{F_\pi |\mathcal{V}^{ud}|} \frac{1}{f_+(0)|\mathcal{V}^{us}|} |\mathcal{V}^{ud}|}_{B_{exp}} + \Delta_{CT}$$

$\Delta_{K\pi} = m_K^2 - m_\pi^2$

$\Delta_{CT} \sim \mathcal{O}(m_{u,d}/4\pi F_\pi)$

$\Delta_{CT} = (-3.5 \pm 8).10^{-3}$

measured very precisely assuming the SM EW couplings from $Br(K_{l2}/\pi_{l2})$, $\Gamma(Ke3)$ and $|\mathcal{V}_{ud}|$ $\Rightarrow$ $\boxed{B_{exp} = 1.2446 \pm 0.0041}$

$\ln C_{SM} = 0.2188(35) + \Delta_{CT}$

  - Relation which tests the Standard Model very accurately for K^0.
    If physics beyond the SM: ~1% difference between C and B_{exp}. Uncertainties from Δ_{CT} and B_{exp} on the permile level $\Rightarrow$ opportunity to see a possible effect.
  - The slope and curvature of the scalar form factor are of special interest to allow for a matching with the 2 loop ChPT calculations.

Computation of K_{l3} form factors at 2 loops in the isospin limit [Bijnens&Talavera'03]

- The scalar form factor at two loops in the isospin limit

$$f_S(t) = f_+(0) + \bar{\Delta}(t) + \frac{F_K/F_\pi - 1}{\Delta_{K\pi}}\, t + \frac{8}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right) t - \frac{8}{F_\pi^4} t^2 C_{12}^r$$

- The vector form factor $f_+(0)$ at 2 loops in the isospin limit is expressed as

$$f_+(0) = 1 + \Delta(0) - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\Delta_{K\pi}^2$$

- In these expressions, no dependence on the L_i at p^4, only via p^6 contribution. Only 2 LECs C_{12} and C_{34} which can be determined by the measurement of the slope and the curvature of the scalar form factor.

- $\bar{\Delta}(t)$ and $\Delta(0)$: contributions from loops: → F_π, the LECs L_i ($L_5 \Leftrightarrow F_K/F_\pi$) can be calculated at $\mathcal{O}(p^6)$ with the knowledge of the L_i at $\mathcal{O}(p^4)$ in the physical region.

Computation of K_{l3} form factors at 2 loops in the isospin limit [Bijnens&Talavera'03]

- The scalar form factor at two loops in the isospin limit

$$f_S(t) = f_+(0) + \overline{\Delta}(t) + \frac{F_K/F_\pi - 1}{\Delta_{K\pi}} t + \frac{8}{F_\pi^4}\left(2C_{12}^r + C_{34}^r\right)\left(m_K^2 + m_\pi^2\right) t - \frac{8}{F_\pi^4} t^2 C_{12}^r$$

- The vector form factor $f_+(0)$ at 2 loops in the isospin limit is expressed as

$$f_+(0) = 1 + \Delta(0) - \frac{8}{F_\pi^4}\left(C_{12}^r + C_{34}^r\right)\Delta_{K\pi}^2$$

- $\overline{\Delta}(t) = -0.25763t + 0.833045t^2 + 1.25252t^3 \quad \left[K_{l3}^0\right]$

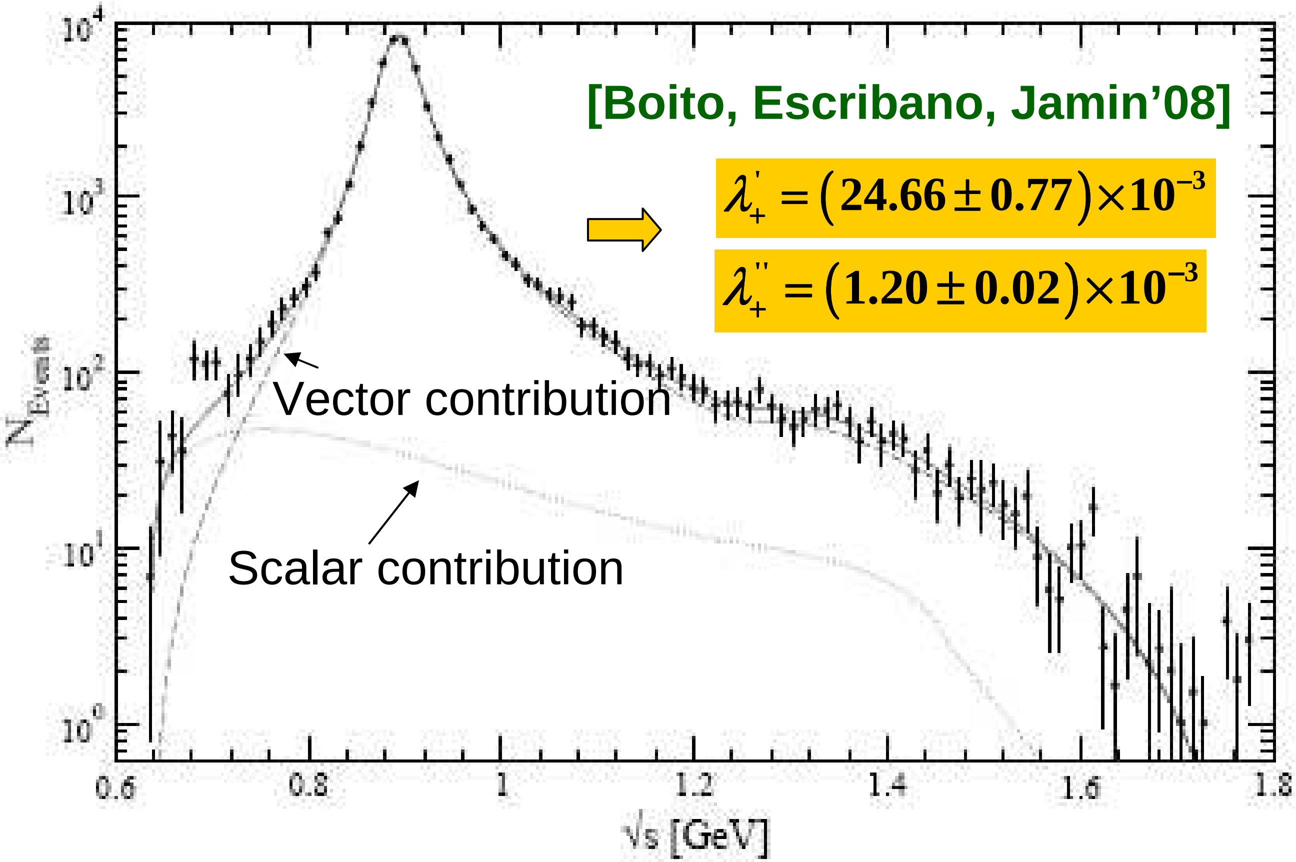
$$\Delta(0) = -0.0080 \pm 0.0057\left[loops\right] \pm 0.0028\left[L_i^r\right]$$

⇨ To be updated with the new experimental inputs (K_{l4})

Extraction of the vector form factor from K_{l3} which can be tested from tau decays

- Tau decay width

$$\frac{d\Gamma_{K\pi}(t)}{d\sqrt{t}} = \frac{|V_{us}f_+(0)|^2 G_F^2 M_\tau^3}{128\pi^3} S_{EW} q_{K\pi}(t) \left(1-\frac{t}{M_\tau^2}\right)^2 \times \left[\left(1+\frac{2t}{M_\tau^2}\right)\frac{4q_{K\pi}^2(t)}{t}|\bar{f}_+(t)|^2 + \frac{3(M_K^2-M_\pi^2)^2}{t^2}|\bar{f}_0(t)|^2\right],$$



[Boito, Escribano, Jamin'08]

$$\lambda_+' = (24.66 \pm 0.77) \times 10^{-3}$$

$$\lambda_+'' = (1.20 \pm 0.02) \times 10^{-3}$$

In good agreement with the K_{l3} analyses

Possibility of a combined Tau & K_{l3} analysis

In progress !

Experiment Ke3+Kμ3	ln C
KTeV+BOPS Prel.	0.192(12)
KLOE'08	0.204(25)
NA48'07 ($K_{\mu 3}$ only)	0.144(14)

- To be compared with

$$\ln C_{SM} = 0.2160(35)(64)$$

KLOE and KTeV in agreement and in agreement with the SM. NA48 4.5σ away !

- A deviation from the SM prediction can be explained :
  - Test of RHCs appearing at NLO of an EW low energy effective theory as a signature of exchange of new particules (W_R,…) at high energy. [Bernard, Oertel, E.P., Stern'06]
  - Presence scalar couplings (charged Higgs) : [Hou]
    MFV + large tanβ **:** hard to explain a 4.5σ effect (~several% level) [Isidori, Paradisi'06]
  - Existence of a complex zero and its complex conjugate for the form factor [Bernard, Oertel, E.P., Stern, work in progress]

3.5 Matching in presence of RHCs

- Change in the values of F_K/F_π and $f_+(0)$ compared to the SM, apparition of V_L and V_R ⟹ $\mathcal{V}_{\text{eff}}$ and $\mathcal{A}_{\text{eff}}$

$$\boxed{\left(\frac{F_K}{F_\pi}\right)^2 = \left(\frac{\widehat{F}_K}{\widehat{F}_\pi}\right)^2 \frac{1+2\left(\varepsilon_S-\varepsilon_{NS}\right)}{1+\frac{2}{\sin^2\hat{\theta}}\left(\delta+\varepsilon_{NS}\right)}}$$

and

$$\boxed{\left[f_+^{K^0\pi^-}(0)\right]^2 = \left[\widehat{f}_+^{K^0\pi^-}(0)\right]^2 \frac{1-2\left(\varepsilon_S-\varepsilon_{NS}\right)}{1+\frac{2}{\sin^2\hat{\theta}}\left(\delta+\varepsilon_{NS}\right)}}$$

with $(\delta+\varepsilon_{NS})$ and $(\varepsilon_S - \varepsilon_{NS})$, combination of new physics parameters.

- Use experimental knowledge of λ_0 and $\Delta\varepsilon$ obtained from dispersive fits to determine F_K/F_π, $f_+(0)$, C_{12}, C_{34}, Δ_{CT}

$$\ln C = 0.2188(35) + \underbrace{2\left(\varepsilon_S-\varepsilon_{NS}\right)+\Delta_{CT}/B_{\exp}}_{\Delta\varepsilon}$$

⟹ KLOE compatible with lattice results + no RHCs

NA48, RHCs + small F_K/F_π ($F_K/F_\pi \sim 1.15$)

4. Conclusion and outlook

- Dispersive parametrization very useful to analyse $K^L_{\mu 3}$ decays: parametrization physically motivated which allows with one parameter to determine the shape of the form factor, quite robust
  - Allows for a test of the SM electroweak couplings via the CT theorem
  - Allows for a matching with the 2 loop ChPT calculation

- Experimental results from dispersive analysis: KLOE and KTeV agree with the SM and NA48 at 4.5σ $\Longrightarrow$ results for K^+

- Matching the K_{l3} two loop computation + experimental results using dispersive representation offer the opportunity to determine $f_+(0)$, C_{12}, C_{34}, Δ_{CT} as a function of F_K/F_π

- Uncertainties too large at the moment to extract these quantities, need of
  - more precise and consistent fits
  - more precise lattice determinations
  - more precise scalar form factor measurements